Exercises 11 – Implicit Functions

Find the first two derivatives of the function $y(x)$ given as the solution of the equation

$$y = 2x \operatorname{arctg} \frac{y}{x}.$$

$$\left[y'(x) = \frac{y}{x}, \quad y''(x) = 0\right]$$

Find the first two derivatives of the function $y(x)$ given as the solution of the equation

$$\ln \sqrt{x^2 + y^2} = \operatorname{arctg} \frac{y}{x}.$$

$$\left[y'(x) = \frac{x+y}{x-y}, \quad y''(x) = \frac{2(x^2+y^2)}{(x-y)^3}\right]$$

Let function $y(x)$ be defined in the neighbourhood of the point $[1, 0]$ by the equation

$$\ln \sqrt{x^2 + y^2} = \operatorname{arctg} \frac{y}{x}.$$

Find its Taylor polynomial of the second degree with the centre in the point $x = 1$.

$$\left[T_2 = (x-1) + (x-1)^2\right]$$

Let function $y(x)$ be defined in the neighbourhood of the point $[+, -1]$ by the equation

$$x^2 + xy + y^2 = \cos(x + y).$$

Find its Taylor polynomial of the second degree with the centre in the point $x = 1$.

$$\left[T_2 = -1 + (x-1) + 5(x-1)^2\right]$$

Let function $y(x)$ be defined in the neighbourhood of the point $[0, 1]$ by the equation

$$x^2 + xy + y^2 - \ln \sqrt{x^2 + y^2} = 1.$$

Find its Taylor polynomial of the second degree with the centre in the point $x = 0$.

$$\left[T_2 = 1 - x - x^2\right]$$

Let $z = z(x, y)$ be a function defined in the neighbourhood of the point $[1, 1, 1]$ as the solution of the equation

$$z^3 + \operatorname{tg}\big((x-y)z\big) = x^2 + y^2 - 1.$$

Find its partial derivative $\dfrac{\partial^2 z}{\partial x \partial y}(1, 1)$.

$$\left[\frac{\partial^2 z}{\partial x \partial y} = -\frac{8}{9}, \quad z'_x = \tfrac{1}{3}, \quad z'_y = 1\right]$$

Let $z = z(x, y)$ be a function defined in the neighbourhood of the point $[1, -1, 1]$ as the solution of the equation

$$z^3 + xy + 1 = z\mathrm{e}^{x^2 - y^2}.$$

Find its partial derivative $\dfrac{\partial^2 z}{\partial x \partial y}(1,-1)$. $\left[\dfrac{\partial^2 z}{\partial x \partial y} = \dfrac{5}{4}\,, \quad z'_x = \tfrac{3}{2}\,, \quad z'_y = \tfrac{1}{2}\right]$

Let $z = z(x,y)$ be a function defined in the neighbourhood of the point $[1,-1,1]$ as the solution of the equation

$$z = 2 + xy + \operatorname{arctg} \frac{x+y}{z}\,.$$

Find its partial derivative $\dfrac{\partial^2 z}{\partial x \partial y}(1,-1)$. $\left[\dfrac{\partial^2 z}{\partial x \partial y} = -1\,, \quad z'_x = 0\,, \quad z'_y = 2\right]$

Let $z = z(x,y)$ be a function defined in the neighbourhood of the point $[0,1,1]$ as the solution of the equation

$$x^2 - y^2 + z^2 + 2xyz + z \ln \sqrt{x^2 + y^2} = 0\,.$$

Find its partial derivative $\dfrac{\partial^2 z}{\partial x \partial y}(0,1)$. $\left[\dfrac{\partial^2 z}{\partial x \partial y} = -\dfrac{1}{2}\,, \quad z'_x = -1\,, \quad z'_y = \tfrac{1}{2}\right]$

Let $z = z(x,y)$ be a function defined in the neighbourhood of the point $[1,1,1]$ as the solution of the equation

$$z^2 + xyz\mathrm{e}^{3-x^2-y^2-z^2} = x^2 + y^2\,.$$

Find its partial derivative $\dfrac{\partial^2 z}{\partial x \partial y}(1,1)$. $\left[\dfrac{\partial^2 z}{\partial x \partial y} = -7\,, \quad z'_x = z'_y = 3\right]$

Let $z = z(x,y)$ be a function defined in the neighbourhood of the point $[1,1,-1]$ as the solution of the equation

$$x^2 - y^2 + z^2 + 2\sqrt{x^2 + y^2 + z^2 + xz + yz} = 3\,.$$

Find its partial derivative $\dfrac{\partial^2 z}{\partial x \partial y}(1,1)$. $\left[\dfrac{\partial^2 z}{\partial x \partial y} = -\dfrac{45}{32}\,, \quad z'_x = \tfrac{5}{4}\,, \quad z'_y = -\tfrac{3}{4}\right]$

Let $z = z(x,y)$ be a function defined in the neighbourhood of the point $[1,-1,2]$ as the solution of the equation

$$x^2 + z^2 + xy + y\ln(z - x) = 4\,,$$

Find its partial derivative $\dfrac{\partial^2 z}{\partial x \partial y}(1,-1)$. $\left[\dfrac{\partial^2 z}{\partial x \partial y} = -\dfrac{1}{9}\,, \quad z'_x = -\tfrac{2}{3}\,, \quad z'_y = -\tfrac{1}{3}\right]$

Let $z = z(x,y)$ be a function defined in the neighbourhood of the point $[1,2,0]$ as the solution of the equation

$$\mathrm{e}^{z\cos(2x-y)} - xy + y^2 \cos z = 3\,.$$

Find its partial derivative $\dfrac{\partial^2 z}{\partial x \partial y}(1,2)$. $\left[\dfrac{\partial^2 z}{\partial x \partial y} = -17\,, \quad z'_x = 2\,, \quad z'_y = -3\right]$

Let $z = z(x,y)$ be a function defined in the neighbourhood of the point $[0,-1,1]$ as the solution of the equation

$$z(2x + y + z) + y\ln(xz + y^2) + y^2 z\mathrm{e}^{-x} = 1\,.$$

Find its partial derivative $\dfrac{\partial^2 z}{\partial x \partial y}(0,-1)$. $\left[\dfrac{\partial^2 z}{\partial x \partial y} = -\dfrac{1}{2}\,, \quad z'_x = 0\,, \quad z'_y = -\tfrac{1}{2}\right]$

Let $z = z(x, y)$ be a function defined in the neighbourhood of the point $[1, -2, 1]$ as the solution of the equation

$$y(x+y)\sin(x-z) + z^2(2x+y) = 0\,.$$

Find its partial derivative $\dfrac{\partial^2 z}{\partial x \partial y}(1,-2)$. $\left[\dfrac{\partial^2 z}{\partial x \partial y} = 5\,, \quad z'_x = 2\,, \quad z'_y = \tfrac{1}{2}\right]$

Let $z = z(x, y)$ be a function defined in the neighbourhood of the point $[2, -1, 1]$ as the solution of the equation

$$xy\ln(x-z) + \mathrm{e}^{x(y+z)} = 1\,.$$

Find its partial derivative $\dfrac{\partial^2 z}{\partial x \partial y}(2,-1)$. $\left[\dfrac{\partial^2 z}{\partial x \partial y} = -\dfrac{5}{8}\,, \quad z'_x = \tfrac{1}{2}\,, \quad z'_y = -\tfrac{1}{2}\right]$

Let $z = z(x, y)$ be a function defined in the neighbourhood of the point $[1, 1, -1]$ as the solution of the equation

$$(x^2+y^2)\ln\sqrt{1+y-z^2} + z(x+2y+3z) = 0\,.$$

Find its partial derivative $\dfrac{\partial^2 z}{\partial x \partial y}(1,1)$. $\left[\dfrac{\partial^2 z}{\partial x \partial y} = -4\,, \quad z'_x = z'_y = -1\right]$

Let $z = z(x, y)$ be a function defined in the neighbourhood of the point $[1, 1, 1]$ as the solution of the equation

$$x^2 + y^2 + xy - x = z^3 + yz\,.$$

Find its Taylor polynomial of the second degree with the centre in the point $[1, 1]$.

$$\left[T_2 = 1 + \tfrac{1}{2}(x-1) + \tfrac{1}{2}(y-1) + \tfrac{1}{16}(x-1)^2 - \tfrac{1}{4}(x-1)(y-1) - \tfrac{1}{16}(y-1)^2\right]$$

Let $z = z(x, y)$ be a function defined in the neighbourhood of the point $[1, 1, 1]$ as the solution of the equation

$$x^2 - y^2 = z^3 - xyz\,.$$

Find its Taylor polynomial of the second degree with the centre in the point $[1, 1]$.

$$\left[T_2 = 1 + \tfrac{3}{2}(x-1) - \tfrac{1}{2}(y-1) - \tfrac{17}{8}(x-1)^2 + \tfrac{13}{4}(x-1)(y-1) - \tfrac{9}{8}(y-1)^2\right]$$

Let $z = z(x, y)$ be a function defined in the neighbourhood of the point $[1, -1, 1]$ as the solution of the equation

$$z^3 = x^2 + y^2 - xz - yz + y\,.$$

Find its Taylor polynomial of the second degree with the centre in the point $[1, -1]$.

$$\left[T_2 = 1 + \tfrac{1}{3}(x-1) - \tfrac{2}{3}(y+1) + \tfrac{1}{9}(x-1)^2 + \tfrac{5}{9}(x-1)(y+1) + \tfrac{1}{9}(y+1)^2\right]$$

Let $z = z(x, y)$ be a function defined in the neighbourhood of the point $[1, -1, 1]$ as the solution of the equation

$$z^3 - zx^2 + zy^2 + xy = 0\,.$$

Find its Taylor polynomial of the second degree with the centre in the point $[1,-1]$.

$$\left[T_2 = 1 + (x-1) + \tfrac{1}{3}\,(y+1) - \tfrac{1}{9}\,(x-1)(y+1) - \tfrac{2}{9}\,(y+1)^2\right]$$

Let $z = z(x, y)$ be a function defined in the neighbourhood of the point $[0, -1, 1]$ as the solution of the equation

$$x^3 + y^3 + z^3 - 3xyz = 0\,.$$

Find its Taylor polynomial of the second degree with the centre in the point $[0,-1]$.

$$\left[T_2 = 1 - x - (y+1)\right]$$

Let $z = z(x, y)$ be a function defined in the neighbourhood of the point $[-1, -1, 1]$ as the solution of the equation

$$x^2 + y^2 = z^3 + xyz\,.$$

Find its Taylor polynomial of the second degree with the centre in the point $[-1,-1]$.

$$\left[T_2 = 1 - \tfrac{1}{4}\,(x+1) - \tfrac{1}{4}\,(y+1) + \tfrac{9}{64}\,(x+1)^2 - \tfrac{15}{32}\,(x+1)(y+1) + \tfrac{9}{64}\,(y+1)^2\right]$$

Let $z = z(x, y)$ be a function defined in the neighbourhood of the point $[1, 0, 1]$ as the solution of the equation

$$z^3 - 3xyz - x^2 + y^2 = 0\,.$$

Find its Taylor polynomial of the second degree with the centre in the point $[1, 0]$.

$$\left[T_2 = 1 + \tfrac{2}{3}\,(x-1) + y - \tfrac{1}{9}\,(x-1)^2 + \tfrac{1}{3}\,(x-1)y - \tfrac{1}{3}\,y^2\right]$$

Let $z = z(x, y)$ be a function defined in the neighbourhood of the point $[0, 1, 0]$ as the solution of the equation

$$x^3 + z^3 + y^2 + xy - xz - yz = 1\,.$$

Find its Taylor polynomial of the second degree with the centre in the point $[0, 1]$.

$$\left[T_2 = x + 2(y-1) - x^2 - 2x(y-1) - (y-1)^2\right]$$

Let $z = z(x, y)$ be a function defined in the neighbourhood of the point $[-1, 0, 1]$ as the solution of the equation

$$z^4 - 4yz - x^2 - y^2 - xy = 0\,.$$

Find its Taylor polynomial of the second degree with the centre in the point $[-1, 0]$.

$$\left[T_2 = 1 - \tfrac{1}{2}\,(x+1) + \tfrac{3}{4}\,y - \tfrac{1}{8}\,(x+1)^2 + \tfrac{7}{8}\,(x+1)y + \tfrac{5}{32}\,y^2\right]$$

Let $z = z(x, y)$ be a function defined in the neighbourhood of the point $[1, -1, -1]$ as the solution of the equation

$$yz^3 + x^2z + xy^2 - y^2 = 0\,.$$

Find its Taylor polynomial of the second degree with the centre in the point $[1,-1]$.

$$\left[T_2 = -1 + \tfrac{3}{2}\,(x-1) - \tfrac{1}{2}\,(y+1) + \tfrac{35}{8}\,(x-1)^2 - \tfrac{3}{2}\,(x-1)(y+1) - \tfrac{3}{8}\,(y+1)^2\right]$$

Let $z = z(x, y)$ be a function defined in the neighbourhood of the point $[1, 1, -1]$ as the solution of the equation

$$z^3 + xyz^2 + x^2 - y^2 = 0\,.$$

Find its Taylor polynomial of the second degree with the centre in the point $[1, 1]$.

$$\left[T_2 = -1 - 3(x-1) + (y-1) + 11(x-1)^2 + 17(x-1)(y-1) + (y-1)^2\right]$$

Let $z = z(x, y)$ be a function defined in the neighbourhood of the point $[1, 0, -1]$ as the solution of the equation

$$x^3 + y^3 + z^3 - 3xyz = 0\,.$$

Find its Taylor polynomial of the second degree with the centre in the point $[1, 0]$.

$$\left[T_2 = -1 - (x-1) - y\right]$$

Let $z = z(x, y)$ be a function defined in the neighbourhood of the point $[1, 1, 1]$ as the solution of the equation

$$z^3 - xyz + x^2 - y^2 = 0\,.$$

Find its Taylor polynomial of the second degree with the centre in the point $[1, 1]$.

$$\left[T_2 = 1 - \tfrac{1}{2}\,(x-1) + \tfrac{3}{2}\,(y-1) - \tfrac{1}{8}\,(x-1)^2 + \tfrac{13}{4}\,(x-1)(y-1) - \tfrac{17}{8}\,(y-1)^2\right]$$

Let $z = z(x, y)$ be a function defined in the neighbourhood of the point $[2, 1, 1]$ as the solution of the equation

$$x^3 - y^3 + z^3 - 3xyz = 2\,.$$

Find its first and second differential in the point $[2, 1]$.

$$\left[\mathrm{d}z = 3\,\mathrm{d}x - 3\,\mathrm{d}y\,, \quad \mathrm{d}^2z = 16\,\mathrm{d}x^2 - 44\,\mathrm{d}x\,\mathrm{d}y + 28\,\mathrm{d}y^2\,.\right]$$

Let $z = z(x, y)$ be a function defined in the neighbourhood of the point $[1, 1, 2]$ as the solution of the equation

$$x^3 - y^3 + z^3 - 3xyz = 2\,.$$

Find its first and second differential in the point $[1, 1]$.

$$\left[\mathrm{d}z = \tfrac{1}{3}\,\mathrm{d}x + \,\mathrm{d}y\,, \quad \mathrm{d}^2z = -\tfrac{16}{27}\,\mathrm{d}x^2 + \tfrac{4}{3}\,\mathrm{d}x\,\mathrm{d}y\,.\right]$$

Let $z = z(x, y)$ be a function defined in the neighbourhood of the point $[1, -1, 1]$ as the solution of the equation

$$z^3 - x^2z + y^2z + xy = 0\,.$$

Find its first and second differential in the point $[1, -1]$.

$$\left[\mathrm{d}z = \,\mathrm{d}x + \tfrac{1}{3}\,\mathrm{d}y\,, \quad \mathrm{d}^2z = -\tfrac{2}{9}\,\mathrm{d}x\,\mathrm{d}y - \tfrac{4}{9}\,\mathrm{d}y^2\,.\right]$$

Let $z = z(x, y)$ be a function defined in the neighbourhood of the point $[1, 1, 1]$ as the solution of the equation

$$z^3 - xyz^2 + x^2z - y^3 = 0\,.$$

Find its first and second differential in the point $[1, 1]$.

$$\left[\mathrm{d}z = -\tfrac{1}{2}\,\mathrm{d}x + 2\,\mathrm{d}y\,, \quad \mathrm{d}^2z = -\tfrac{3}{2}\,\mathrm{d}x^2 + 4\,\mathrm{d}x\,\mathrm{d}y - \,\mathrm{d}y^2\,.\right]$$

Let $z = z(x, y)$ be a function defined in the neighbourhood of the point $[1, 1, 1]$ as the solution of the equation
$$z^3 - xyz^2 + x^2 - y^2 = 0\,.$$
Find its first and second differential in the point $[1, 1]$.

$$\left[\mathrm{d}z = -\,\mathrm{d}x + 3\,\mathrm{d}y\,, \quad \mathrm{d}^2 z = -10\,\mathrm{d}x^2 + 34\,\mathrm{d}x\,\mathrm{d}y - 22\,\mathrm{d}y^2\,.\right]$$

Let $z = z(x, y)$ be a function defined in the neighbourhood of the point $[1, 0, 1]$ as the solution of the equation
$$z^4 - 2xyz^2 + x^4 - y^3 - 2xz = 0\,.$$
Find its first and second differential in the point $[1, 0]$.

$$\left[\mathrm{d}z = -\,\mathrm{d}x + \,\mathrm{d}y\,, \quad \mathrm{d}^2 z = -14\,\mathrm{d}x^2 + 12\,\mathrm{d}x\,\mathrm{d}y - 2\,\mathrm{d}y^2\,.\right]$$

Let functions $y(x)$ and $z(x)$ be defined in the neighbourhood of the point $[2, 1, -1]$ by the system of equations
$$x^2 + y^2 - z^2 = 4\,, \qquad x + yz = 1$$
Find their second derivatives $y''(2)$ and $z''(2)$.

$$\left[y'' = \tfrac{5}{4}\,, \quad z'' = -\tfrac{1}{4}\,; \quad y' = -\tfrac{1}{2}\,, \quad z' = -\tfrac{3}{2}\right]$$

Let functions $y(x)$ and $z(x)$ be defined in the neighbourhood of the point $[1, 1, 1]$ by the system of equations
$$x\mathrm{e}^y + y\mathrm{e}^x - 2\mathrm{e}^z = 0\,, \qquad xy + xz + yz = 3$$
Find their second derivatives $y''(1)$ and $z''(1)$.

$$\left[y'' = 1\,, \quad z'' = 0\,; \quad y' = -1\,, \quad z' = 0\right]$$

Let functions $y(x)$ and $z(x)$ be defined in the neighbourhood of the point $[1, 1, 1]$ by the system of equations
$$x^3 + y^3 + z^3 = 1\,, \qquad x^2 + y^2 + z^2 + 3xyz = 0$$
Find their second derivatives $y''(1)$ and $z''(1)$.

$$\left[y'' = -7\,, \quad z'' = 3\,; \quad y' = 0\,, \quad z' = -1\right]$$

Let functions $y(x)$ and $z(x)$ be defined in the neighbourhood of the point $[1, 1, -1]$ by the system of equations
$$x^2 + y^2 + z^2 = 3\,, \qquad x^2y + y^2z + xz^2 = 1$$
Find their second derivatives $y''(1)$ and $z''(1)$.

$$\left[y'' = 3\,, \quad z'' = 9\,; \quad y' = 1\,, \quad z' = 2\right]$$

Let functions $y(x)$ and $z(x)$ be defined in the neighbourhood of the point $[1,1,1]$ by the system of equations

$$z\mathrm{e}^{x^2-y^2} + (x^2+y^2)\mathrm{e}^z = 2\,, \qquad xy + xz + yz = 1$$

Find their second derivatives $y''(1)$ and $z''(1)$.

$$\left[y'' = -2\,,\quad z'' = 0\,;\quad y' = -1\,,\quad z' = 0\right]$$

Let functions $y(x)$ and $z(x)$ be defined in the neighbourhood of the point $[1,1,1]$ by the system of equations

$$x^2 + 2y^2 + 3z^2 = 6\,, \qquad xyz = 1$$

Find their second derivatives $y''(1)$ and $z''(1)$.

$$\left[y'' = 30\,,\quad z'' = -24\,;\quad y' = -2\,,\quad z' = 1\right]$$

Let functions $y(x)$ and $z(x)$ be defined in the neighbourhood of the point $[2,1,-1]$ by the system of equations

$$xy + xz + yz + 1 = 0\,, \qquad x^2 - y^2 + z^2 = 4$$

Find their second derivatives $y''(2)$ and $z''(2)$.

$$\left[y'' = -\tfrac{15}{2}\,,\quad z'' = \tfrac{1}{2}\,;\quad y' = 3\,,\quad z' = -1\right]$$

Let functions $y(x)$ and $z(x)$ be defined in the neighbourhood of the point $[1,-1,1]$ by the system of equations

$$\frac{x}{y} + \frac{z}{x} + \frac{y}{z} + 1 = 0\,, \qquad x + y + z = 1$$

Find their second derivatives $y''(1)$ and $z''(1)$.

$$\left[y'' = -1\,,\quad z'' = 1\,;\quad y' = -2\,,\quad z' = 1\right]$$

Let functions $y(x)$ and $z(x)$ be defined in the neighbourhood of the point $[1,1,-1]$ by the system of equations

$$y = \mathrm{e}^{x+z}\,, \qquad xy = z^2$$

Find their second derivatives $y''(1)$ and $z''(1)$.

$$\left[y'' = \tfrac{4}{27}\,,\quad z'' = \tfrac{1}{27}\,;\quad y' = \tfrac{1}{3}\,,\quad z' = -\tfrac{2}{3}\right]$$

Nechť jsou v okolí bodu $[1,-1,2]$ pomocí soustavy rovnic

$$z^2 - xy + yz + \sin(x+y) = 3\,, \qquad xy + xz + yz + x - y = 1$$

definovány funkce $y(x)$ a $z(x)$. Najděte jejich druhé derivace $y''(1)$ a $z''(1)$.

$$\left[y'' = 1\,,\quad z'' = -\tfrac{4}{3}\,;\quad y' = -1\,,\quad z' = 0\right]$$

Let functions $y(x)$ and $z(x)$ be defined in the neighbourhood of the point $[1,2,0]$ by the system of equations

$$xy + xz + yz - 2z = 2\,, \qquad y^2 + z^2 - xy + \mathrm{e}^{xz} = 3$$

Find their second derivatives $y''(1)$ and $z''(1)$.

$$\left[y'' = -32\,, \quad z'' = 52\,; \quad y' = 2\,, \quad z' = -4\right]$$

Let functions $y(x)$ and $z(x)$ be defined in the neighbourhood of the point $[0, -1, 1]$ by the system of equations

$$x^2 + y^2 + z^2 + xy - xz = 2\,, \qquad z^2 + 2xz + yz + y \sin xz = 0$$

Find their second derivatives $y''(0)$ and $z''(0)$.

$$\left[y'' = \tfrac{3}{2}\,, \quad z'' = \tfrac{1}{2}\,; \quad y' = -1\,, \quad z' = 0\right]$$

Let functions $y(x)$ and $z(x)$ be defined in the neighbourhood of the point $[2, 1, -1]$ by the system of equations

$$x^3 + y^3 + z^3 + 3xyz = 2\,, \qquad x^2 + y^2 + z^2 + 3yz + xyz = 1$$

Find their second derivatives $y''(2)$ and $z''(2)$.

$$\left[y'' = 1\,, \quad z'' = \tfrac{1}{3}\,; \quad y' = 0\,, \quad z' = -1\right]$$
