Exercise 6 - Monotonicity

Find the intervals of monotony and local extremes of a function $f(x) = \dfrac{x^2 - 3x + 2}{(x+1)^2}$.

$\left[\begin{array}{l} \text{increasing in the intervals } (-\infty, -1) \text{ and } \left(\frac{7}{5}, +\infty\right); \text{ decreasing in the interval } \left(-1, \frac{7}{5}\right); \\ \text{local minimum } -\frac{1}{24} \text{ in a point } x = \frac{7}{5}. \end{array}\right]$

Find the intervals of monotony and local extremes of a function $f(x) = \dfrac{x^2 + 4x + 3}{(x-2)^2}$.

$\left[\begin{array}{l} \text{decreasing in the intervals } \left(-\infty, -\frac{7}{4}\right) \text{ and } \left(2, +\infty\right); \text{ increasing in the interval } \left(-\frac{7}{4}, 2\right); \\ \text{local minimum } -\frac{1}{15} \text{ in a point } x = -\frac{7}{4}. \end{array}\right]$

Find the intervals of monotony and local extremes of a function $f(x) = \sqrt{x - x^2}$.

$\left[\begin{array}{l} \text{increasing in the interval } \left\langle 0, \frac{1}{2}\right); \text{ decreasing in the interval } \left(\frac{1}{2}, 1\right\rangle; \\ \text{local maximum } \frac{1}{2} \text{ in a point } x = \frac{1}{2}. \end{array}\right]$

Find the intervals of monotony and local extremes of a function $f(x) = x\sqrt[3]{1-x}$.

$\left[\begin{array}{l} \text{increasing in the interval } \left(-\infty, \frac{3}{4}\right); \text{ decreasing in the interval } \left(\frac{3}{4}, +\infty\right); \\ \text{local maximum } \frac{3}{8}\sqrt[3]{2} \text{ in a point } x = \frac{3}{4}. \end{array}\right]$

Find the intervals of monotony and local extremes of a function $f(x) = \sqrt{x}\,\ln x$.

$\left[\begin{array}{l} \text{decreasing in the interval } \left(0, \mathrm{e}^{-2}\right); \text{ increasing in the interval } \left(\mathrm{e}^{-2}, +\infty\right); \\ \text{local minimum } -2\mathrm{e}^{-1} \text{ in a point } x = \mathrm{e}^{-2}. \end{array}\right]$

Find the intervals of monotony and local extremes of a function $f(x) = \dfrac{\ln^2 x}{x}$.

$\left[\begin{array}{l} \text{decreasing in the intervals } (0, 1) \text{ and } \left(\mathrm{e}^2, +\infty\right); \text{ increasing in the interval } \left(1, \mathrm{e}^2\right); \\ \text{local minimum 0 in a point } x = 1; \text{ local maximum } 4\mathrm{e}^{-2} \text{ in a point } x = \mathrm{e}^2. \end{array}\right]$

Find the intervals of monotony and local extremes of a function $f(x) = x\ln^2 x$.

$\left[\begin{array}{l} \text{increasing in the intervals } \left(0, \mathrm{e}^{-2}\right) \text{ a } (1, +\infty); \text{ decreasing in the interval } \left(\mathrm{e}^{-2}, 1\right); \\ \text{local maximum } 4\mathrm{e}^{-2} \text{ in a point } x = \mathrm{e}^{-2}; \text{ local minimum 0 in a point } x = 1. \end{array}\right]$

Find the intervals of monotony and local extremes of a function $f(x) = \sqrt[3]{1 - x^3}$.

$\left[\text{decreasing in } \mathbb{R}; \text{ does not have local extremes.}\right]$

Find the intervals of monotony and local extremes of a function $f(x) = \ln\dfrac{3-x}{|x+5|}$.

$\left[\text{increasing in the intervals } (-\infty, -5) \text{ and } (-5, 3); \text{ does not have local extremes.}\right]$

Find the intervals of monotony and local extremes of a function $f(x) = \operatorname{arctg} x - \ln\sqrt{1 + x^2}$.

$$\left[\begin{array}{l}\text{increasing in the interval } (-\infty,1)\text{; decreasing in the interval } (1,+\infty)\,; \\ \text{local maximum } \tfrac14\,\pi-\ln\sqrt{2} \text{ in a point } x=1\,.\end{array}\right]$$

Find the intervals of monotony and local extremes of a function $f(x)=x^3\mathrm{e}^{-x}$.

$$\left[\begin{array}{l}\text{increasing in the interval } (-\infty,3)\text{; decreasing in the interval } (3,+\infty)\,; \\ \text{local maximum } 27\mathrm{e}^{-3} \text{ in a point } x=3\,.\end{array}\right]$$

Find the intervals of monotony and local extremes of a function $f(x)=\dfrac{4}{x}+\dfrac{1}{1-x}$.

$$\left[\begin{array}{l}\text{decreasing in the intervals } (-\infty,0),\ \left(0,\tfrac23\right) \text{ a } (2,\infty)\text{; increasing in the intervals } \left(\tfrac23,1\right) \text{ a } (1,2)\,; \\ \text{local minimum 9 in a point } x=\tfrac23\text{; local maximum 1 in a point } x=2\,.\end{array}\right]$$

Find the intervals of monotony and local extremes of a function $f(x)=\ln x+\dfrac{1}{x^2}$.

$$\left[\begin{array}{l}\text{decreasing in the interval } \left(0,\sqrt{2}\right)\text{; increasing in the interval } \left(\sqrt{2},+\infty\right)\,; \\ \text{local minimum } \tfrac12\,(1+\ln 2) \text{ in a point } x=\sqrt{2}\,.\end{array}\right]$$

Find the intervals of monotony and local extremes of a function $f(x)=x\mathrm{e}^{-x^2+x}$.

$$\left[\begin{array}{l}\text{decreasing in the intervals } (-\infty,-1) \text{ a } \left(\tfrac12,+\infty\right)\text{; increasing in the interval } \left(-1,\tfrac12\right)\,; \\ \text{local minimum } -\mathrm{e}^{-2} \text{ in a point } x=-1\text{, local maximum } \tfrac12\,\sqrt[4]{\mathrm{e}} \text{ in a point } x=\tfrac12\,.\end{array}\right]$$

Find the intervals of monotony and local extremes of a function $f(x)=(x-3)^2\mathrm{e}^{|x|}$.

$$\left[\begin{array}{l}\text{decreasing in the intervals } (-\infty,0) \text{ and } (1,3)\text{; increasing in the intervals } (0,1) \text{ and } (3,+\infty)\,; \\ \text{local minimum 9 in a point } x=0\text{; local maximum } 4\mathrm{e} \text{ in a point } x=1\,; \\ \text{local minimum 0 in a point } x=3\,.\end{array}\right]$$

Find the intervals of monotony and local extremes of a function $f(x)=x\mathrm{e}^{-\sqrt{x}}$.

$$\left[\begin{array}{l}\text{increasing in the interval } \langle 0,4)\text{; decreasing in the interval } (4,+\infty)\,; \\ \text{local maximum } 4\mathrm{e}^{-2} \text{ in a point } x=4\ \ .\end{array}\right]$$

Find intervals in which the function $f(x)=\dfrac{x}{1+x^2}$ is convex and concave, respectively, and determine inflex points of this function.

$$\left[\begin{array}{l}\text{concave in the intervals } \left(-\infty,-\sqrt{3}\right) \text{ and } \left(0,\sqrt{3}\right)\,; \\ \text{convex in the intervals } \left(-\sqrt{3},0\right) \text{ and } \left(\sqrt{3},+\infty\right)\,; \\ \text{inflex points: } x=\pm\sqrt{3} \text{ and } x=0\,.\end{array}\right]$$

Find intervals in which the function $f(x)=\dfrac{x}{1-x^2}$ is convex and concave, respectively, and determine inflex points of this function.

$$\left[\begin{array}{l}\text{convex in the intervals } (-\infty,-1) \text{ a } (0,1)\,; \\ \text{concave in the intervals } (-1,0) \text{ a } (1,+\infty)\,; \\ \text{inflex point } x=0\,.\end{array}\right]$$

Find intervals in which the function $f(x) = \ln(1+x^3)$ is convex and concave, respectively, and determine inflex points of this function.

$$\left[\begin{array}{l}\text{concave in the intervals } (-1,0) \text{ a } \left(\sqrt[3]{2},+\infty\right)\text{; convex in the interval } \left(0,\sqrt[3]{2}\right)\text{;} \\ \text{inflex points: } x=0 \text{ and } x=\sqrt[3]{2}\,.\end{array}\right]$$

Find intervals in which the function $f(x) = x\sin(\ln x)$ is convex and concave, respectively, and determine inflex points of this function.

$$\left[\begin{array}{l}\text{concave in the intervals } \left(\mathrm{e}^{(1/4+2k)\pi},\mathrm{e}^{(5/4+2k)\pi}\right),\ k\in\mathbb{Z}\,; \\ \text{convex in the intervals } \left(\mathrm{e}^{(5/4+2k)\pi},\mathrm{e}^{(9/4+2k)\pi}\right),\ k\in\mathbb{Z}\,; \\ \text{inflex points } x=\mathrm{e}^{(1/4+k)\pi},\ k\in\mathbb{Z}\,.\end{array}\right]$$

Find intervals in which the function $f(x) = \dfrac{x}{\ln x}$ is convex and concave, respectively, and determine inflex points of this function.

$$\left[\begin{array}{l}\text{concave in the intervals } (0,1) \text{ and } \left(\mathrm{e}^2,+\infty\right)\text{; convex in the interval } \left(1,\mathrm{e}^2\right)\text{;} \\ \text{inflex point } x=\mathrm{e}^2\,.\end{array}\right]$$

Find intervals in which the function $f(x) = \mathrm{e}^{\sqrt[3]{x}}$ is convex and concave, respectively, and determine inflex points of this function.

$$\left[\begin{array}{l}\text{convex in the intervals } (-\infty,0) \text{ and } (8,+\infty)\text{; concave in the interval } (0,8)\text{;} \\ \text{inflex points: } x=0 \text{ and } x=8\,.\end{array}\right]$$

Find intervals in which the function $f(x) = \ln\dfrac{|x-1|}{x+3}$ is convex and concave, respectively, and determine inflex points of this function.

$$\left[\begin{array}{l}\text{convex in the interval } (-3,-1)\text{; concave in the intervals } (-1,1) \text{ and } (1,+\infty)\text{;} \\ \text{inflex point } x=-1\,.\end{array}\right]$$

Find intervals in which the function $f(x) = \dfrac{\ln x}{x}$ is convex and concave, respectively, and determine inflex points of this function.

$$\left[\begin{array}{l}\text{concave in the interval } \left(0,\mathrm{e}^{3/2}\right)\text{; convex in the interval } \left(\mathrm{e}^{3/2},+\infty\right)\text{;} \\ \text{inflex point } x=\mathrm{e}^{3/2}\,.\end{array}\right]$$

Find intervals in which the function $f(x) = \ln x + \dfrac{1}{x}$ is convex and concave, respectively, and determine inflex points of this function.

$$\left[\begin{array}{l}\text{convex in the interval } (0,2)\text{; concave in the interval } (2,+\infty)\text{;} \\ \text{inflex point } x=2\,.\end{array}\right]$$

Find intervals in which the function $f(x) = x\ln^2 x$ is convex and concave, respectively, and determine inflex points of this function.

$$\left[\begin{array}{l}\text{concave in the interval } \left(0, \mathrm{e}^{-1}\right); \text{ convex in the interval } \left(\mathrm{e}^{-1}, +\infty\right); \\ \text{inflex point } x = \mathrm{e}^{-1}.\end{array}\right]$$

Find the equation of a tangent to the graph of the function $f(x) = \ln(1 + x^2)$ in its inflex points.

$$\left[x - y - 1 + \ln 2 = 0 \text{ in a point } [1; \ln 2];\ x + y + 1 - \ln 2 = 0 \text{ in the point } [-1; \ln 2].\right]$$

Find the equations of tangents to the graph of the function $f(x) = \dfrac{\ln x}{x}$ in its inflex points.

$$\left[x + \mathrm{e}^3 y - 2\mathrm{e}^{3/2} = 0 \text{ in the point } \left[\mathrm{e}^{3/2}; \tfrac{3}{2}\,\mathrm{e}^{-3/2}\right].\right]$$

Find the equations of the tangent to the graph of the function $f(x) = \ln x + \dfrac{1}{x}$ in its inflex points.

$$\left[x - 4y + 4\ln 2 = 0 \text{ in the point } \left[2; \tfrac{1}{2} + \ln 2\right].\right]$$

Find the set of all $x \in \mathbb{R}$ such that the function $f(x) = 2x^2 + \ln|x|$ is increasing and concave at the same time. $\left[x \in \left(0, \tfrac{1}{2}\right).\right]$

Find the set of all $x \in \mathbb{R}$ such that the function $f(x) = \sqrt[3]{x^2 - 6x}$ is increasing and convex at the same time. $\left[x \in (3, 6).\right]$

Find the smallest and the greatest value of the function $f(x) = \dfrac{x^2 - 3x + 2}{(x + 1)^2}$ on the interval $\langle 0, 4\rangle$.

$$\left[\text{maximum } 2 \text{ for } x = 0;\ \text{minimum } -\tfrac{1}{24} \text{ for } x = \tfrac{7}{5}.\right]$$

Find the smallest and the greatest value of the function $f(x) = \dfrac{x^2 + 4x + 3}{(x - 2)^2}$ on the interval $\langle -4, 1\rangle$.

$$\left[\text{maximum } 8 \text{ for } x = 1;\ \text{minimum } -\tfrac{1}{15} \text{ for } x = -\tfrac{7}{4}.\right]$$

Find the smallest and the greatest value of the function $f(x) = (x-1)^2\mathrm{e}^{-|x|}$ on the interval $\langle -3, 2\rangle$.

$$\left[\text{maximum } 4\mathrm{e}^{-1} \text{ for } x = -1;\ \text{minimum } 0 \text{ for } x = 1.\right]$$

Find the smallest and the greatest value of the function $f(x) = (x + 1)^2\mathrm{e}^{|x-1|}$ on the interval $\langle -2, 3\rangle$.

$$\left[\text{maximum } 16\mathrm{e}^2 \text{ for } x = 3;\ \text{minimum } 0 \text{ for } x = -1.\right]$$

Find the smallest and the greatest value of the function $f(x) = \ln x + \dfrac{2}{x}$ on the interval $\langle 1, \mathrm{e}^2\rangle$.

$\left[\text{maximum } 2+2e^{-1} \text{ for } x=e^2\text{; minimum } 1+\ln 2 \text{ for } x=2\,.\right]$

Find the smallest and the greatest value of the function $f(x) = \operatorname{arccotg}|x^2-2x-8|$ on the interval $\langle -3, 2\rangle$. $\left[\text{maximum } \operatorname{arccotg} 0 = \frac{1}{2}\pi \text{ for } x=-2\text{; minimum } \operatorname{arctg} 9 \text{ for } x=1\,.\right]$

Find the smallest and the greatest value of the function $f(x) = \sqrt[3]{x^2-x^3}$ on the interval $\langle -1, 2\rangle$.

$\left[\text{maximum } \sqrt[3]{2} \text{ for } x=-1\text{, minimum } -\sqrt[3]{4} \text{ for } x=2\,.\right]$

Find the smallest and the greatest value of the function $f(x) = x - |\sin 2x|$ on the interval $\langle 0, \pi\rangle$.

$\left[\text{maximum } \pi \text{ for } x=\pi\text{, minimum } \frac{\pi}{6}-\frac{\sqrt{3}}{2} \text{ for } x=\frac{\pi}{6}\,.\right]$

Find the smallest and the greatest value of the function $f(x) = x + |\sin 2x|$ on the interval $\langle -\frac{1}{2}\pi, \frac{1}{2}\pi\rangle$.

$\left[\text{maximum } \frac{\pi}{3}+\frac{\sqrt{3}}{2} \text{ for } x=\frac{\pi}{3}\text{, minimum } -\frac{\pi}{2} \text{ for } x=-\frac{\pi}{2}\,.\right]$

Find the smallest and the greatest value of the function $f(x) = \left|e^{-x}\sin x\right|$ on the interval $\langle 0, 2\pi\rangle$.

$\left[\text{maximum } \frac{1}{\sqrt{2}}\,e^{-\pi/4} \text{ for } x=\frac{\pi}{4}\text{, minimum 0 in the points } x=0,\ x=\pi \text{ and } x=2\pi\,.\right]$

Find the smallest and the greatest value of the function $f(x) = \left|e^{-x}\cos x\right|$ on the interval $\langle -\pi, \pi\rangle$.

$\left[\text{maximum } e^{\pi} \text{ for } x=-\pi\text{, minimum 0 for } x=\pm\frac{\pi}{2}\,.\right]$

Find the smallest and the greatest value of the function $f(x) = x\ln^2 x$ on the interval $\left\langle e^{-3}, e\right\rangle$.

$\left[\text{maximum e for } x=e\text{; minimum 0 for } x=1\,.\right]$

Find the smallest and the greatest value of the function $f(x) = 2x + e^{-x}$ on the interval $\langle -2, 3\rangle$.

$\left[\text{maximum } 6+e^{-3} \text{ for } x=3\text{; minimum } 2(1-\ln 2) \text{ for } x=-\ln 2\,.\right]$

Find the smallest and the greatest value of the function $f(x) = \dfrac{\ln^2 x}{x}$ in the interval $\left\langle e, e^3\right\rangle$.

$\left[\text{maximum } 4e^{-2} \text{ for } x=e^2\text{; minimum } e^{-1} \text{ for } x=e\,.\right]$

Find the smallest and the greatest value of the function $f(x) = x\sin(\ln x)$ on the interval $\left\langle 1, e^{\pi}\right\rangle$.

$\left[\text{maximum } \frac{1}{\sqrt{2}}\, e^{3\pi/4} \text{ for } x = e^{3\pi/4}\text{; minimum 0 for } x = 1 \text{ and } x = e^{\pi}\,.\right]$

For which number x is the sume of it with its second power minimal? $\left[x = -\frac{1}{2}\,.\right]$

For which positive number x is the sume of it with its inverse value $\dfrac{1}{x}$ minimal?

$\left[x = 1\,.\right]$

For which positive number x is its difference with its second root minimal? $\left[x = \frac{1}{4}\,.\right]$

Determine the numbers x, $y \in \langle 0, 1\rangle$ such that $x+y = 1$ and the value of x^2y^3 is maximal.

$\left[x = \frac{2}{5} \text{ and } y = \frac{3}{5}\,.\right]$

Find a point on the hyperbole $\frac{1}{2}\,x^2 - y^2 = 1$ that is closest to the point $A = [3; 0]$.

$\left[\text{points } [2; 1] \text{ and } [2; -1]\,.\right]$

Which rectangle inscribed to the semicircle with the radius R has the greatest area?

$\left[\text{sides of the rectanle: } \sqrt{2}\,R \text{ and } \frac{1}{\sqrt{2}}\,R\,.\right]$

Find the block with the square base that has the given volume V and the smallest surface.

$\left[\text{cube with the edge } \sqrt[3]{V}\,.\right]$

Which cylinder with the given volume V has the smallest surface?

$\left[\text{radius of the base: } r = \sqrt[3]{\frac{V}{2\pi}},\ \text{height: } h = \sqrt[3]{\frac{4V}{\pi}} = 2r\,.\right]$

Which cylinder with the given surface S has the greatest volume?

$\left[\text{radius of the base: } r = \sqrt{\frac{S}{6\pi}},\ \text{height } h = \sqrt{\frac{2S}{3\pi}} = 2r\,.\right]$

Find the right-angled triangle with the greatest area in which the sum of the length of the hypothenuse and one cathete is equal to one. $\left[a = \frac{1}{3},\ b = \frac{1}{\sqrt{3}},\ c = \frac{2}{3}\,.\right]$
