EXERCISES 6b – Maxima and Minima

1. **Minimizing the Cost of a Fence**

   For insurance purposes, a manufacturer plans to fence in a 10 800 m^2 rectangular storage area adjacent to a building by using the building as one side of the enclosed area. The fencing parallel to the building fraces a highway and will cost 3 EUR per foot installed, whereas the fincing for the other two sides costs 2 EUR per foot installed. Find the amount of each type of fence so that the total cost of the fence will be a minimum. What is the minimum cost? [120 m, 180 m, 720 EUR]

2. **Maximizing Revenue**

   The demand equation for a manufacturer's product is

   $$p = \frac{80 - p}{4}, \qquad 0 \le q \le 80,$$

   where q is the number of units and p is the price per unit. At what value of q will there be maximum revenue? What is the maximum revenue? [$q = 40$, $R = 400$]

3. **Maximizing Average Costs**

   A manufacturer's total-cost function is given by

   $$c = \frac{q^2}{4} + 3q + 400,$$

   where c is the total cost of producing q units. At what level of output will average cost per unit be a minimum? What is the minimum? [$q = 40$, $\overline{c} = 23$]

4. **Maximizing TV Cable Company Revenue**

   The Vista TV Cable Co. currently has 100 000 subscribers who are each paying a monthly rate of 40 EUR. A survey reveals that there will be 1000 more subscribers for each 0.25 EUR decrease in the rate. At what rate will maximum revenue be obtained, and how many subscribers will there be at this rate? [price: $40 - 30 \cdot 0.25 = 32.5$ EUR, number of subscribers: 130000]

5. **Maximizing the Number of Recipients of Health-Care Benefits**

   An article in a sociology journal stated that if a particular health-care program for the elderly where initiated, then t years after its start, n thousand elderly people would receive direct benefits, where

   $$n = \frac{t^3}{3} - 6t^2 + 32t, \qquad 0 \le t \le 12.$$

   For what value of t does the maximum number receive benefits? [$t = 12$]

6. **Maximizing the Profit**

   Suppose that the demand equation for a monopolist's product is $p = 400 - 2q$ and the average-cost function is $\overline{c} = 0.2q + 4 + 400/q$, where q is the number of units, and both p and $\overline{c}$ are expressed in EUR per unit. Determine the level of output at which profit is maximized. Determine the corresponding price and profit. [$q = 90$, $p = 220$ EUR, $P = 17\,420$ EUR]