Exercise 2 – definite integrals

Determine the integral $\displaystyle\int_0^\pi x|\sin 2x|\,\mathrm{d}x$. $\left[\,\pi\,.\right]$

Determine the integral $\displaystyle\int_0^\pi x|\cos x|\,\mathrm{d}x$. $\left[\,\pi\,.\right]$

Determine the integral $\displaystyle\int_{e^{-1}}^{e} x|\ln x|\,\mathrm{d}x$. $\left[\frac{1}{4}\left(e^2+2-3e^{-2}\right).\right]$

Determine the integral $\displaystyle\int_0^4 xe^{|2x-4|}\,\mathrm{d}x$. $\left[2\left(e^4-1\right).\right]$

Determine the integral $\displaystyle\int_0^2 |1-x|e^{-x}\,\mathrm{d}x$. $\left[2\left(e^{-1}-e^{-2}\right).\right]$

Determine the integral $\displaystyle\int_0^\pi \sqrt{1+\sin 2x}\,\mathrm{d}x$. $\left[2\sqrt{2}\,.\right]$

Determine the integral $\displaystyle\int_0^\pi \sqrt{1+\cos 2x}\,\mathrm{d}x$. $\left[2\sqrt{2}\,.\right]$

Determine the integral $\displaystyle\int_0^{\pi/4} \operatorname{tg}^2 x\,\mathrm{d}x$. $\left[1-\frac{1}{4}\pi\,.\right]$

Determine the integral $\displaystyle\int_0^{\pi/2} \sin x \sin 3x\,\mathrm{d}x$. $\left[0\,.\right]$

Determine the integral $\displaystyle\int_0^{\pi/2} \cos 2x \sin 3x\,\mathrm{d}x$. $\left[\frac{3}{5}\,.\right]$

Determine the integral $\displaystyle\int_0^{\pi/2} \cos 2x \cos 5x\,\mathrm{d}x$. $\left[\frac{5}{21}\,.\right]$

Determine the integral $\displaystyle\int_0^{2\pi} \sin^2 2x\,\mathrm{d}x$. $\left[\,\pi\,.\right]$

Determine the integral $\displaystyle\int_{-\pi}^{\pi} \cos^2 3x\,\mathrm{d}x$. $\left[\,\pi\,.\right]$

Determine the integral $\displaystyle\int_1^e \frac{\ln^2 x}{x^3}\,\mathrm{d}x$. $\left[\frac{1}{4}\left(1-5e^{-2}\right).\right]$

Determine the integral $\displaystyle\int_1^{e^2} \sqrt{x}\ln^2 x\,\mathrm{d}x$. $\left[\frac{8}{27}\left(5e^3-2\right).\right]$

Determine the integral $\displaystyle\int_0^1 (3x-2)\ln\sqrt{x+1}\,\mathrm{d}x$. $\left[\frac{11}{8}-2\ln 2\,.\right]$

Determine the integral $\displaystyle\int_0^1 x^3\ln^2 x\,\mathrm{d}x$. $\left[\frac{1}{32}\,.\right]$

Determine the integral $\displaystyle\int_0^1 x(x-1)e^{-2x}\,\mathrm{d}x$. $\left[-\frac{1}{2}e^{-2}\,.\right]$

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Determine the integral $\int_0^\pi (x^2 - \pi^2) \sin 2x \, dx.$	$\left[-\frac{1}{2} \pi^2 . \right]$
Determine the integral $\int_{-\pi/2}^{\pi/2} x(2x - \pi) \cos 2x \, dx.$	$\left[-\pi . \right]$
Determine the integral $\int_0^\pi e^{-3x} \sin 2x \, dx.$	$\left[\frac{2}{13} \left(1 - e^{-3\pi} \right) . \right]$
Determine the integral $\int_{-\pi/2}^{\pi/2} e^{2x} \cos 3x \, dx.$	$\left[-\frac{3}{13} \left(e^\pi + e^{-\pi} \right) . \right]$
Determine the integral $\int_0^\infty e^{-3x} \cos 4x \, dx.$	$\left[\frac{3}{25} . \right]$
Determine the integral $\int_{-\infty}^0 e^{2x} \sin 4x \, dx.$	$\left[-\frac{1}{5} . \right]$
Determine the integral $\int_0^{\sqrt{3}} \operatorname{arctg} x \, dx.$	$\left[\frac{\sqrt{3}}{3} \pi - \ln 2 . \right]$
Determine the integral $\int_{-1}^1 x \operatorname{arctg} x \, dx.$	$\left[\frac{1}{2} \pi - 1 . \right]$
Determine the integral $\int_0^1 \arcsin x \, dx.$	$\left[\frac{1}{2} \pi - 1 . \right]$
Determine the integral $\int_{-2}^2 \arccos \frac{x}{2} \, dx.$	$\left[2\pi . \right]$
Using mathematical induction, show that for each $n \in \mathbb{N}$ it is $\int_0^\infty x^n e^{-x} \, dx = n!.$	
Determine the integral $\int_{-1/2}^{3/2} \frac{dx}{\sqrt{3 + 4x - 4x^2}}.$	$\left[\frac{1}{2} \pi . \right]$
Determine the integral $\int_2^4 \frac{dx}{\sqrt{x(4 - x)}}.$	$\left[\frac{1}{2} \pi . \right]$
Determine the integral $\int_0^3 \frac{1 - \sqrt{1 + x}}{1 + \sqrt{1 + x}} \, dx.$	$\left[1 - 4 \ln \frac{3}{2} . \right]$
Determine the integral $\int_0^\infty x e^{-\sqrt{x}} \, dx.$	$\left[12 . \right]$
Determine the integral $\int_0^1 (x - x^3) e^{x^2} \, dx.$	$\left[\frac{1}{2} e - 1 . \right]$
Determine the integral $\int_{-1}^0 \frac{e^{1/x}}{x^3} \, dx.$	$\left[-2e^{-1} . \right]$
Determine the integral $\int_1^9 x \sqrt[3]{x - 1} \, dx.$	$\left[\frac{468}{7} . \right]$

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Determine the integral $\displaystyle\int_0^\infty \frac{\mathrm{d}x}{\cosh x}$. $\qquad$ $\left[\frac{1}{2}\pi\,.\right]$

Determine the integral $\displaystyle\int_0^{\ln 2} \sqrt{\mathrm{e}^x - 1}\,\mathrm{d}x$. $\qquad$ $\left[2 - \frac{1}{2}\pi\,.\right]$

Determine the integral $\displaystyle\int_0^{\mathrm{e}^{-1}} \frac{1 + \ln x}{x \ln^3 x}\,\mathrm{d}x$. $\qquad$ $\left[\frac{1}{2}\,.\right]$

Determine the integral $\displaystyle\int_1^{\mathrm{e}} \frac{\ln^2 x - 1}{x\sqrt{\ln x}}\,\mathrm{d}x$. $\qquad$ $\left[-\frac{8}{5}\,.\right]$

Determine the integral $\displaystyle\int_0^{\pi/4} \frac{\sin^3 x}{\cos^4 x}\,\mathrm{d}x$. $\qquad$ $\left[\frac{1}{3}\left(2 - \sqrt{2}\right).\right]$

Determine the integral $\displaystyle\int_0^{\pi/4} \frac{\sin^2 x}{\cos^4 x}\,\mathrm{d}x$. $\qquad$ $\left[\frac{1}{3}\,.\right]$

Determine the integral $\displaystyle\int_0^{\pi} \mathrm{e}^{-\cos x} \sin 2x\,\mathrm{d}x$. $\qquad$ $\left[-4\mathrm{e}^{-1}\,.\right]$

Determine the integral $\displaystyle\int_0^1 \frac{x^3\,\mathrm{d}x}{x^2 - x - 2}$. $\qquad$ $\left[\frac{3}{2} - \frac{7}{3}\ln 2\,.\right]$

Determine the integral $\displaystyle\int_{-1}^1 \frac{x(x^2 + 1)}{x^2 + x - 6}\,\mathrm{d}x$. $\qquad$ $\left[6\ln 2 - 2\ln 3 - 2\,.\right]$

Determine the integral $\displaystyle\int_{-2}^0 \frac{x^2 + x + 1}{3 - 2x - x^2}\,\mathrm{d}x$. $\qquad$ $\left[\frac{5}{2}\ln 3 - 2\,.\right]$

Determine the integral $\displaystyle\int_2^\infty \frac{(2x + 1)\,\mathrm{d}x}{(x + 1)(x - 1)^2}$. $\qquad$ $\left[\frac{3}{2} + \frac{1}{4}\ln 3\,.\right]$

Determine the integral $\displaystyle\int_0^1 \frac{(x^2 + 1)\,\mathrm{d}x}{(x - 2)(x + 1)^2}$. $\qquad$ $\left[-\frac{1}{3} - \frac{1}{9}\ln 2\,.\right]$

Determine the integral $\displaystyle\int_2^3 \frac{x(x + 2)\,\mathrm{d}x}{(x + 1)(x - 1)^2}$. $\qquad$ $\left[\frac{9}{8} + \frac{3}{4}\ln 2 + \frac{1}{4}\ln 3\,.\right]$

Determine the integral $\displaystyle\int_0^1 \frac{(2x^2 + 1)\,\mathrm{d}x}{(x + 1)(x - 2)^2}$. $\qquad$ $\left[\frac{3}{2} - \frac{4}{3}\ln 2\,.\right]$

Determine the integral $\displaystyle\int_0^\infty \frac{(3x + 8)\,\mathrm{d}x}{(x + 1)(x + 2)^2}$. $\qquad$ $\left[5\ln 2 - 1\,.\right]$

Determine the integral $\displaystyle\int_0^1 \frac{(3x + 8)\,\mathrm{d}x}{(x + 1)(x + 2)^2}$. $\qquad$ $\left[5\ln\frac{4}{3} - \frac{1}{3}\,.\right]$

Determine the integral $\displaystyle\int_{-1}^1 \frac{x^2 + x + 1}{(x + 2)(x - 2)^2}\,\mathrm{d}x$. $\qquad$ $\left[\frac{7}{6} - \frac{5}{8}\ln 3\,.\right]$

Determine the integral $\displaystyle\int_0^2 \frac{(4x + 3)\,\mathrm{d}x}{(x - 3)(x + 2)^2}$. $\qquad$ $\left[\frac{1}{4} - \frac{3}{5}\ln 6\,.\right]$

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Determine the integral $\displaystyle\int_3^\infty \frac{(4x-3)\,\mathrm{d}x}{(x+3)(x-2)^2}$.	$\left[1+\frac{3}{5}\ln 6\,.\right]$
Determine the integral $\displaystyle\int_{-1}^0 \frac{2x^2+8x+1}{(x+3)(x-1)^2}\,\mathrm{d}x$.	$\left[\frac{11}{8}-2\ln 2-\frac{5}{16}\ln 3\,.\right]$
Determine the integral $\displaystyle\int_0^2 \frac{x^2+3x+4}{(x-3)(x+1)^2}\,\mathrm{d}x$.	$\left[-\frac{1}{3}-\frac{7}{4}\ln 3\,.\right]$
Determine the integral $\displaystyle\int_{-2}^0 \frac{x^2+4x+8}{(x+3)(x-1)^2}\,\mathrm{d}x$.	$\left[\frac{13}{6}-\frac{3}{8}\ln 3\,.\right]$
Determine the integral $\displaystyle\int_{-1}^0 \frac{2x^2+3x+1}{(x+2)(x-1)^2}\,\mathrm{d}x$.	$\left[1-\frac{4}{3}\ln 2\,.\right]$
Determine the integral $\displaystyle\int_0^1 \frac{(x+1)(x+2)}{(x-3)(x-2)^2}\,\mathrm{d}x$.	$\left[39\ln 2-20\ln 3-6\,.\right]$
Determine the integral $\displaystyle\int_0^\infty \frac{(7x+2)\,\mathrm{d}x}{(x+3)(x+2)^2}$.	$\left[19\ln\frac{3}{2}-6\,.\right]$
Determine the integral $\displaystyle\int_1^\infty \frac{(x-2)\,\mathrm{d}x}{x^3+2x^2+x}$.	$\left[\frac{3}{2}-\ln 4\,.\right]$
Determine the integral $\displaystyle\int_{-\infty}^{-1} \frac{(x-3)\,\mathrm{d}x}{x^3-2x^2+x}$.	$\left[\ln 8-1\,.\right]$
Determine the integral $\displaystyle\int_0^1 \frac{x^2+3x+1}{(x+1)(x^2+x-6)}\,\mathrm{d}x$.	$\left[-\frac{11}{30}\ln 2-\frac{1}{10}\ln 3\,.\right]$
Determine the integral $\displaystyle\int_{-1}^0 \frac{x^2+x+1}{(x-1)(x^2+x-2)}\,\mathrm{d}x$.	$\left[\frac{1}{2}-\ln 2\,.\right]$
Determine the integral $\displaystyle\int_{-1}^1 \frac{(x^2+4)\,\mathrm{d}x}{(x+2)(x^2-4)}$.	$\left[-\frac{4}{3}\,.\right]$
Determine the integral $\displaystyle\int_2^\infty \frac{x\,\mathrm{d}x}{(x-1)(x^2-1)}$.	$\left[\frac{1}{2}+\frac{1}{4}\ln 3\,.\right]$
Determine the integral $\displaystyle\int_0^1 \frac{\mathrm{d}x}{(x-2)(x^2+4x+3)}$.	$\left[-\frac{1}{30}\ln 2-\frac{1}{10}\ln 3\,.\right]$
Determine the integral $\displaystyle\int_2^\infty \frac{\mathrm{d}x}{(x-1)(x^2+4x+3)}$.	$\left[\frac{1}{8}\ln\frac{9}{5}\,.\right]$
Determine the integral $\displaystyle\int_{-1}^0 \frac{(x+1)^2\,\mathrm{d}x}{(x-1)(x^2-4x+3)}$.	$\left[4\ln 3-5\ln 2-1\,.\right]$
Determine the integral $\displaystyle\int_3^\infty \frac{x\,\mathrm{d}x}{(x+3)(x^2-3x+2)}$.	$\left[\frac{1}{4}\ln 2+\frac{3}{20}\ln 6\,.\right]$
Determine the integral $\displaystyle\int_0^1 \frac{x^2-x+1}{(x+1)(x^2-2x-3)}\,\mathrm{d}x$.	$\left[\ln 2-\frac{7}{16}\ln 3-\frac{3}{8}\,.\right]$

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Determine the integral $\displaystyle\int_0^\infty \frac{\mathrm{d}x}{(x+1)(x^2+4x+3)}$. $\qquad\left[\frac{1}{2}-\frac{1}{4}\ln 3\,.\right]$

Determine the integral $\displaystyle\int_{-\infty}^0 \frac{(x+2)\,\mathrm{d}x}{(x-2)(x^2-4x+3)}$. $\qquad\left[\frac{5}{2}\ln 3-4\ln 2\,.\right]$

Determine the integral $\displaystyle\int_2^\infty \frac{\mathrm{d}x}{(x-1)(x^2+x-2)}$. $\qquad\left[\frac{1}{3}-\frac{1}{9}\ln 4\,.\right]$

Determine the integral $\displaystyle\int_0^\infty \frac{(3x+1)\,\mathrm{d}x}{(x+1)(x^2+3x+2)}$. $\qquad\left[5\ln 2-2\,.\right]$

Determine the integral $\displaystyle\int_{-\infty}^0 \frac{\mathrm{d}x}{(x-2)(x^2-6x+8)}$. $\qquad\left[\frac{1}{4}\ln 2-\frac{1}{4}\,.\right]$

Determine the integral $\displaystyle\int_0^1 \frac{x^2-x+1}{(x-2)(x^2-6x+8)}\,\mathrm{d}x$. $\qquad\left[\frac{13}{4}\ln 3-\frac{17}{4}\ln 2-\frac{3}{4}\,.\right]$

Determine the integral $\displaystyle\int_0^1 \frac{(4x-5)\,\mathrm{d}x}{(x-2)(x^2-x-2)}$. $\qquad\left[\frac{1}{2}-\ln 4\,.\right]$

Determine the integral $\displaystyle\int_0^2 \frac{(x-1)\,\mathrm{d}x}{x^2-4x+16}$. $\qquad\left[\frac{1}{2}\ln\frac{3}{4}+\frac{1}{12\sqrt{3}}\,\pi\,.\right]$

Determine the integral $\displaystyle\int_{-1}^1 \frac{x\,\mathrm{d}x}{x^2-2x+5}$. $\qquad\left[tfrac18\,\pi-\frac{1}{2}\ln 2\,.\right]$

Determine the integral $\displaystyle\int_0^1 \frac{(2x+3)\,\mathrm{d}x}{x^2-6x+13}$. $\qquad\left[\ln\frac{8}{13}-\frac{9}{8}\,\pi+\frac{9}{8}\arctan\frac{3}{2}\,.\right]$

Determine the integral $\displaystyle\int_{-1}^1 \frac{x\,\mathrm{d}x}{x^2+x+1}$. $\qquad\left[\frac{1}{2}\ln 3-\frac{1}{2\sqrt{3}}\,\pi\,.\right]$

Determine the integral $\displaystyle\int_0^2 \frac{(2x-1)\,\mathrm{d}x}{x^2-4x+8}$. $\qquad\left[\frac{3}{8}\,\pi-\ln 2\,.\right]$

Determine the integral $\displaystyle\int_{-1}^1 \frac{(2x-3)\,\mathrm{d}x}{x^2+6x+25}$. $\qquad\left[\ln\frac{8}{5}-\frac{9}{16}\,\pi+\frac{9}{4}\arctan\frac{1}{2}\,.\right]$

Determine the integral $\displaystyle\int_0^\infty \frac{2\,\mathrm{d}x}{(x+1)(x^2+1)}$. $\qquad\left[\frac{1}{2}\,\pi\,.\right]$

Determine the integral $\displaystyle\int_{-\infty}^{-1} \frac{(x+1)\,\mathrm{d}x}{x(x^2+1)}$. $\qquad\left[\frac{1}{4}\,\pi-\frac{1}{2}\ln 2\,.\right]$

Determine the integral $\displaystyle\int_{-1}^1 \frac{(3x-1)\,\mathrm{d}x}{(x+2)(x^2+3)}$. $\qquad\left[\frac{1}{3\sqrt{3}}\,\pi-\ln 3\,.\right]$

Determine the integral $\displaystyle\int_1^\infty \frac{\mathrm{d}x}{\mathrm{e}^{2x}+\mathrm{e}^x-2}$. $\qquad\left[\frac{1}{2}-\frac{1}{3}\ln(\mathrm{e}-1)-\frac{1}{6}\ln(\mathrm{e}+2)\,.\right]$

Determine the integral $\displaystyle\int_{\ln 2}^\infty \frac{4\mathrm{e}^x\,\mathrm{d}x}{(\mathrm{e}^x+1)(\mathrm{e}^{2x}-1)}$. $\qquad\left[\ln 3-\frac{2}{3}\,.\right]$

Determine the integral $\displaystyle\int_0^{\ln 2} \frac{(2\mathrm{e}^x + 3)\,\mathrm{d}x}{\mathrm{e}^{2x} - \mathrm{e}^x - 6}.$ $\qquad$ $\left[\frac{1}{10}\ln 3 - \frac{13}{10}\ln 2\,.\right]$

Determine the integral $\displaystyle\int_0^\infty \frac{2\mathrm{e}^x + 3}{\mathrm{e}^{2x} + 2\mathrm{e}^x}\,\mathrm{d}x.$ $\qquad$ $\left[\frac{3}{2} + \frac{1}{2}\ln 3\,.\right]$

Determine the integral $\displaystyle\int_{-\infty}^\infty \frac{\mathrm{e}^x\,\mathrm{d}x}{\mathrm{e}^{2x} - 2\mathrm{e}^x + 4}.$ $\qquad$ $\left[\frac{2}{3\sqrt{3}}\,\pi\,.\right]$

Determine the integral $\displaystyle\int_0^{\ln 2} \frac{\mathrm{e}^{2x} + \mathrm{e}^x}{\mathrm{e}^{2x} - 4\mathrm{e}^x + 5}\,\mathrm{d}x.$ $\qquad$ $\left[\frac{3}{4}\,\pi - \frac{1}{2}\ln 2\,.\right]$

Determine the integral $\displaystyle\int_1^{\mathrm{e}} \frac{(\ln x + 4)\,\mathrm{d}x}{x(\ln x + 2)(\ln^2 x - 4)}.$ $\qquad$ $\left[-\frac{1}{12} - \frac{3}{8}\ln 3\,.\right]$

Determine the integral $\displaystyle\int_{\mathrm{e}}^\infty \frac{\mathrm{d}x}{x\ln x(2\ln^2 x + 3\ln x - 2)}.$ $\qquad$ $\left[\frac{2}{5}\ln 2 - \frac{1}{10}\ln 3\,.\right]$

Determine the integral $\displaystyle\int_{\mathrm{e}^{-1}}^{\mathrm{e}} \frac{(\ln x + 2)\,\mathrm{d}x}{x(\ln^2 x + 2\ln x + 5)}.$ $\qquad$ $\left[\frac{1}{2}\ln 2 + \frac{1}{8}\,\pi\,.\right]$

Determine the integral $\displaystyle\int_0^{\pi/2} \frac{\sin x + \sin 2x}{1 + \cos x + \sin^2 x}\,\mathrm{d}x.$ $\qquad$ $\left[\frac{4}{3}\ln 2\,.\right]$

Determine the integral $\displaystyle\int_0^{\pi/4} \frac{\mathrm{d}x}{2\cos x + \sin 2x}.$ $\qquad$ $\left[\frac{1}{4}\ln(\sqrt{2}+1) + \frac{1}{4}\left(\sqrt{2}-1\right).\right]$

Determine the integral $\displaystyle\int_{-\pi/4}^{\pi/4} \frac{\mathrm{d}x}{1 + 5\sin x\cos x + 5\cos^2 x}.$ $\qquad$ $\left[\ln\frac{3}{2}\,.\right]$

Determine the integral $\displaystyle\int_0^1 \frac{\ln x}{(x+1)^2}\,\mathrm{d}x.$ $\qquad$ $\left[-\ln 2\,.\right]$

Determine the integral $\displaystyle\int_1^\infty \frac{\ln x}{(x+1)^2}\,\mathrm{d}x.$ $\qquad$ $\left[\ln 2\,.\right]$

Determine the integral $\displaystyle\int_1^\infty \frac{\operatorname{arctg} x}{x^2}\,\mathrm{d}x.$ $\qquad$ $\left[\frac{1}{4}\,\pi + \ln 2\,.\right]$