Exercises 1 – indefinite integrals

Determine the integral $\int \frac{\sqrt{x}-2\sqrt[3]{x^2}+1}{\sqrt[4]{x}}\,\mathrm{d}x$. $\left[\frac{4}{5}\,x^{5/4}-\frac{24}{17}\,x^{17/12}+\frac{4}{3}\,x^{3/4}\,.\right]$

Determine the integral $\int \frac{(1-x)^3}{x\sqrt[3]{x}}\,\mathrm{d}x$. $\left[-3x^{-1/3}-\frac{9}{2}\,x^{2/3}+\frac{9}{5}\,x^{5/3}-\frac{3}{8}\,x^{8/3}\,.\right]$

Determine the integral $\int \frac{(x-1)(x+2)}{x^2}\sqrt{\sqrt{x}}\,\mathrm{d}x$. $\left[\frac{4}{5}\,x^{5/4}+4x^{1/4}+\frac{8}{3}\,x^{-3/4}\,.\right]$

Determine the integral $\int \frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1-x^4}}\,\mathrm{d}x$. $\left[\arcsin x+\ln\left(x+\sqrt{1+x^2}\right)\,.\right]$

Determine the integral $\int \frac{2^{2x+1}-5^{x-2}}{10^{x-1}}\,\mathrm{d}x$. $\left[\frac{20}{\ln(2/5)}\left(\frac{2}{5}\right)^x+\frac{2}{5\ln 2}\,\frac{1}{2^x}\,.\right]$

Determine the integral $\int \operatorname{cotg}^2 x\,\mathrm{d}x$. $\left[-x-\operatorname{cotg} x\,.\right]$

Determine the integral $\int \operatorname{tgh}^2 x\,\mathrm{d}x$. $\left[x-\operatorname{tgh} x\,.\right]$

Determine the integral $\int \sin 2x\sin 3x\,\mathrm{d}x$. $\left[\frac{1}{2}\,\sin x-\frac{1}{10}\,\sin 5x\,.\right]$

Determine the integral $\int \sin 2x\cos 5x\,\mathrm{d}x$. $\left[\frac{1}{6}\,\cos 3x-\frac{1}{14}\,\cos 7x\,.\right]$

Determine the integral $\int \cos 4x\cos 3x\,\mathrm{d}x$. $\left[\frac{1}{2}\,\sin x+\frac{1}{14}\,\sin 7x\,.\right]$

Determine the integral $\int \frac{\ln^2 x}{x^2}\,\mathrm{d}x$. $\left[-\frac{\ln^2 x+2\ln x+2}{x}\,.\right]$

Determine the integral $\int \sqrt{x}\,\ln^3 x\,\mathrm{d}x$. $\left[x^{3/2}\left(\frac{2}{3}\,\ln^3 x-\frac{4}{3}\,\ln^2 x+\frac{16}{9}\,\ln x-\frac{32}{27}\right)\,.\right]$

Determine the integral $\int \ln\left(x+\sqrt{1+x^2}\right)\,\mathrm{d}x$. $\left[x\ln\left(x+\sqrt{1+x^2}\right)-\sqrt{1+x^2}\,.\right]$

Determine the integral $\int \ln\left|\frac{x}{2-x}\right|\,\mathrm{d}x$. $\left[x\ln|x|+(2-x)\ln|2-x|\,.\right]$

Determine the integral $\int \ln\sqrt{1-x^2}\,\mathrm{d}x$. $\left[\frac{1}{2}\,(1+x)\ln(1+x)-\frac{1}{2}\,(1-x)\ln(1-x)-x\,.\right]$

Determine the integral $\int (2x+1)\ln\sqrt{x+1}\,\mathrm{d}x$. $\left[x(x+1)\ln\sqrt{x+1}-\frac{1}{4}\,x^2\,.\right]$

Determine the integral $\int x\ln\frac{1-x}{1+x}\,\mathrm{d}x$. $\left[\frac{x^2-1}{2}\,\ln\frac{1-x}{1+x}-x\,.\right]$

Determine the integral $\int (x^2+x)\mathrm{e}^{-4x+1}\,\mathrm{d}x$. $\left[-\left(\frac{1}{4}\,x^2+\frac{3}{8}\,x-\frac{3}{32}\right)\mathrm{e}^{-4x+1}\,.\right]$

Determine the integral $\int (x-2x^2)\cos 3x$. $\left[\frac{1}{27}\left(4+9x-18x^2\right)\sin 3x+\frac{1}{9}\left(1-4x\right)\cos 3x\,.\right]$

Determine the integral $\int (x^2+x+1)\sin\frac{1}{2}\,x\,\mathrm{d}x$. $\left[2(7-x-x^2)\cos\frac{1}{2}\,x+4(2x+1)\sin\frac{1}{2}\,x\,.\right]$

Determine the integral $\int \mathrm{e}^{-3x}\sin 2x\,\mathrm{d}x$. $\left[-\frac{1}{13}\,\mathrm{e}^{-3x}(2\cos 2x+3\sin 2x)\,.\right]$

Determine the integral $\int \mathrm{e}^{-4x}\cos 3x\,\mathrm{d}x$. $\left[\frac{1}{25}\,\mathrm{e}^{-4x}(3\sin 3x-4\cos 3x)\,.\right]$

Determine the integral $\int \arcsin 2x\,\mathrm{d}x$. $\left[x\arcsin 2x+\frac{1}{2}\,\sqrt{1-4x^2}\,.\right]$

Determine the integral $\int \arccos\frac{x}{3}\,\mathrm{d}x$. $\left[x\arccos\frac{1}{3}\,x-\frac{1}{3}\,\sqrt{9-x^3}\,.\right]$

Determine the integral $\int (x-1)\operatorname{arctg} x\,\mathrm{d}x$. $\left[\frac{1}{2}\,(x-1)^2\operatorname{arctg} x+\frac{1}{2}\,\ln(1+x^2)-\frac{1}{2}\,x\,.\right]$

Determine the integral $\int \frac{\ln(1+x^2)}{x^2}\,\mathrm{d}x$. $\left[\frac{\ln(1+x^2)}{x}+2\operatorname{arctg} x\,.\right]$

Determine the integral $\int \frac{\mathrm{d}x}{\sqrt{1+2x-x^2}}$. $\left[\arcsin\frac{1}{\sqrt{2}}\,(x-1)\,.\right]$

Determine the integral $\int \frac{\mathrm{d}x}{x+\sqrt[3]{x^2}}$. $\left[3\ln\left(1+x^{1/3}\right)\,.\right]$

Determine the integral $\int \frac{1+\sqrt{x+1}}{\sqrt{x+1}-1}\,\mathrm{d}x$. $\left[x+4\sqrt{x+1}+4\ln\left|\sqrt{1+x}-1\right|\,.\right]$

Determine the integral $\int \operatorname{tg}\frac{x}{2}\,\mathrm{d}x$. $\left[-2\ln\left|\cos\frac{1}{2}\,x\right|\,.\right]$

Determine the integral $\int \operatorname{cotg} 3x\,\mathrm{d}x$. $\left[\frac{1}{3}\ln\left|\sin 3x\right|\,.\right]$

Determine the integral $\int \frac{\mathrm{d}x}{\cosh x}$. $\left[2\operatorname{arctg}\mathrm{e}^x\,.\right]$

Determine the integral $\int \sin^5 x\,\mathrm{d}x$. $\left[-\cos x+\frac{2}{3}\,\cos^3 x-\frac{1}{5}\,\cos^5 x\,.\right]$

Determine the integral $\int \frac{\sin x-\cos x}{\sin x+\cos x}\,\mathrm{d}x$. $\left[-\ln\left|\sin x+\cos x\right|\,.\right]$

Determine the integral $\int \frac{\mathrm{d}x}{\mathrm{e}^x+\sqrt{\mathrm{e}^x}}$. $\left[2\ln\left(1+\mathrm{e}^{-x/2}\right)-2\mathrm{e}^{-x/2}\,.\right]$

Determine the integral $\int \cos\sqrt{x}\,\mathrm{d}x$. $\left[2\left(\sqrt{x}\sin\sqrt{x}+\cos\sqrt{x}\right)\,.\right]$

Determine the integral $\int \sqrt{x}\,\mathrm{e}^{\sqrt{x}}\,\mathrm{d}x$. $\left[2\left(x+2-2\sqrt{x}\right)\mathrm{e}^{\sqrt{x}}\,.\right]$

Determine the integral $\int \frac{\sin^3 x}{\cos^4 x}\,\mathrm{d}x$. $\left[-\frac{1}{\cos x}+\frac{1}{3\cos^3 x}\,.\right]$

Determine the integral $\int \frac{\sin^2 x}{\cos^4 x}\,\mathrm{d}x$. $\left[\frac{1}{3}\ \mathrm{tg}^3\, x\,.\right]$

Determine the integral $\int (x^3+x)\mathrm{e}^{-x^2}\,\mathrm{d}x$. $\left[-\left(\frac{1}{2}\,x^2+1\right)\mathrm{e}^{-x^2}\,.\right]$

Determine the integral $\int \mathrm{arctg}\,\sqrt{x}\,\mathrm{d}x$. $\left[(x+1)\,\mathrm{arctg}\,\sqrt{x}-\sqrt{x}\,.\right]$

Determine the integral $\int \mathrm{e}^{\sin x}\sin 2x\,\mathrm{d}x$. $\left[2(\sin x-1)\mathrm{e}^{\sin x}\,.\right]$

Determine the integral $\int \frac{x^3+1}{x^2+x-2}\,\mathrm{d}x$. $\left[\frac{1}{2}\,x^2-x+\frac{2}{3}\ln|x-1|+\frac{7}{3}\ln|x+2|\,.\right]$

Determine the integral $\int \frac{x(x^2+1)}{x^2+x-6}\,\mathrm{d}x$. $\left[\frac{1}{2}\,x^2-x+2\ln|x-2|+6\ln|x+3|\,.\right]$

Determine the integral $\int \frac{x^2+x+1}{x^2+2x-3}\,\mathrm{d}x$. $\left[x+\frac{3}{4}\ln|x-1|-\frac{7}{4}\ln|x+3|\,.\right]$

Determine the integral $\int \frac{(x+2)\,\mathrm{d}x}{(x+1)^2(x-1)}$. $\left[\frac{3}{4}\ln\left|\frac{x-1}{x+1}\right|+\frac{1}{2(x+1)}\,.\right]$

Determine the integral $\int \frac{(x^2+1)\,\mathrm{d}x}{(x+2)^2(x-1)}$. $\left[\frac{2}{9}\ln|x-1|+\frac{7}{9}\ln|x+2|+\frac{5}{3(x+2)}\,.\right]$

Determine the integral $\int \frac{x(x+2)\,\mathrm{d}x}{(x-1)^2(x+1)}$. $\left[\frac{5}{4}\ln|x-1|-\frac{1}{4}\ln|x+1|-\frac{3}{2(x-1)}\,.\right]$

Determine the integral $\int \frac{(2x^2+1)\,\mathrm{d}x}{(x-2)^2(x+1)}$. $\left[\frac{1}{3}\ln|x+1|+\frac{5}{3}\ln|x-2|-\frac{3}{x-2}\,.\right]$

Determine the integral $\int \frac{(3x+8)\,\mathrm{d}x}{(x+2)^2(x+1)}$. $\left[5\ln\left|\frac{x+1}{x+2}\right|+\frac{2}{x+2}\,.\right]$

Determine the integral $\int \frac{x^2+3x+1}{(x-2)^2(x+2)}\,\mathrm{d}x$. $\left[\frac{17}{16}\ln|x-2|-\frac{1}{16}\ln|x+2|-\frac{11}{4(x-2)}\,.\right]$

Determine the integral $\int \frac{(2x^2+3)\,\mathrm{d}x}{(x-2)^2(x-1)}$. $\left[5\ln|x-1|-3\ln|x-2|-\frac{11}{x-2}\,.\right]$

Determine the integral $\int \frac{(4x+3)\,\mathrm{d}x}{(x+2)^2(x-3)}$. $\left[\frac{3}{5}\ln\left|\frac{x-3}{x+2}\right|-\frac{1}{x+2}\,.\right]$

Determine the integral $\int \frac{2x^2+8x-2}{(x-1)^2(x+3)}\,\mathrm{d}x$. $\left[\frac{5}{2}\ln|x-1|-\frac{1}{2}\ln|x+3|-\frac{2}{x-1}\,.\right]$

Determine the integral $\int \frac{x^2+3x+6}{(x+1)^2(x-3)}\,\mathrm{d}x$. $\left[\frac{3}{2}\ln|x-3|-\frac{1}{2}\ln|x+1|+\frac{1}{x+1}\,.\right]$

Determine the integral $\int \frac{x^2+4x+8}{(x+3)(x-1)^2}\,\mathrm{d}x$. $\left[\frac{5}{16}\ln|x+3|+\frac{11}{16}\ln|x-1|-\frac{13}{4(x-1)}\,.\right]$

Determine the integral $\displaystyle\int \frac{2x^2+3x+1}{(x+2)(x-1)^2}\,\mathrm{d}x.$ $\left[\frac{1}{3}\ln|x+2|+\frac{5}{3}\ln|x-1|-\frac{2}{x-1}\,.\right]$

Determine the integral $\displaystyle\int \frac{x^2+3x+2}{(x-3)(x-2)^2}\,\mathrm{d}x.$ $\left[20\ln|x-3|-19\ln|x-2|+\frac{12}{x-2}\,.\right]$

Determine the integral $\displaystyle\int \frac{(7x+2)\,\mathrm{d}x}{(x+3)(x+2)^2}.$ $\left[19\ln\left|\frac{x+2}{x+3}\right|+\frac{12}{x+2}\,.\right]$

Determine the integral $\displaystyle\int \frac{(x+2)\,\mathrm{d}x}{x^3+2x^2+x}.$ $\left[2\ln\left|\frac{x}{x+1}\right|+\frac{1}{x+1}\,.\right]$

Determine the integral $\displaystyle\int \frac{(x-3)\,\mathrm{d}x}{x^3-2x^2+x}.$ $\left[3\ln\left|\frac{x-1}{x}\right|+\frac{2}{x-1}\,.\right]$

Determine the integral $\displaystyle\int \frac{x^2+3x+1}{(x^2+2x-3)(x+1)}\,\mathrm{d}x.$ $\left[\frac{5}{8}\ln|x-1|+\frac{1}{4}\ln|x+1|+\frac{1}{8}\ln|x+3|\,.\right]$

Determine the integral $\displaystyle\int \frac{x^2+x+1}{(x^2+x-2)(x-1)}\,\mathrm{d}x.$ $\left[\frac{1}{3}\ln|x+2|+\frac{2}{3}\ln|x-1|-\frac{1}{x-1}\,.\right]$

Determine the integral $\displaystyle\int \frac{(x^2+1)\,\mathrm{d}x}{(x+1)(x^2-1)}.$ $\left[\frac{1}{2}\ln\left|x^2-1\right|+\frac{1}{x+1}\,.\right]$

Determine the integral $\displaystyle\int \frac{x\,\mathrm{d}x}{(x-1)(x^2-1)}.$ $\left[\frac{1}{4}\ln\left|\frac{x-1}{x+1}\right|-\frac{1}{2(x-1)}\,.\right]$

Determine the integral $\displaystyle\int \frac{\mathrm{d}x}{(x^2+4x+3)(x-1)}.$ $\left[\frac{1}{8}\ln\left|\frac{(x-1)(x+3)}{(x+1)^2}\right|.\right]$

Determine the integral $\displaystyle\int \frac{(x+1)^2\,\mathrm{d}x}{(x-1)(x^2-4x+3)}.$ $\left[4\ln|x-3|-3\ln|x-1|+\frac{2}{x-1}\,.\right]$

Determine the integral $\displaystyle\int \frac{x\,\mathrm{d}x}{(x^2-3x+2)(x+3)}.$ $\left[\frac{2}{5}\ln|x-2|-\frac{1}{4}\ln|x-1|-\frac{3}{20}\ln|x+3|\,.\right]$

Determine the integral $\displaystyle\int \frac{x^2+x+1}{(x-1)(x^2+2x-3)}\,\mathrm{d}x.$ $\left[\frac{7}{16}\ln|x+3|+\frac{9}{16}\ln|x-1|-\frac{3}{4(x-1)}\,.\right]$

Determine the integral $\displaystyle\int \frac{\mathrm{d}x}{(x+1)(x^2+4x+3)}.$ $\left[\frac{1}{4}\ln\left|\frac{x+3}{x+1}\right|-\frac{1}{2(x+1)}\,.\right]$

Determine the integral $\displaystyle\int \frac{(x+2)\,\mathrm{d}x}{(x-2)(x^2-4x+3)}.$ $\left[\frac{3}{2}\ln|x-1|-4\ln|x-2|+\frac{5}{2}\ln|x-3|\,.\right]$

Determine the integral $\displaystyle\int \frac{\mathrm{d}x}{(x-1)(x^2+x-2)}.$ $\left[\frac{1}{9}\ln\left|\frac{x+2}{x-1}\right|-\frac{1}{3(x-1)}\,.\right]$

Determine the integral $\displaystyle\int \frac{(3x+1)\,\mathrm{d}x}{(x+1)(x^2+3x+2)}.$ $\left[5\ln\left|\frac{x+1}{x+2}\right|+\frac{2}{x+1}\,.\right]$

Determine the integral $\displaystyle\int \frac{\mathrm{d}x}{(x-2)(x^2-6x+8)}.$ $\left[\frac{1}{4}\ln\left|\frac{x-4}{x-2}\right|+\frac{1}{2(x-2)}\,.\right]$

Determine the integral $\int \frac{(4x-5)\,\mathrm{d}x}{(x-2)(x^2-x-2)}$. $\left[\ln\left|\frac{x-2}{x+1}\right| - \frac{1}{x-2}\,.\right]$

Determine the integral $\int \frac{(x+1)\,\mathrm{d}x}{x^2+4x+13}$. $\left[\ln\sqrt{x^2+4x+13} - \frac{1}{3}\operatorname{arctg}\frac{1}{3}(x+2)\,.\right]$

Determine the integral $\int \frac{x\,\mathrm{d}x}{x^2-2x+2}$. $\left[\ln\sqrt{x^2-2x+2} + \operatorname{arctg}(x-1)\,.\right]$

Determine the integral $\int \frac{(2x+3)\,\mathrm{d}x}{x^2-6x+13}$. $\left[\ln\left(x^2-6x+13\right) + \frac{9}{2}\operatorname{arctg}\frac{1}{2}(x-3)\,.\right]$

Determine the integral $\int \frac{x\,\mathrm{d}x}{x^2+x+2}$. $\left[\ln\sqrt{x^2+x+2} - \frac{1}{\sqrt{7}}\operatorname{arctg}\frac{1}{\sqrt{7}}(2x+1)\,.\right]$

Determine the integral $\int \frac{(2x-1)\,\mathrm{d}x}{x^2-4x+8}$. $\left[\ln\left(x^2-4x+8\right) + \frac{3}{2}\operatorname{arctg}\frac{1}{2}(x-2)\,.\right]$

Determine the integral $\int \frac{(2x-3)\,\mathrm{d}x}{x^2+6x+25}$. $\left[\ln\left(x^2+6x+25\right) - \frac{9}{4}\operatorname{arctg}\frac{1}{4}(x+3)\,.\right]$

Determine the integral $\int \frac{2\,\mathrm{d}x}{(1+x)(1+x^2)}$. $\left[\ln|x+1| - \ln\sqrt{x^2+1} + \operatorname{arctg} x\,.\right]$

Determine the integral $\int \frac{\mathrm{d}x}{2-\mathrm{e}^x-\mathrm{e}^{2x}}$. $\left[\frac{1}{2}x - \frac{1}{3}\ln\left|\mathrm{e}^x-1\right| - \frac{1}{6}\ln\left|\mathrm{e}^x+2\right|\,.\right]$

Determine the integral $\int \frac{4\mathrm{e}^x\,\mathrm{d}x}{(\mathrm{e}^x+1)(\mathrm{e}^{2x}-1)}$. $\left[\ln\left|\frac{\mathrm{e}^x-1}{\mathrm{e}^x+1}\right| + \frac{2}{\mathrm{e}^x+1}\,.\right]$

Determine the integral $\int \frac{(2\mathrm{e}^x+3)\,\mathrm{d}x}{\mathrm{e}^{2x}-\mathrm{e}^x-6}$. $\left[\frac{3}{5}\ln\left|\mathrm{e}^x-3\right| - \frac{1}{10}\ln\left|\mathrm{e}^x+2\right| - \frac{1}{2}x\,.\right]$

Determine the integral $\int \frac{2\mathrm{e}^x+3}{\mathrm{e}^{2x}+2\mathrm{e}^x}\,\mathrm{d}x$. $\left[-\frac{1}{4}\ln\left|1+2\mathrm{e}^{-x}\right| + \frac{3}{2}\mathrm{e}^{-x}\,.\right]$

Determine the integral $\int \frac{\mathrm{e}^x(\mathrm{e}^x+1)\,\mathrm{d}x}{\mathrm{e}^{2x}+4\mathrm{e}^x+5}$. $\left[\ln\sqrt{\mathrm{e}^{2x}+4\mathrm{e}^x+5} - \operatorname{arctg}\left(\mathrm{e}^x+2\right)\,.\right]$

Determine the integral $\int \frac{(\ln x+4)\,\mathrm{d}x}{x(\ln x+2)(\ln^2 x-4)}$. $\left[\frac{3}{8}\ln\left|\frac{\ln x-2}{\ln x+2}\right| + \frac{1}{2(\ln x+2)}\,.\right]$

Determine the integral $\int \frac{\mathrm{d}x}{x\ln x(2\ln^2 x+3\ln x-2)}$.

$\left[\frac{1}{10}\ln|\ln x+2| + \frac{2}{5}\ln|2\ln x-1| - \frac{1}{2}\ln|\ln x|\,.\right]$

Determine the integral $\int \frac{(\ln x+2)\,\mathrm{d}x}{x(\ln^2 x+2\ln x+5)}$. $\left[\ln\sqrt{\ln^2 x+2\ln x+5} + \frac{1}{2}\operatorname{arctg}(\ln x+1)\,.\right]$

Determine the integral $\int \frac{\sin x+\sin 2x}{1+\cos x+\sin^2 x}\,\mathrm{d}x$. $\left[\frac{1}{3}\ln(1+\cos x) + \frac{5}{3}\ln(2-\cos x)\,.\right]$

Determine the integral $\int \frac{\mathrm{d}x}{2\cos x + \sin 2x}$. $\left[\frac{1}{8}\ln\left|\frac{1+\sin x}{1-\sin x}\right| - \frac{1}{4(1+\sin x)}\,.\right]$

Determine the integral $\int \frac{\mathrm{d}x}{1 + 2\sin x\cos x - 4\cos^2 x}$. $\left[\frac{1}{4}\ln\left|\frac{\operatorname{tg} x - 1}{\operatorname{tg} x + 3}\right|.\right]$

Determine the integral $\int \frac{\ln x\,\mathrm{d}x}{(x+1)^2}$. $\left[\ln\left|\frac{x}{x+1}\right| - \frac{\ln x}{x+1}\,.\right]$

Determine the integral $\int \frac{\operatorname{arctg} x}{x^2}\,\mathrm{d}x$. $\left[\ln\frac{|x|}{\sqrt{x^2+1}} - \frac{\operatorname{arctg} x}{x}\,.\right]$
