Exercises 3a – Multiple Integrals

Express the integral $\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y$, where the region $\Omega\subset\mathbb{R}^2$ is given by the inequalities

$$2x+y+2>0\,,\qquad 2x+3y<2\,,\qquad y>0\,,$$

in the terms of two simple integrals in at least one integration order.

$$\left[\begin{aligned}\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y &= \int_0^2 \mathrm{d}y \int_{-\frac12(y+2)}^{\frac12(2-3y)} f(x,y)\,\mathrm{d}x = \\ &= \int_{-2}^{-1}\mathrm{d}x\int_{-2(x+1)}^{\frac23(1-x)} f(x,y)\,\mathrm{d}y + \int_{-1}^{1}\mathrm{d}x\int_0^{\frac23(1-x)} f(x,y)\,\mathrm{d}y\,.\end{aligned}\right]$$

Express the integral $\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y$, where $\Omega\subset\mathbb{R}^2$ is a finite region bounded by the lines

$$-\tfrac12\,x+\tfrac14\,y=1\,,\qquad \tfrac13\,x+\tfrac14\,y=1\,,\qquad y=0\,,$$

in the terms of two simple integrals in at least one integration order.

$$\left[\begin{aligned}\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y &= \int_0^4 \mathrm{d}y \int_{\frac12(y-4)}^{\frac34(4-y)} f(x,y)\,\mathrm{d}x = \\ &= \int_{-2}^{0}\mathrm{d}x\int_{0}^{2(2+x)} f(x,y)\,\mathrm{d}y + \int_{0}^{3}\mathrm{d}x\int_0^{\frac43(3-x)} f(x,y)\,\mathrm{d}y\,.\end{aligned}\right]$$

Express the integral $\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y$, where the region $\Omega\subset\mathbb{R}^2$ is given by the inequalities

$$3x+y+1<0\,,\qquad x<2y+2\,,\qquad x+1>0\,,$$

in the terms of two simple integrals in at least one integration order.

$$\left[\begin{aligned}\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y &= \int_{-1}^0 \mathrm{d}x \int_{\frac12(x-2)}^{-3x-1} f(x,y)\,\mathrm{d}y = \\ &= \int_{-\frac32}^{-1} dy\int_{-1}^{2(y+1)} f(x,y)\,\mathrm{d}x + \int_{-1}^{2}\mathrm{d}y\int_{-1}^{-\frac13(y+1)} f(x,y)\,\mathrm{d}x\,,\end{aligned}\right]$$

Express the integral $\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y$, where $\Omega\subset\mathbb{R}^2$ is a finite region bounded by the lines

$$-\tfrac12\,x-\tfrac15\,y=1\,,\qquad \tfrac14\,x-\tfrac15\,y=1\,,\qquad y=0\,,$$

in the terms of two simple integrals in at least one integration order.

$$\left[\begin{aligned}\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y &= \int_{-5}^0 \mathrm{d}y \int_{-\frac25(y+5)}^{\frac45(y+5)} f(x,y)\,\mathrm{d}x = \\ &= \int_{-2}^{0}\mathrm{d}x\int_{-\frac52(2+x)}^{0} f(x,y)\,\mathrm{d}y + \int_{0}^{4}\mathrm{d}x\int_{\frac54(x-4)}^{0} f(x,y)\,\mathrm{d}y\,,\end{aligned}\right]$$

Express the integral $\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y$, where the region $\Omega\subset\mathbb{R}^2$ is given by the inequalities

$$2y<x+1\,,\qquad x<y+1\,,\qquad 0<y<1\,,$$

in the terms of two simple integrals in at least one integration order.

$$\left[\begin{aligned}\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y &= \int_0^1 \mathrm{d}y\int_{2y-1}^{y+1} f(x,y)\,\mathrm{d}x = \\ &= \int_{-1}^1 \mathrm{d}x\int_0^{\frac12(x+1)} f(x,y)\,\mathrm{d}y + \int_1^2 \mathrm{d}x\int_{x-1}^1 f(x,y)\,\mathrm{d}y\,.\end{aligned}\right]$$

Express the integral $\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y$, where $\Omega\subset\mathbb{R}^2$ is a finite region bounded by the lines

$$-\tfrac12\,x+\tfrac15\,y=1\,,\qquad -\tfrac12\,x-\tfrac13\,y=1\,,\qquad x=0\,,$$

in the terms of two simple integrals in at least one integration order.

$$\left[\begin{aligned}\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y &= \int_{-2}^0 \mathrm{d}x\int_{-\frac32(x+2)}^{\frac52(x+2)} f(x,y)\,\mathrm{d}y = \\ &= \int_{-3}^0 \mathrm{d}y\int_{-\frac23(y+3)}^{0} f(x,y)\,\mathrm{d}x + \int_0^5 \mathrm{d}y\int_{\frac25(y-5)}^{0} f(x,y)\,\mathrm{d}x\,,\end{aligned}\right]$$

Express the integral $\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y$, where the region $\Omega\subset\mathbb{R}^2$ is given by the inequalities

$$6y<x+2\,,\qquad x+2y<2\,,\qquad y>0\,,$$

in the terms of two simple integrals in at least one integration order.

$$\left[\begin{aligned}\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y &= \int_0^{\frac12} \mathrm{d}y\int_{6y-2}^{2-2y} f(x,y)\,\mathrm{d}x = \\ &= \int_{-2}^1 \mathrm{d}x\int_0^{\frac16(x+2)} f(x,y)\,\mathrm{d}y + \int_1^2 \mathrm{d}x\int_0^{\frac12(2-x)} f(x,y)\,\mathrm{d}y\,.\end{aligned}\right]$$

Express the integral $\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y$, where $\Omega\subset\mathbb{R}^2$ is a finite region bounded by the lines

$$\tfrac13\,x+\tfrac14\,y=1\,,\qquad \tfrac13\,x-\tfrac12\,y=1\,,\qquad x=0\,,$$

in the terms of two simple integrals in at least one integration order.

$$\left[\begin{aligned}\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y &= \int_0^3 \mathrm{d}x\int_{\frac23(x-3)}^{\frac43(3-x)} f(x,y)\,\mathrm{d}y = \\ &= \int_{-2}^0 \mathrm{d}y\int_0^{\frac32(2+y)} f(x,y)\,\mathrm{d}x + \int_0^4 \mathrm{d}y\int_0^{\frac34(4-y)} \mathrm{d}x\,.\end{aligned}\right]$$

Express the integral $\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y$, where the region $\Omega\subset\mathbb{R}^2$ is given by the inequalities

$$x<2y+1\,,\qquad 3y<x+2\,,\qquad 0<y<1\,,$$

in the terms of two simple integrals in at least one integration order.

$$\left[\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y = \int\limits_0^1 \mathrm{d}y \int\limits_{3y-2}^{2y+1} f(x,y)\,\mathrm{d}x = \int\limits_{-2}^{1} \mathrm{d}x \int\limits_{0}^{\frac{1}{3}(x+2)} f(x,y)\,\mathrm{d}y + \int\limits_{1}^{3} \mathrm{d}x \int\limits_{\frac{1}{2}(x-1)}^{1} f(x,y)\,\mathrm{d}y\,.\right]$$

Express the integral $\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y$, where $\Omega \subset \mathbb{R}^2$ is a finite region bounded by the lines

$$-\tfrac{1}{7}\,x + \tfrac{1}{5}\,y = 1\,, \qquad -\tfrac{1}{7}\,x + \tfrac{1}{2}\,y = 1\,, \qquad x = 0\,,$$

in the terms of two simple integrals in at least one integration order.

$$\left[\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y = \int\limits_{-7}^{0} \mathrm{d}x \int\limits_{\frac{2}{7}(x+7)}^{\frac{5}{7}(x+7)} f(x,y)\,\mathrm{d}y = \int\limits_{0}^{2} \mathrm{d}y \int\limits_{\frac{7}{5}(y-5)}^{\frac{7}{2}(y-2)} f(x,y)\,\mathrm{d}x + \int\limits_{2}^{5} \mathrm{d}y \int\limits_{\frac{7}{5}(y-5)}^{0} f(x,y)\,\mathrm{d}x\,.\right]$$

Express the integral $\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y$, where the region $\Omega \subset \mathbb{R}^2$ is given by the inequalities

$$x < 2y + 3\,, \qquad 3y < 2x + 1\,, \qquad -1 < y < 1\,,$$

in the terms of two simple integrals in at least one integration order.

$$\left[\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y = \int\limits_{-1}^{1} \mathrm{d}y \int\limits_{\frac{1}{2}(3y-1)}^{2y+3} f(x,y)\,\mathrm{d}x = \int\limits_{-2}^{1} \mathrm{d}x \int\limits_{-1}^{\frac{1}{3}(2x+1)} f(x,y)\,\mathrm{d}y + \int\limits_{1}^{5} \mathrm{d}x \int\limits_{\frac{1}{2}(x-3)}^{1} f(x,y)\,\mathrm{d}y\,.\right]$$

Express the integral $\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y$, where $\Omega \subset \mathbb{R}^2$ is a finite region bounded by the lines

$$-\tfrac{1}{5}\,x - \tfrac{1}{4}\,y = 1\,, \qquad -\tfrac{1}{5}\,x - \tfrac{1}{2}\,y = 1\,, \qquad x = 0\,,$$

in the terms of two simple integrals in at least one integration order.

$$\left[\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y = \int\limits_{-5}^{0} \mathrm{d}x \int\limits_{-\frac{4}{5}(x+5)}^{-\frac{2}{5}(x+5)} f(x,y)\,\mathrm{d}y = \int\limits_{-4}^{-2} \mathrm{d}y \int\limits_{-\frac{5}{4}(y+4)}^{0} f(x,y)\,\mathrm{d}x + \int\limits_{-2}^{0} \mathrm{d}y \int\limits_{-\frac{5}{4}(y+4)}^{-\frac{5}{2}(y+2)} f(x,y)\,\mathrm{d}x\,.\right]$$

Express the integral $\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y$, where the region $\Omega \subset \mathbb{R}^2$ is given by the inequalities

$$x + 2y < 1\,, \qquad 0 < x + y + 1\,, \qquad 0 < y < 1\,,$$

in the terms of two simple integrals in at least one integration order.

$$\left[\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y = \int\limits_{0}^{1} \mathrm{d}y \int\limits_{-1-y}^{1-2y} f(x,y)\,\mathrm{d}x = \int\limits_{-2}^{-1} \mathrm{d}x \int\limits_{-x-1}^{1} f(x,y)\,\mathrm{d}y + \int\limits_{-1}^{1} \mathrm{d}x \int\limits_{0}^{\frac{1}{2}(1-x)} f(x,y)\,\mathrm{d}y\,.\right]$$

Express the integral $\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y$, where $\Omega \subset \mathbb{R}^2$ is a finite region bounded by the lines

$$\tfrac{1}{2}\,x - \tfrac{1}{3}\,y = 1\,, \qquad \tfrac{1}{2}\,x - \tfrac{1}{5}\,y = 1\,, \qquad x = 0\,,$$

in the terms of two simple integrals in at least one integration order.

$$\left[\begin{aligned} \iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y &= \int_0^2 \mathrm{d}x \int_{\frac{5}{2}(x-2)}^{\frac{3}{2}(x-2)} f(x,y)\,\mathrm{d}y = \\ &= \int_{-3}^{0} \mathrm{d}y \int_{\frac{2}{3}(y+3)}^{\frac{2}{5}(y+5)} f(x,y)\,\mathrm{d}x + \int_{-5}^{-3} \mathrm{d}y \int_{0}^{\frac{2}{5}(y+5)} f(x,y)\,\mathrm{d}x\,. \end{aligned}\right]$$

Express the integral $\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y$, where the region $\Omega \subset \mathbb{R}^2$ is given by the inequalities

$$x < y + 2\,, \qquad y < 2x + 1\,, \qquad -1 < x < 1\,,$$

in the terms of two simple integrals in at least one integration order.

$$\left[\begin{aligned} \iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y &= \int_{-1}^{1} \mathrm{d}x \int_{x-2}^{2x+1} f(x,y)\,\mathrm{d}y = \\ &= \int_{-3}^{-1} \mathrm{d}y \int_{-1}^{y+2} f(x,y)\,\mathrm{d}x + \int_{-1}^{3} \int_{\frac{1}{2}(y-1)}^{1} f(x,y)\,\mathrm{d}x\,. \end{aligned}\right]$$

Express the integral $\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y$, where $\Omega \subset \mathbb{R}^2$ is a finite region bounded by the lines

$$\tfrac{1}{3}\,x - \tfrac{1}{4}\,y = 1\,, \qquad \tfrac{1}{5}\,x - \tfrac{1}{4}\,y = 1\,, \qquad y = 0\,,$$

in the terms of two simple integrals in at least one integration order.

$$\left[\begin{aligned} \iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y &= \int_{-4}^{0} \mathrm{d}y \int_{\frac{3}{4}(y+4)}^{\frac{5}{4}(y+4)} f(x,y)\,\mathrm{d}x = \\ &= \int_{0}^{3} \mathrm{d}x \int_{\frac{4}{5}(x-5)}^{\frac{4}{3}(x-3)} f(x,y)\,\mathrm{d}y + \int_{3}^{5} \mathrm{d}x \int_{\frac{4}{5}(x-5)}^{0} f(x,y)\,\mathrm{d}y\,. \end{aligned}\right]$$

Express the integral $\iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y$, where the region $\Omega \subset \mathbb{R}^2$ is given by the inequalities

$$3y < 5x + 4\,, \qquad 5x + 4y < 17\,, \qquad y + 2 > 0\,,$$

in the terms of two simple integrals in at least one integration order.

$$\left[\begin{aligned} \iint_\Omega f(x,y)\,\mathrm{d}x\,\mathrm{d}y &= \int_{-2}^{3} \mathrm{d}y \int_{\frac{1}{5}(3y-4)}^{\frac{1}{5}(17-4y)} f(x,y)\,\mathrm{d}x = \\ &= \int_{-2}^{1} \mathrm{d}x \int_{-2}^{\frac{1}{3}(5x+4)} f(x,y)\,\mathrm{d}y + \int_{1}^{5} \mathrm{d}x \int_{-2}^{\frac{1}{4}(17-5x)} f(x,y)\,\mathrm{d}y\,. \end{aligned}\right]$$