Diferenciální rovnice

Najděte obecný tvar $u(t,c)$ řešení homogenní lineární rovnice

$\dot x = -2tx.$ $\qquad [u(t,c) = c\mathrm{e}^{-t^2},\ t \in \mathbb{R}]$

$\dot x = x \sin t.$ $\qquad [u(t,c) = c\mathrm{e}^{-\cos t},\ t \in \mathbb{R}]$

$\dot x = \dfrac{x}{1-t}.$ $\qquad \left[u(t,c) = \dfrac{c}{1-t}, t > 1 \text{ nebo } t < 1\right]$

$\dot x = \dfrac{3t}{t^2+1}x.$ $\qquad \left[u(t,c) = c(t^2+1)^{3/2},\ t \in \mathbb{R}\right]$

Nalezněte obecný tvar řešení rovnice

$\dot x = -\dfrac{3}{t}x + \dfrac{2}{t^3}.$ $\qquad \left[v(t,c) = \dfrac{c}{t^3} + \dfrac{2}{t^2},\ t \neq 0\right]$

$\dot x = -\dfrac{4t}{t^2+1}x + \dfrac{1}{t^2+1}.$ $\qquad \left[v(t,c) = \dfrac{c}{(t^2+1)^2} + \dfrac{t^3+3t}{3(t^2+1)^2},\ t \in \mathbb{R}\right]$

$\dot x = x \operatorname{tg} t + \dfrac{1}{\cos t}.$ $\qquad \left[v(t,c) = \dfrac{c}{\cos t} + \dfrac{t}{\cos t},\ t \in \left((2k-1)\dfrac{\pi}{2}, (2k+1)\dfrac{\pi}{2}\right)\right]$

$\dot x = -2tx + 2t\mathrm{e}^{-t^2}.$ $\qquad \left[v(t,c) = c\mathrm{e}^{-t^2} + t^2\mathrm{e}^{-t^2},\ t \in \mathbb{R}\right]$

$\dot x = x \operatorname{cotg} t + 2t \sin t.$ $\qquad \left[v(t,c) = c\sin t + t^2 \sin t,\ t \in (k\pi, (k+1)\pi)\right]$

$\dot x - x = \mathrm{e}^t.$ $\qquad [x = C\mathrm{e}^t + t\mathrm{e}^t\,;\ t \in \mathbb{R},\ C \in \mathbb{R}]$

$\dot x = \dfrac{2x}{t} + \dfrac{t-1}{t}.$ $\qquad \left[x = -t + \dfrac{1}{2} + Ct^2\,;\ t \in \mathbb{R},\ C \in \mathbb{R}\right]$

$\dot x = t(x+1).$ $\qquad \left[x = -1 + C\exp\left(t^2/2\right)\,;\ C \in \mathbb{R}\right]$

$\dot x + \dfrac{x}{t+1} = -t^2.$ $\qquad \left[x = \left(C - \dfrac{t^4}{4} - \dfrac{t^3}{3}\right)(t+1)^{-1}\,;\ C \in \mathbb{R}\right]$

$\dot x + t^2 x = t^2.$ $\qquad \left[x = 1 + C\exp(-t^3/3)\,;\ C \in \mathbb{R}\right]$

$\dot x + x = \dfrac{1}{\mathrm{e}^t(1-t)}.$ $\qquad \left[x = \mathrm{e}^{-t}\left(C - \ln|1-t|\right)\,;\ C \in \mathbb{R}\right]$

$\dot x + \dfrac{x}{t} = \dfrac{3}{t}.$ $\qquad \left[x = 3 + \dfrac{C}{t}\,;\ C \in \mathbb{R}\right]$

Metodou odhadu řešte rovnice

$\dot x + x = 2t + 4.$ $\qquad [x = C\mathrm{e}^{-t} + 2t + 2\,;\ t \in \mathbb{R},\ C \in \mathbb{R}]$

$\dot x + 2x = 3t\mathrm{e}^{-t}.$ $\qquad [x = C\mathrm{e}^{-2t} + 3\mathrm{e}^{-t}(t-1)\,;\ t \in \mathbb{R},\ C \in \mathbb{R}]$

$\dot x + x = 4t\mathrm{e}^{-t}.$ $\qquad [x = C\mathrm{e}^{-t} + 2t^2\mathrm{e}^{-t}\,;\ t \in \mathbb{R},\ C \in \mathbb{R}]$

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$\dot{x} + 3x = 2\cos 2t.$ $\left[x = C\mathrm{e}^{-3t} + \frac{6}{13}\cos 2t + \frac{4}{13}\sin 2t\,;\; t \in \mathbb{R}\,,\; C \in \mathbb{R}\right]$

$\dot{x} + 4x = 2\mathrm{e}^{-t}\sin 3t.$ $\left[x = C\mathrm{e}^{-4t} + \mathrm{e}^{-t}\left(-\frac{1}{3}\cos 3t + \frac{1}{3}\sin 3t\right)\;;\; t \in \mathbb{R}\,,\; C \in \mathbb{R}\right]$

$\dot{x} - 2x = 3t + 2.$ $\left[x = C\mathrm{e}^{2t} - \frac{3}{2}t - \frac{7}{4}\right]$

$\dot{x} + x = (t^2 - 1)\mathrm{e}^{2t}.$ $\left[x = C\mathrm{e}^{-t} + \mathrm{e}^{2t}\left(\frac{t^2}{3} - \frac{2t}{9} - \frac{7}{27}\right)\right]$

$\dot{x} + 3x = (2t - 1)\mathrm{e}^{-3t}.$ $\left[x = \mathrm{e}^{-3t}(C + t^2 - t\right]$

$\dot{x} + 2x = 3\sin 2t - 4\cos 2t.$ $\left[x = C\mathrm{e}^{-2t} - \frac{1}{4}\sin 2t - \frac{7}{4}\cos 2t\right]$

$\dot{x} + x = 3\sin 4t.$ $\left[x = C\mathrm{e}^{-t} + \frac{3}{17}\sin 4t - \frac{12}{17}\cos 4t\right]$

$\dot{x} + 3x = \mathrm{e}^{-t}(2\sin t - \cos t).$ $\left[x = C\mathrm{e}^{-3t} + \frac{\mathrm{e}^{-t}}{5}(3\sin t - 4\cos t)\right]$

$\dot{x} + 2x = 4\mathrm{e}^{-t}\cos 2t.$ $\left[x = C\mathrm{e}^{-2t} + \frac{4}{5}\mathrm{e}^{-t}(\cos 2t + 2\sin 2t)\right]$

$\dot{x} + 4x = 3\cos^2 t.$ $\left[x = C\mathrm{e}^{-4t} + \frac{3}{8} + \frac{3}{20}\sin 2t + \frac{3}{10}\cos 2t\right]$

$\dot{x} + 2x = 4\sin^2 t.$ $\left[x = C\mathrm{e}^{-2t} + 1 - \frac{1}{2}(\cos 2t + \sin 2t)\right]$

$\dot{x} + 3x = \sin 2t\cos 3t.$ $\left[x = C\mathrm{e}^{-3t} - \frac{3}{20}\sin t + \frac{1}{20}\cos t + \frac{3}{68}\sin 5t - \frac{5}{68}\cos 5t\right]$

$\dot{x} + x = 2\cos t\cos 2t.$ $\left[x = C\mathrm{e}^{-t} + \frac{1}{2}(\cos t + \sin t) + \frac{1}{10}(\cos 3t + 3\sin 3t)\right]$

Hledejte řešení Cauchyovy úlohy.

$\dot{x} = -\frac{3}{t}x + \frac{2}{t^3},\; x(2) = 3.$ $\left[v(t;2,3) = \frac{20}{t^3} + \frac{2}{t^2}\,,\; t > 0\right]$

$\dot{x} = -\frac{4t}{t^2+1}x + \frac{1}{t^2+1},\; x(0) = 0.$ $\left[v(t;0,0) = \frac{t^3 + 3t}{3(t^2+1)^2}\,,\; t \in \mathbb{R}\right]$

$\dot{x} = x\,\mathrm{tg}\,t + \frac{1}{\cos t},\; x(0) = 1.$ $\left[v(t;0,1) = \frac{1+t}{\cos t}\,,\; t \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$

$\dot{x} = -2tx + 2t\mathrm{e}^{-t^2},\; x(\tau) = \xi.$ $\left[v(t;\tau,\xi) = (\xi\mathrm{e}^{\tau^2} + t^2 - \tau^2)\mathrm{e}^{-t^2}\,,\; t \in \mathbb{R}\right]$

$\dot{x} = x\,\mathrm{cotg}\,t + 2t\sin t,\; x(\tau) = \xi,\; (\tau,\xi) \in \mathbb{R}\times\mathbb{R}.$

$\left[v(t;\tau,\xi) = \left(\frac{\xi}{\sin\tau} + t^2 - \tau^2\right)\sin t\,,\; t \in (k\pi, (k+1)\pi)\right]$

Nalezněte řešení následujících Cauchyových úloh

$\dot{x} = -4x,\; x(0) = 2.$ $[u(t;0,2) = 2\mathrm{e}^{-4t}\,,\; t \in \mathbb{R}]$

$\dot{x} = \dfrac{x}{2-t}$, $x(0) = 1$. $\left[u(t;0,1) = \dfrac{2}{2-t}\,,\ t < 2\right]$

$\dot{x} = \dfrac{2x}{1+t} + (1+t)^3$,

a) $x(0) = -3$. $[v(t;0,-3) = \frac{1}{2}\,(1+t)^2(t^2+2t-6),\ t \in (-1,\infty)]$

b) $x(-2) = 1$. $[v(t;-2,1) = \frac{1}{2}\,(1+t)^2(t^2+2t+2),\ t \in (-\infty,-1)]$

$\dot{x} = (1-x)\operatorname{tg} t$

a) $x(0) = 4$. $[v(t;0,4) = 3\cos t + 1,\ t \in (-\frac{\pi}{2},\frac{\pi}{2})]$

b) $x(-\pi) = 1$. $[v(t;-\pi,1) = 1,\ t \in (-\frac{3\pi}{2},-\frac{\pi}{2})]$

$\dot{x} = x\sin t + \sin t\cos t$, $x(\tau) = \xi$, $(\tau,\xi) \in \mathbb{R}\times\mathbb{R}$.

$[v(t;\tau,\xi) = (\xi + \cos\tau - 1)\mathrm{e}^{\cos\tau - \cos t} - \cos t + 1\,,\ t \in \mathbb{R}]$

$\dot{x} = \dfrac{2t}{1+t^2}\,x + t^2 + 1$, $x(\tau) = \xi$, $(\tau,\xi) \in \mathbb{R}\times\mathbb{R}$. $\left[v(t;\tau,\xi) = \left(\dfrac{\xi}{1+\tau^2} - \tau + t\right)(1+t^2)\,,\ t \in \mathbb{R}\right]$

$\dot{x} = -\dfrac{2tx}{1+t^2} + \dfrac{2t^2}{1+t^2}$, $x(\tau) = \xi$, $(\tau,\xi) \in \mathbb{R}\times\mathbb{R}$. $\left[v(t;\tau,\xi) = \dfrac{1+\tau^2}{1+t^2}\,\xi + \dfrac{2}{3}\,\dfrac{t^3-\tau^3}{1+t^2}\,,\ t \in \mathbb{R}\right]$

$\dot{x} = -\dfrac{x}{t} + t$, $x(\tau) = \xi$, $(\tau,\xi) \in \mathbb{R}\times\mathbb{R}$, $\tau \neq 0$. $\left[v(t;\tau,\xi) = \dfrac{3\tau\xi + t^3 - \tau^3}{3t}\,,\ t < 0 \text{ nebo } t > 0\right]$

$\dot{x} = (1-x)\cos t$, $x(\tau) = \xi$, $(\tau,\xi) \in \mathbb{R}\times\mathbb{R}$. $[u(t;\tau,\xi) = (\xi - 1)\mathrm{e}^{\sin\tau - \sin t} + 1\,,\ t \in \mathbb{R}]$

$\dot{x} + 2x = f(t)$, kde $f(t) = \begin{cases} \sin t & \text{pro } t \in \langle 0,\pi\rangle \\ 0 & \text{pro } t \notin \langle 0,\pi\rangle \end{cases}$, $x(0) = 0$.

,

$$\left[x = \begin{cases} 0 & \text{pro } t \in (-\infty,0) \\ \dfrac{1}{5}\left(\mathrm{e}^{-2t} - \cos t + 2\sin t\right) & \text{pro } t \in \langle 0,\pi\rangle \\ \dfrac{1}{5}\left(1 + \mathrm{e}^{2\pi}\right)\mathrm{e}^{-2x} & \text{pro } (\pi,+\infty) \end{cases} ;\ t \in \mathbb{R}\right]$$

Určete obecné řešení diferenciálních rovnic

$tx\dot{x} - \sqrt{x^2+1} = 0$. $[\sqrt{x^2+1} = \ln|t| + C\,;\ t \neq 0\,,\ C \in \mathbb{R}]$

$(t^2-1)\dot{x} + 2tx = 0$. $\left[x = \dfrac{C}{t^2-1}\,;\ t \neq \pm 1\,,\ C \in \mathbb{R}\right]$

Řešte diferenciální rovnice

$\dot{x} = 4t\sqrt{x}$, $x(1) = 1$. $\left[\begin{array}{l} u(t;1,1) = t^4\,,\ t \in \mathbb{R} \\ u(t;1,1) = \begin{cases} t^4 & t \in \langle 0,\infty) \\ 0 & t \in (-\infty,0,) \end{cases} \end{array}\right]$

$\dot{x} = -x^2$,

a) $x(0) = 0$. $[u(t;0,0) = 0,\ t \in \mathbb{R}]$

b) $x(1) = 1$. $[u(t;1,1) = \frac{1}{t},\ t > 0]$

$\dot{x} - x^2 = 1$, $x(\frac{5\pi}{4}) = 1$. $[u(t;\frac{5\pi}{4},1) = \operatorname{tg} t,\ t \in (\frac{\pi}{2},\frac{3\pi}{2})]$

$t\dot{x} + x = x\ln x$, $x(1) = 1$. $[u(t;1,1) = \mathrm{e}^{1-t},\ t \in (0,\infty)]$

$(1+\mathrm{e}^t)x\dot{x}=\mathrm{e}^t,$

a) $x(0)=1.$ $\quad\left[u(t;0,1)=\sqrt{1+2\ln\frac{1}{2}(1+\mathrm{e}^t)},\ t\in\mathbb{R}\right]$

b) $x(0)=-1.$ $\quad\left[u(t;0,-1)=-\sqrt{1+2\ln\frac{1}{2}(1+\mathrm{e}^t)},\ t\in\mathbb{R}\right]$

$\dot{x}-x^2=0,$

a) $x(0)=0.$ $\quad[u(t;0,0)=0,\ t\in\mathbb{R}]$

b) $x(1)=1.$ $\quad\left[u(t;1,1)=\dfrac{1}{2-t},\ t\in(-\infty,2)\right]$

c) $x(\tau)=\xi.$ $\quad\left[\begin{array}{l}\text{pro}\quad \xi=0,\quad u(t;\tau,\xi)=0\,,\ t\in\mathbb{R}\\ \text{pro}\ \xi\neq 0\ u(t;\tau,\xi)=\frac{\xi}{1+\xi(\tau-t)}\,,\ t\in\left(-\infty,\frac{1+\xi\tau}{\xi}\right)\cup\left(\frac{1+\xi\tau}{\xi},\infty\right)\end{array}\right]$

$\dot{x}=\dfrac{2-x}{\operatorname{cotg}t}\,,\ x\left(\dfrac{\pi}{4}\right)=2-\dfrac{3}{\sqrt{2}}.$ $\quad\left[u\left(t;\dfrac{\pi}{4},2+\dfrac{3}{\sqrt{2}}\right)=2-3\cos t\,,\ t\in\left(0,\dfrac{\pi}{2}\right)\right]$

$\dot{x}=\dfrac{-t}{x+1},\ x(0)=0.$ $\quad[u(t;0,0)=-1+\sqrt{1-t^2},\ t\in(-1,1)]$

$\dot{x}=\dfrac{1}{\sqrt{1-t^2}},$

a) $x(1)=\frac{\pi}{2}.$ $\quad[u(t;1,\frac{\pi}{2})\ \text{neexistuje}]$

b) $x(0)=0.$ $\quad[u(t;0,0)=\arcsin t,\ t\in(-1,1)]$

$\dot{x}=x\cos t,\ x(0)=1.$ $\quad[u(t;0,1)=\mathrm{e}^{\sin t},\ t\in\mathbb{R}]$

$\dot{x}=(x-1)\operatorname{tg}t\,,\ x(\tau)=\xi\,,\ \tau\in\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right).$ $\quad\left[u(t;\tau,\xi)=1+(\xi-1)\dfrac{\cos\tau}{\cos t}\,,\ t\in\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)\right]$

$\dot{x}=\dfrac{2tx}{t^2-4},$

a) $x(-5)=0.$ $\quad[u(t;-5,0)=0,\ t\in(-\infty,-2)]$

b) $x(0)=1.$ $\quad[u(t;0,1)=1-\frac{1}{4}t^2,\ t\in(-2,2)]$

c) $x(0)=-1.$ $\quad[u(t;0,-1)=\frac{1}{4}t^2-1,\ t\in(-2,2)]$

d) $x(3)=1.$ $\quad[u(t;3,1)=\frac{1}{5}t^2-\frac{4}{5},\ t\in(2,\infty)]$

$\dot{x}=-\dfrac{1+3x}{t},\ x(1)=2.$ $\quad\left[u(t;1,2)=\dfrac{7-t^3}{3t^3}\,,\ t\in(0,\infty)\right]$

$\dot{x}=\dfrac{1-t^2}{tx},\ x(1)=2.$ $\quad[u(t;1,2)=\sqrt{5+2\ln t-t^2},\ t\in(0,\infty)]$

$\dot{x}=-\dfrac{2tx^2}{t^2-1},\ x(0)=1.$ $\quad\left[u(t;0,1)=\dfrac{1}{1+\ln(1-t^2)}\,,\ t\in\left(-\sqrt{1-\dfrac{1}{\mathrm{e}}},\sqrt{1-\dfrac{1}{\mathrm{e}}}\right)\right]$

• Určete obecné řešení rovnice

$\dot{x}=\dfrac{t}{x}+\dfrac{x}{t}.$ $\quad[x^2=t^2\ln t^2+Ct^2\,;\ C\in\mathbb{R}]$

$(\sqrt{xt}-t)\,\dot{x}+x=0.$ $\quad\left[x=C\operatorname{sgn}t\exp\left(-2\operatorname{sgn}t\sqrt{\dfrac{t}{x}}\right)\right]$

$x^2+t^2-2tx\dot{x}=0.$ $\quad[x^2=t^2+Ct\,;\ C\in\mathbb{R}]$

$\dot{x}(x+t) = x - t.$ $\left[\ln(x^2+t^2) + 2\,\mathrm{arctg}\,\frac{x}{t} = C\,;\ C \in \mathbb{R}\right]$

• Přesvědčte se, že výraz na pravé straně je úplná derivace a najděte obecné řešení diferenciální rovnice

$3(1-x^2)y^{-4}y' + (2xy^{-3} - 3x^2) = 0.$ $\left[x^2 - 1 - x^3y^3 + Cy^3 = 0\right]$

$\mathrm{e}^{x/y}\left(1 - \frac{x}{y}\right)y' + \left(1 + \mathrm{e}^{x/y}\right) = 0.$ $\left[x + y\mathrm{e}^{x/y} + C = 0\right]$

$(x\mathrm{e}^y - 2y)y' + \mathrm{e}^y = 0.$ $\left[x\mathrm{e}^y - y^2 = C\right]$

$\left(\frac{y}{\sqrt{x^2+y^2}} + \frac{1}{x}\right)y' + \frac{x}{\sqrt{x^2+y^2}} - \frac{y}{x^2} = 0.$ $\left[\sqrt{x^2+y^2} + \frac{y}{x} = C\right]$

$(2x - 2y + 3)y' - 6x + 2y = 1.$ $\left[3x^2 - 2xy + x + y^2 - 3y = C\right]$

$\left(y + \frac{x}{x^2+y^2}\right)y' + x - \frac{y}{x^2+y^2} = 0.$ $\left[x^2 + y^2 - 2\,\mathrm{arctg}\,\frac{x}{y} = C\right]$

$xy'\cos\frac{y}{x} + 2x\sin\frac{y}{x} - y\cos\frac{y}{x} = 0.$ $\left[x^2\sin\frac{y}{x} = C\right]$

Najděte obecné řešení diferenciální rovnice

$y^{(5)} = \mathrm{e}^{2x}.$ $\left[y = \frac{1}{32}\,\mathrm{e}^{2x} + C_1x^4 + C_2x^3 + C_3x^2 + C_4x + C_5\right]$

$y'''\sin^4 x - \sin 2x = 0.$ $\left[x = \ln|\sin x| + C_1x^2 + C_2x + C_3\right]$

$y'' = \mathrm{arctg}\,x.$ $\left[y = \frac{1}{2}\,(x^2-1)\,\mathrm{arctg}\,x - \frac{1}{2}\,x\ln(1+x^2) + C_1x + C_2\right]$

$y'' = \ln x.$ $\left[y = \frac{1}{2}\,x^2\left(\ln x - \frac{3}{2}\right) + C_1x + C_2\right]$

$y'' = \frac{1}{(x-1)^3} - \frac{1}{(1+x)^3}.$ $\left[y = \frac{1}{x^2-1} + C_1x + C_2\right]$

$y''' = \cos^2 x - \sin^2 x.$ $\left[y = -\frac{1}{4}\,\sin x\cos x + C_1x^2 + C_2x + C_3\right]$

$xy^{(4)} = 1.$ $\left[y = \frac{1}{6}\,x^3\ln x + C_1x^3 + C_2x^2 + C_3x + C_4\right]$

$y''' = \mathrm{e}^x\left(x^2 + 6x + 6\right).$ $\left[y = \mathrm{e}^x x^2 + C_1x^2 + C_2x + C_3\right]$

$y^{(4)} = 16(\cosh^2 x + \sinh^2 x).$ $\left[y = \cosh^2 x + \sinh^2 x + C_1x^3 + C_2x^2 + C_3x + C_4\right]$

$y^{(5)} = 4\mathrm{e}^x(\sin x - \cos x).$ $\left[y = \mathrm{e}^x\cos x + C_1x^4 + C_2x^3 + C_3x^2 + C_4x + C_5\right]$

Najděte řešení diferenciální rovnice, které vyhovuje daným počátečním podmínkám

$y'' = 2(3x + 2x^3)\exp\left(x^2\right),\ y(0) = 1,\ y'(0) = 2.$ $\left[y = x\left(\exp(x^2) + 1\right) + 1\right]$

$y'' = \frac{2x}{1+x^2} + 2\,\mathrm{arctg}\,x,\ y(0) = 1,\ y'(0) = -1.$ $\left[y = (1+x^2)\,\mathrm{arctg}\,x - 2x + 1\right]$

Najděte obecné řešení diferenciální rovnice

$yy'' + (y')^2 = 1.$ $\qquad [y^2 - (x + C_2)^2 = C_1]$

$2yy'' - 3(y')^2 = 4y^2.$ $\qquad [y\cos^2(x + C_1) = C_2]$

$y^3y'' - 1 = 0.$ $\qquad [C_1y^2 - (C_1x + C_2)^2 = 1]$

$1 + (y')^2 = 2yy''.$ $\qquad [4C_1y - 4 = (C_1x + C_2)^2]$

$y'' = \dfrac{1}{4\sqrt{y}}.$ $\qquad \left[x + C_2 = \dfrac{4}{3}\left(C_1 + \sqrt{y}\right)^{3/2} - 4C_1\left(C_1 + \sqrt{y}\right)^{1/2}\right]$

$yy'' - (y')^2 - 4yy' = 0.$ $\qquad [\ln|C_1y| = C_2\mathrm{e}^{4x}]$

$y(1 - \ln y)y'' + (1 + \ln y)(y')^2 = 0.$ $\qquad [x + C_1 = (x + C_2)\ln y]$

$y'' + 2(y')^2 \operatorname{tg} y = 0.$ $\qquad [y = \operatorname{arctg}(C_1 + C_2x)]$

$2yy'' + (y')^2 + (y')^4 = 0.$ $\qquad [2(C_1y - 1)^{3/2} = 3C_1x + C_2]$

Najděte řešení diferenciální rovnice, které vyhovuje daným počátečním podmínkám

$y'' + \dfrac{2}{1-y}(y')^2 = 0,\ y(0) = 2,\ y'(0) = 1.$ $\qquad \left[y = 1 - \dfrac{1}{x-1}\right]$

$(y')^2 + 2yy'' = 0,\ y(1) = 1,\ y'(1) = \dfrac{2}{3}.$ $\qquad [y = x^{2/3}]$

$yy'' = (y')^2,\ y(0) = 1,\ y'(0) = 2.$ $\qquad [y = \mathrm{e}^{2x}]$

Najděte obecné řešení diferenciální rovnice

$y'' = \dfrac{y'}{x} + x.$ $\qquad \left[y = \dfrac{1}{3}x^3 + C_1x^2 + C_2\right]$

$x^4y''' + 2x^3y'' - 1 = 0.$ $\qquad \left[y = -\dfrac{1}{2x} - C_1\ln|x| + C_2x + C_3\right]$

$y''\left(1 + x^2\right) + (y')^2 + 1 = 0.$ $\qquad \left[y = \left(1 + C_2^2\right)\ln|x + C_1| - C_1x + C_2\right]$

$xy'' = y'\ln\dfrac{y'}{x}.$ $\qquad \left[y = \left(C_1x - C_1^2\right)\exp\left(\dfrac{x}{C_1} + 1\right) + C_2\right]$

$y'' + y'\operatorname{tg} x = \sin 2x.$ $\qquad \left[y = -x - \dfrac{1}{2}\sin 2x + C_1\sin x + C_2\right]$

$xy'' = y' + x\sin\dfrac{y'}{x}.$ $\qquad \left[y = \left(x^2 + C_1^{-2}\right)\operatorname{arctg} C_1x - C_1^{-1}x + C_2\right]$

$y'y''' = 3(y'')^2.$ $\qquad [x = C_1y^2 + C_2y + C_3]$

$y''' = (y'')^3.$ $\qquad \left[y = \dfrac{1}{3}(C_1 - 2x)^{3/2} + C_2x + C_3\right]$

$xy^{(5)} = y^{(4)}.$ $\qquad [y = C_1x^5 + C_2x^3 + C_3x^2 + C_4x + C_5]$

Najděte řešení diferenciální rovnice, které vyhovuje daným počátečním podmínkám

$(y'')^2 = y'$, $y(-1) = 1$, $y'(-1) = 0$. $\left[y = \frac{1}{12}(x+1)^3 + 1\right]$

$xy'' - y' = x^3$, $y(1) = \frac{1}{2}$, $y'(1) = 1$. $\left[y = \frac{1}{8}(x^2+1)^2\right]$

Najděte obecné řešení diferenciální rovnice, znáte-li její jedno partikulární řešení

$x^2(\ln x - 1)y'' - xy' + y = 0$; $y_1 = x$. $[y = C_1 x + C_2 \ln x]$

$xy'' - (2x-1)y' + (x-1)y = 0$; $y_1 = \mathrm{e}^x$. $[y = \mathrm{e}^x(C_1 + C_2 \ln |x|)]$

$(1+x^2)y'' + 2xy' - 2y = 0$; $y_1 = x$. $[y = C_1 x + C_2(1 + x \operatorname{arctg} x)]$

$x(1-x)^2 y'' = 2y$; $y_1 = \frac{x}{1-x}$. $\left[y = \frac{C_1 x}{1-x} + C_2\left(x + 1 + \frac{2x \ln|x|}{1-x}\right)\right]$

$(x^3 - 2x^2)y'' - (x^3 + 2x^2 - 6x)y' + (3x^2 - 6)y = 0$; $y_1 = x^m$. $[y = C_1 x^3 + C_2 x\mathrm{e}^x]$

$(2x+1)y'' + (4x-2)y' - 8y = 0$; $y_1 = \mathrm{e}^{mx}$. $[y = C_1 \mathrm{e}^{-2x} + C_2(1 + 4x^2)]$

$xy'' + 2y' - xy = 0$; $y_1 = \frac{1}{x}\mathrm{e}^x$. $\left[y = \frac{1}{x}(C_1\mathrm{e}^x + C_2\mathrm{e}^{-x})\right]$

$y'' \sin^3 x = 4y \sin 3x$; $y_1 = \sin^4 x$. $\left[y = C_1 \sin^4 x + C_2 \frac{\cos x}{\sin^3 x}(5 + 6\sin^2 x + 8\sin^4 x + 16\sin^6 x)\right]$

$(1+x^2)y'' + xy' - n^2 y = 0$; $y_1 = \left(x + \sqrt{x^2+1}\right)^n$. $\left[y = C_1\left(x + \sqrt{x^2+1}\right)^n + C_2\left(\sqrt{x^2+1} - x\right)^n\right]$

Znáte-li jedno partikulární řešení diferenciální rovnice, najděte její řešení, které vyhovuje daným počátečním podmínkám

$x(2x+1)y'' + 2(x+1)y' - 2y = 0$; $y_1 = 1 + x$; $y(1) = 3$, $y'(1) = 0$. $[y = x + 1 + x^{-1}]$

$xy'' - (2x+1)y' + (x+1)y = 0$; $y_1 = \mathrm{e}^x$; $y(1) = 1$, $y'(1) = 3$. $[y = x^2\mathrm{e}^{x-1}]$

• Za předpokladu, že lze řešení diferenciální rovnice vyjádřit mocninnou řadou, najděte obecné řešení rovnice

$(1-x^2)y'' - 2xy' + 6y = 0$. $\left[y = C_1(1 - 3x^2) + C_2\left(x - \sum_{n=1}^{\infty} \frac{n+1}{(2n-1)(2n+1)} x^{2n+1}\right)\right]$

$y'' - xy = 0$. $\left[y = C_1 \sum_{n=0}^{\infty} \frac{1 \cdot 4 \cdot \ldots \cdot (3n-2)}{(3n)!} x^{3n} + C_2 \sum_{n=0}^{\infty} \frac{2 \cdot 5 \cdot \ldots \cdot (3n-1)}{(3n+1)!} x^{3n+1}\right]$

$y'' + 4xy' + (4x^2 + 2)y = 0$. $[y = (C_1 + C_2 x)\exp(-x^2)]$

$x(x+4)y'' - (2x+4)y' + 2y = 0$. $[y = C_1(x+2) + C_2 x^2]$

$(x^2 - x)y'' + (x-3)y' - 4y = 0$. $[y = C_1 x^{-2} + C_2(3x^2 - 8x + 6)]$

$xy'' + 2y' + xy = 0$. $\left[y = C_1 \frac{\sin x}{x} + C_2 \frac{\cos x}{x}\right]$

$(2x - x^2)y'' + (x^2 - 2)y' + 2(1-x)y = 0$. $[y = C_1\mathrm{e}^x + C_2 x^2]$

$xy'' - (1+x)y' + y = 0.$ $[y = C_1 \mathrm{e}^x + C_2(1+x)]$

$xy'' + (2x-1)y' + (x-1)y = 0.$ $\left[y = \mathrm{e}^{-x}\left(C_1 + C_2 x^2\right)\right]$

• Za předpokladu, že řešení lze vyjádřit mocninnou řadou, najděte řešení diferenciální rovnice, které vyhovuje daným počátečním podmínkám
$(2x+1)x^2y'' - 2x(5x+2)y' + 2(8x+3)y = 0;\ y(1) = 3,\ y'(1) = 9.$ $\left[y = x^2 + x^3 + x^4\right]$

$(1-x)y'' + xy' - y = 0;\ y(0) = 1,\ y'(0) = 1.$ $[y = \mathrm{e}^x]$

$xy'' - y' - x^3y = 0;\ y(1) = \sqrt{\mathrm{e}},\ y'(1) = 0.$ $\left[y = \frac{1}{2}\left(\exp(x^2/2) + \exp(1-x^2/2)\right)\right]$

Najděte řešení Cauchyovy úlohy
$\ddot{x} + \dot{x} - 2x = 0,\ x(0) = 0,\ \dot{x}(0) = 3.$ $[u(t;0,(0,3)) = \mathrm{e}^t - \mathrm{e}^{-2t}\,,\ t \in \mathbb{R}]$

$\ddot{x} + 8\dot{x} + 15x = 0,\ x(0) = -2,\ \dot{x}(0) = 2.$ $[u(t;0,(-2,2)) = 2\mathrm{e}^{-5t} - 4\mathrm{e}^{-3t}\,,\ t \in \mathbb{R}]$

$\ddot{x} - 5\dot{x} + 6x = 0,\ x(0) = 1,\ \dot{x}(0) = 3.$ $[u(t;0,(1,3)) = \mathrm{e}^{3t}\,,\ t \in \mathbb{R}]$

$\ddot{x} + 8\dot{x} + 16x = 0,\ x(0) = 4,\ \dot{x}(0) = 0.$ $[u(t;0,(4,0)) = (4+16t)\mathrm{e}^{-4t}\,,\ t \in \mathbb{R}]$

$\ddot{x} - 6\dot{x} + 9x = 0,\ x(0) = 0,\ \dot{x}(0) = 13.$ $[u(t;0,(0,13)) = 13t\mathrm{e}^{3t}\,,\ t \in \mathbb{R}]$

$\ddot{x} + x = 0,\ x(0) = 3,\ \dot{x}(0) = 4.$ $[u(t;0,(3,4)) = 3\cos t + 4\sin t\,,\ t \in \mathbb{R}]$

$\ddot{x} + 4x = 0,\ x(0) = 1,\ \dot{x}(0) = -4.$ $[u(t;0,(1,-4)) = \cos 2t - 2\sin 2t\,,\ t \in \mathbb{R}]$

$\ddot{x} + 6\dot{x} + 13x = 0,\ x(0) = 0,\ \dot{x}(0) = 2.$ $[u(t;0,(0,2)) = \mathrm{e}^{-3t}\sin 2t\,,\ t \in \mathbb{R}]$

$\ddot{x} - 2\dot{x} + 5x = 0,\ x(\pi/2) = 0,\ \dot{x}(\pi/2) = -1.$ $[u(t;\pi/2,(0,-1)) = \frac{1}{2}\mathrm{e}^{t-\pi/2}\sin 2t\,,\ t \in \mathbb{R}]$

Uveďte, v jakém tvaru budete hledat partikulární řešení následujících rovnic.
$\ddot{x} - 5\dot{x} + 6x = 3t^3 + 8t.$ $[w(t) = at^3 + bt^2 + ct + d]$

$\ddot{x} - 3\dot{x} + 2x = t\mathrm{e}^{2t} + \mathrm{e}^{-t}.$ $[w(t) = t(at+b)\mathrm{e}^{2t} + c\mathrm{e}^{-t}]$

$\dddot{x} + \ddot{x} = 1 - 2t + t\mathrm{e}^{-t}.$ $[w(t) = t^2(at+b) + t(ct+d)\mathrm{e}^{-t}]$

$\dddot{x} + 2\ddot{x} + \dot{x} = 1 + 2t^2\mathrm{e}^t.$ $[w(t) = at + (bt^2 + ct + d)\mathrm{e}^t]$

$\dddot{x} - 3\ddot{x} + 3\dot{x} - x = (1-t)\mathrm{e}^t + 2.$ $[w(t) = t^3(at+b)\mathrm{e}^t + c]$

Najděte obecný tvar řešení následujících rovnic
$\ddot{x} - 3\dot{x} + 2x = 2t^3 - 30.$ $\left[v(t;c_1,c_2) = c_1\mathrm{e}^t + c_2\mathrm{e}^{2t} + \frac{1}{4}(4t^3 + 18t^2 + 42t - 15)\,,\ t, c_1, c_2 \in \mathbb{R}\right]$

$\ddot{x} - 4\dot{x} + 4x = 3\mathrm{e}^{2t} + \mathrm{e}^{-t} + 1.$ $\left[v(t;c_1,c_2) = (c_1 + c_2 t)\mathrm{e}^{2t} + \dfrac{3}{2}t^2\mathrm{e}^{2t} + \dfrac{1}{9}\mathrm{e}^{-t} + \dfrac{1}{4}\,,\ t, c_1, c_2 \in \mathbb{R}\right]$

$\ddot{x} + x = 2t^3 - t + 2 - 2\mathrm{e}^{-t}$

$[v(t;c_1,c_2) = c_1 \cos t + c_2 \sin t + 2t^3 - 13t + 2 - \mathrm{e}^{-t}\,,\ t, c_1, c_2 \in \mathbb{R}]$

$\dddot{x} - 3\dot{x} + 2x = (4t^2 + 4t - 10)\mathrm{e}^{-t}.$

$[v(t;c_1,c_2,c_3) = (c_1 + c_2 t)\mathrm{e}^{t} + c_3\mathrm{e}^{-2t} + (t^2 + t - 1)\mathrm{e}^{-t}\,,\ t, c_1, c_2, c_3 \in \mathbb{R}]$

$\dddot{x} - 6\ddot{x} + 11\dot{x} - 6x = 12t^2\mathrm{e}^{3t} - \mathrm{e}^{2t}.$

$[v(t;c_1,c_2,c_3) = c_1\mathrm{e}^{t} + c_2\mathrm{e}^{2t} + c_3\mathrm{e}^{3t} + (2t^3 - 9t^2 + 21t)\mathrm{e}^{3t} + t\mathrm{e}^{2t}\,,\ t, c_1, c_2, c_3 \in \mathbb{R}]$

Nalezněte řešení následujících Cauchyových úloh.

$\ddot{x} - \dot{x} = 2(1 - t),\ x(0) = 1,\ \dot{x}(0) = 1.$ $[u(t;0,(1,1)) = \mathrm{e}^{t} + t^2\,,\ t \in \mathbb{R}]$

$\ddot{x} - 2\dot{x} = (t^2 + t - 3)\mathrm{e}^{t},\ x(0) = \dot{x}(0) = 2.$ $[u(t;0,(2,2)) = \mathrm{e}^{2t} - (t^2 + t - 1)\mathrm{e}^{t}\,,\ t \in \mathbb{R}]$

$\ddot{x} - 4x = 4\mathrm{e}^{2t},\ x(0) = \dot{x}(0) = 0.$ $\left[u(t;0,(0,0)) = t\mathrm{e}^{2t} - \dfrac{\sinh 2t}{2}\,,\ t \in \mathbb{R}\right]$

$\ddot{x} + x = t^3 + 6t + \mathrm{e}^{-t},\ x(0) = \dot{x}(0) = 0.$ $\left[u(t;0,(0,0)) = t^3 + \dfrac{1}{2}\left(\sin t - \cos t + \mathrm{e}^{-t}\right)\,,\ t \in \mathbb{R}\right]$

$\dddot{x} - \dot{x} = 6 - 3t^2,\ x(0) = \dot{x}(0) = \ddot{x}(0) = 1.$ $[u(t;0,(1,1,1)) = t^3 + \mathrm{e}^{t}\,,\ t \in \mathbb{R}]$

Uveďte, v jakém tvaru budete hledat partikulární řešení následujících rovnic; $\Re z$, resp. $\Im z$, je reálná, resp. imaginární, část komplexního čísla z.

$\ddot{x} + 4\dot{x} + 3x = \mathrm{e}^{-t}\cos t + 5\sin 3t.$

$[w(t) = \Re(p\mathrm{e}^{(-1+i)t}) + \Im(q\mathrm{e}^{3it}) = a\mathrm{e}^{-t}\cos t + b\mathrm{e}^{-t}\sin t + c\cos 3t + d\sin 3t]$

$\ddot{x} + 6\dot{x} + 10x = \mathrm{e}^{-3t} + 2\mathrm{e}^{-3t}\cos t.$

$[w(t) = \Re\left(p\mathrm{e}^{-3t} + tq\mathrm{e}^{-(3+i)t}\right) = a\mathrm{e}^{-3t} + bt\mathrm{e}^{-3t}\cos t + ct\mathrm{e}^{-3t}\sin t]$

$\dddot{x} + 4\ddot{x} + 5\dot{x} + 2x = \mathrm{e}^{-t}\cos 2t + t\mathrm{e}^{-2t}\sin t.$

$$\left[\begin{aligned} w(t) = \Re(p\mathrm{e}^{(-1+2i)t}) + \Im((qt + r)\mathrm{e}^{(-2+i)t}) = \\ = a\mathrm{e}^{-t}\cos 2t + b\mathrm{e}^{-t}\sin 2t + (ct + d)\mathrm{e}^{-2t}\sin t + (ft + g)\mathrm{e}^{-2t}\cos t \end{aligned}\right]$$

$\dddot{x} + 2\ddot{x} + \dot{x} = \sin^2 t + \mathrm{e}^{t}\cosh t.$ $[w(t) = \Re(pt + q\mathrm{e}^{2t} + r\mathrm{e}^{2it}) = ta + b\mathrm{e}^{2t} + c\sin 2t + d\cos 2t]$

Najděte obecný tvar řešení následujících rovnic

$\ddot{x} + x = 6\sin 2t.$ $[v(t;c_1,c_2) = c_1\cos t + c_2\sin t - 2\sin 2t\,,\ t, c_1, c_2 \in \mathbb{R}]$

$\ddot{x} + 4x = \sin 2t.$ $[v(t;c_1,c_2) = c_1\cos 2t + c_2\sin 2t - \frac{1}{4}t\cos 2t\,,\ t, c_1, c_2 \in \mathbb{R}]$

$\ddot{x} - x = 2\sin t - 4\cos t.$ $[v(t;c_1,c_2) = c_1\mathrm{e}^{t} + c_2\mathrm{e}^{-t} + 2\cos t - \sin t\,,\ t, c_1, c_2 \in \mathbb{R}]$

$\ddot{x} - x = \cos^2 t.$ $[v(t;c_1,c_2) = c_1\mathrm{e}^{t} + c_2\mathrm{e}^{-t} - \frac{1}{10}(\cos 2t + 5)\,,\ t, c_1, c_2 \in \mathbb{R}]$

$\ddot{x} + x = \cos t + \cos 2t.$ $[v(t;c_1,c_2) = c_1\cos t + c_2\sin t + \frac{1}{6}(3t\sin t - 2\cos 2t)\,,\ t, c_1, c_2 \in \mathbb{R}]$

$\ddot{x} + x = 2\sin t + 4t\cos t.$ $[v(t;c_1,c_2) = c_1\cos t + c_2\sin t + t^2\sin t\,,\ t, c_1, c_2 \in \mathbb{R}]$

$\ddot{x} - 4x = (4-4t)\mathrm{e}^t \cos t - (2t+6)\mathrm{e}^t \sin t.$
$[v(t; c_1, c_2) = c_1 \mathrm{e}^{2t} + c_2 \mathrm{e}^{-2t} + t\mathrm{e}^t \cos t + \mathrm{e}^t \sin t\,,\ t, c_1, c_2 \in \mathbb{R}]$

$\ddot{x} - 2\dot{x} + 2x = 2\mathrm{e}^t \cos t - 4t\mathrm{e}^t \sin t.$
$[v(t; c_1, c_2) = c_1 \mathrm{e}^t \cos t + c_2 \mathrm{e}^t \sin t + t^2 \mathrm{e}^t \cos t\,,\ t, c_1, c_2 \in \mathbb{R}]$

Nalezněte řešení následujících Cauchyových úloh.

$\ddot{x} + x = 6\sin 2t,\ x(0) = 0,\ \dot{x}(0) = -4.$ $[u(t; 0, (0, -4)) = -2\sin 2t\,,\ t \in \mathbb{R}]$

$\ddot{x} + 4x = 2\cos 2t,\ x(0) = 0,\ \dot{x}(0) = 4.$ $[u(t; 0, (0, 4)) = \frac{1}{2}(t+4)\sin 2t\,,\ t \in \mathbb{R}]$

$\ddot{x} + 4x = \sin 2t,\ x(0) = 0,\ \dot{x}(0) = 0.$ $[u(t; 0, (0, 0)) = \frac{1}{8}(\sin 2t - 2t\cos 2t)\,,\ t \in \mathbb{R}]$

$\ddot{x} + x = \cos t + \sin 2t,\ x(0) = 0,\ \dot{x}(0) = 0.$ $[u(t; 0, (0, 0)) = \frac{1}{6}((3t+4)\sin t - 2\sin 2t)\,,\ t \in \mathbb{R}]$

$\ddot{x} + x = \sin 2t,\ x(0) = 0,\ \dot{x}(0) = 0.$ $[u(t; 0, (0, 0)) = \frac{1}{3}(2\sin t - \sin 2t)\,,\ t \in \mathbb{R}]$

Najděte obecný tvar řešení $v(t; c_1, c_2)$ následujících rovnic.

$\ddot{x} + x = \operatorname{cotg} t.$ $\left[v(t; c_1, c_2) = c_1 \cos t + c_2 \sin t + \sin t \ln \left|\operatorname{tg} \frac{t}{2}\right|\right]$

$\ddot{x} - 6\dot{x} + 9x = \dfrac{9t^2 + 6t + 2}{t^3}.$ $\left[v(t; c_1, c_2) = c_1 \mathrm{e}^{3t} + c_2 t\mathrm{e}^{3t} + \frac{1}{t}\right]$

$\ddot{x} + x = \sin^2 t.$ $\left[v(t; c_1, c_2) = c_1 \cos t + c_2 \sin t + \frac{1}{3}(1 + \cos^2 t)\right]$

• Najděte obecné řešení Eulerovy rovnice pro $x > 0$

$x^2 y'' + 4xy' + 2y = 0.$ $[y = C_1 x^{-1} + C_2 x^{-2}]$

$x^2 y'' + 3xy' + y = 0.$ $[y = C_1 x^{-1} + C_2 x^{-1} \ln x]$

$x^2 y'' - xy' - 3y = 0.$ $[y = C_1 x^3 + C_2 x^{-1}]$

$x^2 y''' = 2y'.$ $[y = C_1 + C_2 \ln x + C_3 x^3]$

$x^3 y''' - 3x^2 y'' + 6xy' - 6y = 0.$ $[y = C_1 x + C_2 x^2 + C_3 x^3]$

$x^3 y''' + 2x^2 y'' - xy' + y = 0.$ $[y = C_1 x^{-1} + C_2 x + C_3 x \ln x]$

$x^4 y^{(4)} + 6x^3 y''' + 9x^2 y'' + 3xy' + y = 0.$ $[y = C_1 \cos \ln x + C_2 \sin \ln x + \ln x (C_3 \cos \ln x + C_4 \sin \ln x)]$

• Najděte obecné řešení diferenciální rovnice

$x^3 y'' - x^2 y' - 3xy + 16 \ln x = 0.$ $[C_1 x^3 + x^{-1}(C_2 + \ln x + 2\ln^2 x)]$

$x^2 y'' - xy' + y = 8x^3.$ $[y = x(C_1 + C_2 \ln x) + 2x^3]$

$x^2 y'' + xy' + 4y = 10x.$ $[y = C_1 \cos\left(\ln x^2\right) + C_2 \sin\left(\ln x^2\right) + 2x]$

$x^2y'' - 3xy' + 5y = 3x^2.$ $\left[y = x^2\big(C_1\cos(\ln x) + C_2\sin(\ln x) + 3\big)\right]$

$x^3y'' - 2xy + 6\ln x = 0.$ $\left[y = C_1x^2 + \dfrac{1}{x}\left(C_2 + \dfrac{2}{3}\ln x + \ln^2 x\right)\right]$

Nalezněte charakteristické hodnoty a charakteristické vektory soustavy $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$, kde $\mathbf{A}$ je matice:

$\mathbf{A} = \begin{pmatrix} 1, & 5 \\ 2, & 4 \end{pmatrix}$ $[\lambda_1 = 6\,,\ \mathbf{p} = (t,t)\,,\ \lambda_2 = -1\,,\ \mathbf{q} = (-5t, 2t)]$

$\mathbf{A} = \begin{pmatrix} 1, & -1 \\ 1, & -1 \end{pmatrix}$ $[\lambda_1 = \lambda_2 = 0\,,\ \mathbf{p} = (t,t)]$

$\mathbf{A} = \begin{pmatrix} 0, & 1 \\ -2, & 2 \end{pmatrix}$ $[\lambda_1 = \overline{\lambda}_2 = 1 + i\,,\ \mathbf{p} = (w, (1+i)w) = \overline{\mathbf{q}}]$

$\mathbf{A} = \begin{pmatrix} 0, & 1, & 0 \\ 0, & 0, & 1 \\ 0, & 1, & 0 \end{pmatrix}$ $[\lambda_1 = 0\,,\ \mathbf{p} = (t,0,0)\,,\ \lambda_2 = 1\,,\ \mathbf{q} = (t,t,t)\,,\ \lambda_3 = -1\,, \mathbf{r} = (t,-t,t)]$

$\mathbf{A} = \begin{pmatrix} 2, & -1, & -1 \\ 0, & -1, & 0 \\ 0, & 2, & 1 \end{pmatrix}$ $[\lambda_1 = 2\,,\ \mathbf{p} = (t,0,0)\,,\ \lambda_2 = 1\,,\ \mathbf{q} = (t,0,t)\,,\ \lambda_3 = -1\,,\ \mathbf{r} = (0,t,-t)]$

$\mathbf{A} = \begin{pmatrix} -1, & -2, & 2 \\ 0, & 1, & 0 \\ 0, & 0, & 1 \end{pmatrix}$ $[\lambda_1 = -1\,,\ \mathbf{p} = (t,0,0)\,,\ \lambda_2 = \lambda_3 = 1\,,\ \mathbf{q} = (t,0,t)\,,\ \mathbf{r} = (0,t,t)]$

$\mathbf{A} = \begin{pmatrix} 3, & 5, & 3 \\ -4, & -9, & -6 \\ 6, & 15, & 10 \end{pmatrix}$ $[\lambda_1 = 2\,,\ \mathbf{p} = (t,-2t,3t)\,,\ \lambda_2 = \lambda_3 = 1\,,\ \mathbf{q} = (3t,-6t,8t)\,,\ \mathbf{r} = (t,-t,t)]$

$\mathbf{A} = \begin{pmatrix} 7, & -12, & 6 \\ 10, & -19, & 10 \\ 12, & -24, & 13 \end{pmatrix}$ $[\lambda_1 = -1\,,\ \mathbf{p} = (3t,5t,6t)\,,\ \lambda_2 = \lambda_3 = 1\,,\ \mathbf{q} = (2t,t,0)\,,\ \mathbf{r} = (t,0,-t)]$

$\mathbf{A} = \begin{pmatrix} 1, & -1, & 0 \\ 0, & 1, & -4 \\ -1, & 0, & 4 \end{pmatrix}$ $[\lambda_1 = 0\,,\ \mathbf{p} = (4t,4t,t)\,,\ \lambda_2 = \lambda_3 = 3\,,\ \mathbf{q} = (t,-2t,t)]$

$\mathbf{A} = \begin{pmatrix} 4, & 4, & 0 \\ -1, & 0, & 0 \\ -2, & -4, & 2 \end{pmatrix}$ $[\lambda_1 = \lambda_2 = \lambda_3 = 2\,,\ \mathbf{p} = (2t,-t,0)\,,\ \mathbf{q} = (0,0,t)]$

$\mathbf{A} = \begin{pmatrix} 0, & 1, & 1 \\ 2, & 0, & -2 \\ 2, & 2, & 0 \end{pmatrix}$ $[\lambda_1 = \lambda_2 = \lambda_3 = 0\,,\ \mathbf{p} = (t,-t,t)]$

$\mathbf{A} = \begin{pmatrix} 13, & -28, & 3 \\ 4, & -8, & 1 \\ -1, & 4, & 1 \end{pmatrix}$ $[\lambda_1 = \lambda_2 = \lambda_3 = 2\,,\ \mathbf{p} = (2t,t,2t)]$

$\mathbf{A} = \begin{pmatrix} 2, & 1, & 0 \\ 1, & 3, & -1 \\ -1, & 2, & 3 \end{pmatrix}$ $[\lambda_1 = 2\,,\ \mathbf{p} = (t,0,t)\,,\ \lambda_2 = \overline{\lambda}_3 = 3 + i\,,\ \mathbf{q} = (w, (1+i)w, (2-i)w) = \overline{\mathbf{r}}]$

$\mathbf{A} = \begin{pmatrix} -7, & 2, & -1, & 0 \\ -23, & 3, & -8, & 1 \\ 10, & -2, & 3, & 0 \\ -24, & 4, & -7, & 1 \end{pmatrix}$ $[\lambda_1 = \lambda_2 = \overline{\lambda}_3 = \overline{\lambda}_4 = i\,,\ \mathbf{p} = (5w, 2(7+i)w, -(7+i)w, 15w) = \overline{\mathbf{q}}]$

Nalezněte fundamentální systém řešení soustavy $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$, kde $\mathbf{A}$ je matice

$\mathbf{A} = \begin{pmatrix} 0, & 1 \\ 12, & -1 \end{pmatrix}$ např. $\mathbf{u}_1 = (1,3)\mathrm{e}^{3t}$, $\mathbf{u}_2 = (1,-4)\mathrm{e}^{-4t}$, $t \in \mathbb{R}$]

$\mathbf{A} = \begin{pmatrix} 0, & 1 \\ -2, & 2 \end{pmatrix}$ např. $\mathbf{u}_1(t) = (\cos t, \cos t - \sin t)\mathrm{e}^t$, $\mathbf{u}_2(t) = (\sin t, \sin t + \cos t)\mathrm{e}^t$, $t \in \mathbb{R}$]

$\mathbf{A} = \begin{pmatrix} 7, & -18 \\ 3, & -8 \end{pmatrix}$ např. $\mathbf{u}_1(t) = (3\mathrm{e}^t - 2\mathrm{e}^{-2t}, \mathrm{e}^t - \mathrm{e}^{-2t})$, $\mathbf{u}_2(t) = (6\mathrm{e}^{-2t} - 6\mathrm{e}^t, 3\mathrm{e}^{-2t} - 2\mathrm{e}^t)$, $t \in \mathbb{R}$]

$\mathbf{A} = \begin{pmatrix} 2, & 1, & 0 \\ 1, & 3, & -1 \\ -1, & 2, & 3 \end{pmatrix}$ [např. $\mathbf{u}_1(t) = (\mathrm{e}^{2t}, 0, \mathrm{e}^{2t})$, $t \in \mathbb{R}$
$\mathbf{u}_2(t) = (\cos t, \cos t - \sin t, 2\cos t + \sin t)\mathrm{e}^{3t}$, $t \in \mathbb{R}$
$\mathbf{u}_3(t) = (\sin t, \cos t + \sin t, 2\sin t - \cos t)\mathrm{e}^{3t}$, $t \in \mathbb{R}$]

$\mathbf{A} = \begin{pmatrix} 0, & 1, & 0 \\ 0, & 0, & 1 \\ 0, & 1, & 0 \end{pmatrix}$ [např. $\mathbf{u}_1(t) = (1,0,0)$, $t \in \mathbb{R}$
$\mathbf{u}_2(t) = (1,1,1)\mathrm{e}^t$, $t \in \mathbb{R}$
$\mathbf{u}_3(t) = (1,-1,1)\mathrm{e}^{-t}$, $t \in \mathbb{R}$]

$\mathbf{A} = \begin{pmatrix} 0, & 1, & 0 \\ 0, & 0, & 1 \\ 0, & -1, & 0 \end{pmatrix}$ [např. $\mathbf{u}_1(t) = (1,0,0)$, $t \in \mathbb{R}$
$\mathbf{u}_2(t) = (\cos t, -\sin t, -\cos t)$, $t \in \mathbb{R}$
$\mathbf{u}_3(t) = (\sin t, \cos t, -\sin t)$, $t \in \mathbb{R}$]

Nalezněte obecný tvar řešení soustavy $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$, kde $\mathbf{A}$ je matice

$\mathbf{A} = \begin{pmatrix} -3, & 9 \\ -1, & 3 \end{pmatrix}$ [např. $\mathbf{u}(t, c_1, c_2) = (3c_1 + 2c_2 + 3c_2 t, c_1 + c_2 + c_2 t)$, $t \in \mathbb{R}$]

$\mathbf{A} = \begin{pmatrix} -7, & 9 \\ -1, & -1 \end{pmatrix}$ [např. $\mathbf{u}(t, c_1, c_2) = (3c_1 + 2c_2 + 3c_2 t, c_1 + c_2 + c_2 t)\mathrm{e}^{-4t}$, $t \in \mathbb{R}$]

$\mathbf{A} = \begin{pmatrix} 0, & 1, & 0 \\ -1, & 2, & 0 \\ 0, & 0, & 1 \end{pmatrix}$ [např. $\mathbf{u}(t, c_1, c_2, c_3) = (c_2 + c_3 t, c_2 + c_3 + c_3 t, c_1)\mathrm{e}^t$, $t \in \mathbb{R}$]

Nalezněte řešení následujících Cauchyových úloh.

$\dot{x}_1 = x_1 - 3x_2$, $x_1(0) = 1$
$\dot{x}_2 = 4x_1 - 6x_2$, $x_2(0) = 0$
[$\mathbf{u}(t;(1,0)) = (4\mathrm{e}^{-2t} - 3\mathrm{e}^{-3t}, 4\mathrm{e}^{-2t} - 4\mathrm{e}^{-3t})$, $t \in \mathbb{R}$]

$\dot{x}_1 = 2x_2$, $x_1(0) = -3$
$\dot{x}_2 = -5x_1 + 7x_2$, $x_2(0) = 6$
[$\mathbf{u}(t;(-3,6)) = (6\mathrm{e}^{5t} - 9\mathrm{e}^{2t}, 15\mathrm{e}^{5t} - 9\mathrm{e}^{2t})$, $t \in \mathbb{R}$]

$\dot{x}_1 = 7x_1 - 18x_2$, $x_1(0) = -1$
$\dot{x}_2 = 3x_1 - 8x_2$, $x_2(0) = 1$
[$\mathbf{u}(t;(-1,1)) = (8\mathrm{e}^{-2t} - 9\mathrm{e}^t, 4\mathrm{e}^{-2t} - 3\mathrm{e}^t)$, $t \in \mathbb{R}$]

$\dot{x}_1 = -3x_1$, $x_1(0) = 5$
$\dot{x}_2 = -3x_2$, $x_2(0) = -3$
[$\mathbf{u}(t;(5,-3)) = (5\mathrm{e}^{-3t}, -3\mathrm{e}^{-3t})$, $t \in \mathbb{R}$]

$\dot{x}_1 = 2x_1 + x_2$, $x_1(0) = -1$
$\dot{x}_2 = 2x_2$, $x_2(0) = 2$
[$\mathbf{u}(t;(-1,2)) = ((2t-1)\mathrm{e}^{2t}, 2\mathrm{e}^{2t})$, $t \in \mathbb{R}$]

$\dot{x}_1 = x_2$, $x_1(0) = 3$
$\dot{x}_2 = -x_1 - 2x_2$, $x_2(0) = 4$
[$\mathbf{u}(t;(3,4)) = ((3+7t)\mathrm{e}^{-t}, (4-7t)\mathrm{e}^{-t})$, $t \in \mathbb{R}$]

$\dot{x}_1 = x_2$, $x_1(0) = 2$
$\dot{x}_2 = -9x_1$, $x_2(0) = -3$
[$\mathbf{u}(t;(2,-3)) = (2\cos 3t - \sin 3t, -6\sin 3t - 3\cos 3t)$, $t \in \mathbb{R}$]

$\dot{x}_1 = x_1 + x_2$, $x_1(0) = -1$
$\dot{x}_2 = -x_1 + x_2$, $x_2(0) = 3$
[$\mathbf{u}(t;(-1,3)) = \big((3\sin t - \cos t)\mathrm{e}^t, (\sin t + 3\cos t)\mathrm{e}^t\big)$, $t \in \mathbb{R}$]

$\dot{x}_1 = 2x_1 - 3x_2$, $x_1(0) = 2$
$\dot{x}_2 = 3x_1 + 2x_2$, $x_2(0) = -1$
[$\mathbf{u}(t;(2,-1)) = \big((2\cos 3t + \sin 3t)\mathrm{e}^{2t}, (2\sin 3t - \cos 3t)\mathrm{e}^{2t}\big)$, $t \in \mathbb{R}$]

$$\begin{array}{ll} \dot{x}_1 = \phantom{x_1 +{}} x_2 + x_3\,, & x_1(0) = 3 \\ \dot{x}_2 = x_1 \phantom{{}+ x_2} + x_3\,, & x_2(0) = 0 \\ \dot{x}_3 = x_1 + x_2 \phantom{{}+ x_3}\,, & x_1(0) = -3 \end{array} \qquad [\mathbf{u}(t;(3,0,-3)) = (3\mathrm{e}^{-t}, 0, -3\mathrm{e}^{-t})\,,\ t \in \mathbb{R}]$$

$$\begin{array}{ll} \dot{x}_1 = 2x_1 + x_2 - 2x_3\,, & x_1(0) = 0 \\ \dot{x}_2 = -x_1\,, & x_2(0) = 1 \\ \dot{x}_3 = x_1 + x_2 - x_3\,, & x_3(0) = 1 \end{array} \qquad [\mathbf{u}(t;(0,1,1)) = (\cos t - \mathrm{e}^t, -\sin t + \mathrm{e}^t, \cos t)\,,\ t \in \mathbb{R}]$$

Najděte standardní fundamentální matici soustavy $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$, kde

$$\mathbf{A} = \begin{pmatrix} 7, & -18 \\ 3, & -8 \end{pmatrix} \qquad \left[\mathbf{U}(t) = \begin{pmatrix} 3\mathrm{e}^t - 2\mathrm{e}^{-2t} & 6\mathrm{e}^{-2t} - 6\mathrm{e}^t \\ \mathrm{e}^t - \mathrm{e}^{-2t} & 3\mathrm{e}^{-2t} - 2\mathrm{e}^t \end{pmatrix}\,,\ t \in \mathbb{R}\right]$$

$$\mathbf{A} = \begin{pmatrix} 0, & 3 \\ -3, & 0 \end{pmatrix} \qquad \left[\mathbf{U}(t) = \begin{pmatrix} \cos 3t & \sin 3t \\ -\sin 3t & \cos 3t \end{pmatrix}\,,\ t \in \mathbb{R}\right]$$

$$\mathbf{A} = \begin{pmatrix} 1, & 1 \\ -1, & 1 \end{pmatrix} \qquad \left[\mathbf{U}(t) = \begin{pmatrix} \mathrm{e}^t \cos t & \mathrm{e}^t \sin t \\ -\mathrm{e}^t \sin t & \mathrm{e}^t \cos t \end{pmatrix}\,,\ t \in \mathbb{R}\right]$$

$$\mathbf{A} = \begin{pmatrix} 0, & 1, & 1 \\ 1, & 0, & 1 \\ 1, & 1, & 0 \end{pmatrix} \qquad \left[\mathbf{U}(t) = \frac{1}{3} \begin{pmatrix} \mathrm{e}^{2t} + 2\mathrm{e}^{-t} & \mathrm{e}^{2t} - \mathrm{e}^{-t} & \mathrm{e}^{2t} - \mathrm{e}^{-t} \\ \mathrm{e}^{2t} - \mathrm{e}^{-t} & \mathrm{e}^{2t} + 2\mathrm{e}^{-t} & \mathrm{e}^{2t} - \mathrm{e}^{-t} \\ \mathrm{e}^{2t} - \mathrm{e}^{-t} & \mathrm{e}^{2t} - \mathrm{e}^{-t} & \mathrm{e}^{2t} + 2\mathrm{e}^{-t} \end{pmatrix}\,,\ t \in \mathbb{R}\right]$$

$$\mathbf{A} = \begin{pmatrix} -1, & 1, & 1 \\ 1, & -1, & 1 \\ 1, & 1, & 1 \end{pmatrix}$$

$$\left[\mathbf{U}(t) = \frac{1}{6} \begin{pmatrix} 2\mathrm{e}^{-t} + 3\mathrm{e}^{-2t} + \mathrm{e}^{2t} & \mathrm{e}^{2t} + 2\mathrm{e}^{-t} - 3\mathrm{e}^{-2t} & 2\mathrm{e}^{2t} - 2\mathrm{e}^{-t} \\ \mathrm{e}^{2t} + 2\mathrm{e}^{-t} - 3\mathrm{e}^{-2t} & \mathrm{e}^{2t} + 2\mathrm{e}^{-t} + 3\mathrm{e}^{-2t} & 2\mathrm{e}^{2t} - 2\mathrm{e}^{-t} \\ 2\mathrm{e}^{2t} - 2\mathrm{e}^{-t} & 2\mathrm{e}^{2t} - 2\mathrm{e}^{-t} & 4\mathrm{e}^{2t} + 2\mathrm{e}^{-t} \end{pmatrix}\,,\ t \in \mathbb{R}\right]$$

$$\mathbf{A} = \begin{pmatrix} 2, & 1, & -2 \\ -1, & 0, & 0 \\ 1, & 1, & -1 \end{pmatrix} \qquad \left[\mathbf{U}(t) = \begin{pmatrix} \mathrm{e}^t + \sin t & \sin t & \cos t - \sin t - \mathrm{e}^t \\ \cos t - \mathrm{e}^t & \cos t & \mathrm{e}^t - \sin t - \cos t \\ \sin t & \sin t & \cos t - \sin t \end{pmatrix}\,,\ t \in \mathbb{R}\right]$$

$$\mathbf{A} = \begin{pmatrix} 4, & 3, & 2, & -3 \\ 6, & 9, & 4, & -8 \\ -3, & -4, & -1, & 4 \\ 9, & 9, & 6, & -8 \end{pmatrix}$$

$$\left[\mathbf{U}(t) = \mathrm{e}^t \begin{pmatrix} 1 + 3t - 3t^2 & 3t - t^2 & 2t - 2t^2 & -3t + t^2 \\ 6t - 9t^2 & 1 + 8t - 3t^2 & 4t - 6t^2 & -8t + 3t^2 \\ -3t + \frac{9}{2}t^2 & -4t + \frac{3}{2}t^2 & 1 - 2t + 3t^2 & 4t - \frac{3}{2}t^2 \\ 9t - 9t^2 & 9t - 3t^2 & 6t - 6t^2 & 1 - 9t + 3t^2 \end{pmatrix}\,,\ t \in \mathbb{R}\right]$$

$$\mathbf{A} = \begin{pmatrix} -13, & 5, & 4, & 2 \\ 0, & -1, & 0, & 0 \\ -30, & 12, & 9, & 5 \\ -12, & 6, & 4, & 1 \end{pmatrix} \qquad \left[\mathbf{U}(t) = \mathrm{e}^{-t} \begin{pmatrix} 1 - 12t & 5t & 4t & 2t \\ 0 & 1 & 0 & 0 \\ -30t & 12t & 1 + 10t & 5t \\ -12t & 6t & 4t & 1 + 2t \end{pmatrix}\,,\ t \in \mathbb{R}\right]$$

Řešte soustavu rovnic

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 + \frac{1}{\cos t} \end{aligned} \qquad \left[\begin{aligned} x_1 &= (t + C_1)\sin t + (C_2 + \ln|\cos t|)\cos t \\ x_2 &= (t + C_1)\cos t - (C_2 + \ln|\cos t|)\sin t \end{aligned}\right]$$

$$\begin{aligned} \dot{x}_1 &= -x_2 + \operatorname{tg} t \\ \dot{x}_2 &= x_1 + \operatorname{tg}^2 t - 1 \end{aligned} \qquad \left[\begin{aligned} x_1 &= 2 + C_1 \cos t + C_2 \sin t \\ x_2 &= \operatorname{tg} t + C_1 \sin t - C_2 \cos t \end{aligned}\right]$$

$$\begin{aligned} \dot{x}_1 &= 2x_1 + 3x_2 + 4\mathrm{e}^{-t} \\ \dot{x}_2 &= 3x_1 + 2x_2 + 5t \end{aligned} \qquad \left[\begin{aligned} x_1 &= C_1\mathrm{e}^{5t} + C_2\mathrm{e}^{-t} + \frac{12}{5} - 3t + 2t\mathrm{e}^{-t} \\ x_2 &= C_1\mathrm{e}^{5t} - C_2\mathrm{e}^{-t} - \frac{13}{5} + 2t - \frac{2}{3}\,\mathrm{e}^{-t} - 2t\mathrm{e}^{-t} \end{aligned}\right]$$

$$\begin{aligned}\dot{x}_1 - 6x_1 + 16x_2 &= 48\cos 2t\\ \dot{x}_2 - x_1 + 2x_2 &= 6\cos 2t - 2\sin 2t\end{aligned}$$

$$\left[\begin{gathered}x_1 = C_1\mathrm{e}^{2t} + C_2 t\mathrm{e}^{2t} - 8\cos 2t\\ x_2 = -\sin 2t + \frac{1}{4}(C_1 + C_2 t)\mathrm{e}^{2t} - \frac{1}{16}C_2\mathrm{e}^{2t}\end{gathered}\right]$$

$$\begin{aligned}\dot{x}_1 + 2x_1 - 2x_2 &= t\\ \dot{x}_2 - 3x_1 + x_2 &= \mathrm{e}^t\end{aligned}$$

$$\left[\begin{gathered}x_1 = C_1\mathrm{e}^t + C_2\mathrm{e}^{-4t} - \frac{t}{4} - \frac{7}{16} + \frac{2}{5}t\mathrm{e}^t\\ x_2 = \frac{1}{2}\left(3C_1\mathrm{e}^t - 2C_2\mathrm{e}^{-4t} - \frac{3}{2}t - \frac{9}{8} + \frac{2}{5}(1+3t)\mathrm{e}^t\right)\end{gathered}\right]$$

$$\begin{aligned}\dot{x}_1 + 2x_1 - x_2 &= t\\ \dot{x}_2 - 3x_1 + 4x_2 &= 0\end{aligned}$$

$$\left[\begin{gathered}x_1 = C_1\mathrm{e}^{-t} + C_2\mathrm{e}^{-5t} + \frac{4}{5}t - \frac{19}{25}\\ x_2 = C_1\mathrm{e}^{-t} - 3C_2\mathrm{e}^{-5t} + \frac{3}{5}t - \frac{18}{25}\end{gathered}\right]$$

$$\begin{aligned}\dot{x}_1 + 2x_1 - 3x_2 &= t\\ -3x_1 + \dot{x}_2 + 2x_2 &= \mathrm{e}^{2t}\end{aligned}$$

$$\left[\begin{gathered}x_1 = C_1\mathrm{e}^t + C_2\mathrm{e}^{-5t} - \frac{2}{5}t - \frac{13}{25} + \frac{3}{7}\mathrm{e}^{2t}\\ x_2 = C_1\mathrm{e}^t - C_2\mathrm{e}^{-5t} - \frac{12}{25} - \frac{3}{5}t + \frac{4}{7}\mathrm{e}^{2t}\end{gathered}\right]$$

$$\begin{aligned}\dot{x}_1 + n^2 x_2 &= \cos nt\\ \dot{x}_2 + n^2 x_1 &= \sin nt\end{aligned}$$

$$\left[\begin{gathered}x_1 = C_1\exp\left(n^2 t\right) + C_2\exp\left(-n^2 t\right) + \frac{1+n}{n(1+n^2)}\sin nt\\ x_2 = -C_1\exp\left(n^2 t\right) + C_2\exp\left(-n^2 t\right) + \frac{n-1}{n(1+n^2)}\cos nt\end{gathered}\right]$$

$$\begin{aligned}\dot{x}_1 + x_1 + 2x_2 &= \cos t + \sin t + \mathrm{e}^{-t}\\ \dot{x}_2 - 2x_1 + x_2 &= \sin t - \cos t\end{aligned}$$

$$\left[\begin{gathered}x_1 = \mathrm{e}^{-t}(C_1\cos 2t + C_2\sin 2t) + \cos t\\ x_2 = \mathrm{e}^{-t}\left(C_1\sin 2t - C_2\cos 2t + \frac{1}{2}\right) + \sin t\end{gathered}\right]$$

$$\begin{aligned}\dot{x}_1 &= 2x_2 - x_1\\ \dot{x}_2 &= 4x_2 - 3x_1 + \frac{\mathrm{e}^{3t}}{\mathrm{e}^{2t}+1}\end{aligned}$$

$$\left[\begin{gathered}x_1 = C_1\mathrm{e}^t + 2C_2\mathrm{e}^{2t} - \mathrm{e}^t\ln(\mathrm{e}^{2t}+1) + 2\mathrm{e}^{2t}\operatorname{arctg}\mathrm{e}^t\\ x_2 = C_1\mathrm{e}^t + 3C_2\mathrm{e}^{2t} - \mathrm{e}^t\ln(\mathrm{e}^{2t}+1) + 3\mathrm{e}^{2t}\operatorname{arctg}\mathrm{e}^t\end{gathered}\right]$$

$$\begin{aligned}\dot{x}_1 &= -4x_1 - 2x_2 + \frac{2}{\mathrm{e}^t - 1}\\ \dot{x}_2 &= 6x_1 + 3x_2 - \frac{3}{\mathrm{e}^t - 1}\end{aligned}$$

$$\left[\begin{gathered}x_1 = C_1 + 2C_2\mathrm{e}^{-t} + 2\mathrm{e}^{-t}\ln|\mathrm{e}^t - 1|\\ x_2 = -2C_1 - 3C_2\mathrm{e}^{-t} - 3\mathrm{e}^{-t}\ln|\mathrm{e}^t - 1|\end{gathered}\right]$$

$$\begin{aligned}\dot{x}_1 &= 3x_1 - 2x_2\\ \dot{x}_2 &= 2x_1 - x_2 + 15\mathrm{e}^t\sqrt{t}\end{aligned}$$

$$\left[\begin{gathered}x_1 = \left(C_1 + 2C_2 t - 8t^{5/2}\right)\mathrm{e}^t\\ x_2 = \left(C_1 + 2C_2 t - C_2 - 8t^{5/2} + 10t^{3/2}\right)\mathrm{e}^t\end{gathered}\right]$$

Řešte soustavu

$$\begin{aligned}\dot{x}_1 + \dot{x}_2 &= 2x_2\\ 3\dot{x}_1 + \dot{x}_2 &= x_1 + 9x_2\end{aligned}$$

$$\left[\begin{gathered}x_1 = \left((-2C_1 - \sqrt{3}C_2)\cos\frac{\sqrt{3}}{2}t + (\sqrt{3}C_1 - 2C_2)\sin\frac{\sqrt{3}}{2}t\right)\mathrm{e}^{-t/2}\\ x_2 = \left(C_1\cos\frac{\sqrt{3}}{2}t + C_2\sin\frac{\sqrt{3}}{2}t\right)\mathrm{e}^{-t/2}\end{gathered}\right]$$

$$\begin{aligned}\ddot{x}_1 &= x_2\\ \ddot{x}_2 &= x_1\end{aligned}$$

$$\left[\begin{gathered}x_1 = C_1\mathrm{e}^t + C_2\mathrm{e}^{-t} + C_3\cos t + C_4\sin t\\ x_2 = C_1\mathrm{e}^t + C_2\mathrm{e}^{-t} - C_3\cos t - C_4\sin t\end{gathered}\right]$$

$$\begin{aligned}\dot{x}_1 + 2x_1 - \dot{x}_2 + x_2 &= 0\\ 5\dot{x}_1 + \dot{x}_2 + 3x_2 &= 0\end{aligned}$$

$$\left[\begin{gathered}x_1 = (3C_1 - C_2)\cos t - (C_1 + 3C_2)\sin t\\ x_2 = 5C_1\sin t + 5C_2\cos t\end{gathered}\right]$$

$$\begin{aligned}\ddot{x}_1 - x_1 + 4x_2 &= 0\\ \ddot{x}_2 + x_1 - x_2 &= 0\end{aligned}$$

$$\left[\begin{gathered}x_1 = 2C_1\mathrm{e}^{\sqrt{3}t} + 2C_2\mathrm{e}^{-\sqrt{3}t} + 2C_3\cos t + 2C_4\sin t\\ x_2 = -C_1\mathrm{e}^{\sqrt{3}t} - C_2\mathrm{e}^{\sqrt{3}t} + C_3\cos t + C_4\sin t\end{gathered}\right]$$

$$\begin{aligned}\ddot{x}_1 - 3x_1 - 4x_2 &= 0\\ x_1 + \ddot{x}_2 + x_2 &= 0\end{aligned}$$

$$\left[\begin{gathered}x_1 = (C_1 + C_2 t)\mathrm{e}^t + (C_3 + C_4 t)\mathrm{e}^{-t}\\ x_2 = \frac{1}{2}\left((C_2 - C_1 - C_2 t)\mathrm{e}^t - (C_3 + C_4 + C_4 t)\mathrm{e}^{-t}\right)\end{gathered}\right]$$

$$\begin{aligned}\ddot{x}_1 + 3\ddot{x}_2 - x_1 &= 0\\ \dot{x}_1 + 3\dot{x}_2 - 2x_2 &= 0\end{aligned}$$

$$\left[\begin{gathered}x_1 = C_1\mathrm{e}^{t/2} + C_2\mathrm{e}^{-2t}\\ x_2 = C_1\mathrm{e}^{t/2} - \frac{1}{4}C_2\mathrm{e}^{-2t}\end{gathered}\right]$$

$$\begin{aligned} \ddot{x}_1 - 2\dot{x}_2 + 2x_1 &= 0 \\ \ddot{x}_2 + 3\dot{x}_1 - 8x_2 &= 0 \end{aligned} \qquad \left[\begin{aligned} x_1 &= 2C_1\mathrm{e}^{2t} + 2C_2\mathrm{e}^{-2t} + 2C_3\cos 2t + 2C_4\sin 2t \\ x_2 &= 3C_1\mathrm{e}^{2t} - 3C_2\mathrm{e}^{-2t} - C_3\sin 2t + C_4\cos 2t \end{aligned}\right]$$

$$\begin{aligned} \ddot{x}_1 &= 3x_1 - x_2 - x_3 \\ \ddot{x}_2 &= -x_1 + 3x_2 - x_3 \\ \ddot{x}_3 &= -x_1 - x_2 + 3x_3 \end{aligned} \qquad \left[\begin{aligned} x_1 &= C_1\mathrm{e}^{t} + C_2\mathrm{e}^{-t} + C_3\mathrm{e}^{2t} + C_5\mathrm{e}^{-2t} \\ x_2 &= C_1\mathrm{e}^{t} + C_2\mathrm{e}^{-t} + C_4\mathrm{e}^{2t} + C_6\mathrm{e}^{-2t} \\ x_2 &= C_1\mathrm{e}^{t} + C_2\mathrm{e}^{-t} - (C_3 + C_4)\mathrm{e}^{2t} - (C_5 + C_6)\mathrm{e}^{-2t} \end{aligned}\right]$$

Řešte soustavu

$$\begin{aligned} \dddot{x}_2 + 3\ddot{x}_1 - t^2 &= 0 \\ \dot{x}_2 - \dot{x}_1 + 2x_1 &= 0 \end{aligned} \qquad \left[\begin{aligned} x_1 &= C_1\mathrm{e}^{-t} + C_2 + C_3 t + t^2 - \frac{1}{3}t^3 + \frac{1}{12}t^4 \\ x_2 &= 3C_1\mathrm{e}^{-t} + (C_3 - 2C_2)t + (1 - C_3)t^2 - t^3 + \frac{t^4}{4} - \frac{t^5}{30} + C_4 \end{aligned}\right]$$

$$\begin{aligned} 2\ddot{x}_2 - \dot{x}_1 - 4x_2 &= 2t \\ 4\dot{x}_1 + 2\dot{x}_2 - 3x_1 &= 0 \end{aligned} \qquad \left[\begin{aligned} x_1 &= \mathrm{e}^{t}(C_1 + C_2 t) + C_3\mathrm{e}^{-3t/2} - \frac{1}{3} \\ x_2 &= \mathrm{e}^{t}\left(-\frac{1}{2}C_1 - \frac{3}{2}C_2 - \frac{1}{2}C_2 t\right) - 3C_3\mathrm{e}^{-3t/2} - \frac{1}{2}t \end{aligned}\right]$$

$$\begin{aligned} \dot{x}_2 - \dot{x}_3 + x_1\sqrt{3} &= \sin t \\ -\dot{x}_1 + \dot{x}_3 + x_2\sqrt{3} &= \sin t \\ \dot{x}_1 - \dot{x}_2 + x_3\sqrt{3} &= \sin t \end{aligned} \qquad \left[\begin{aligned} x_1 &= C_1\cos t + C_2\sin t \\ x_2 &= \frac{1}{2}\left((C_2\sqrt{3} - C_1 - 1)\cos t + (\sqrt{3} - C_1\sqrt{3} - C_2)\sin t\right) \\ x_3 &= -\frac{1}{2}\left((C_1 + C_2\sqrt{3} - 1)\cos t + (C_2 - \sqrt{3} - C_1\sqrt{3})\sin t\right) \end{aligned}\right]$$
