Exercise 4 – line integrals

Determine the length of a curve $\mathcal{C}$ given by parametric equations

$$x = \mathrm{e}^{-4\varphi}\cos 3\varphi\,, \quad y = \mathrm{e}^{-4\varphi}\sin 3\varphi\,, \qquad \varphi \in (0,\infty)\,.$$

$\left[\frac{5}{4}\right]$

Determine the length of a curve $\mathcal{C}$ given by parametric equations

$$x = \frac{t}{1+t^2}\,, \quad y = \frac{1}{1+t^2}\,, \qquad t \in \mathbb{R}\,.$$

$\left[\pi\right]$

Determine the length of a curve $\mathcal{C}$ given by parametric equations

$$x = \cos\varphi\sin\varphi\,, \quad y = \sin^2\varphi\,, \qquad 0 \le \varphi \le \pi\,.$$

$\left[\pi\right]$

Determine the length of a curve $\mathcal{C}$ given by the equations

$$x^2 + y^2 = 2y\,, \qquad y \le x\,.$$

$\left[\frac{1}{2}\pi\right]$

Determine the length of a curve $\mathcal{C}$ given by the equations

$$x^2 + y^2 + z^2 = 6\,, \qquad y + z = 2\,.$$

$\left[4\pi\right]$

Determine the length of a curve $\mathcal{C}$ given by the equations

$$x^2 + y^2 + z^2 = 25\,, \qquad x + 2y = 5\,.$$

$\left[4\sqrt{5}\,\pi\right]$

Find the length of a curve $\mathcal{C}$ given by parametric equations

$$x = t - \sin t\,, \quad y = 1 - \cos t\,, \qquad 0 \le t \le 4\pi\,, \qquad \big(\text{Hint: it is } 1-\cos t = 2\sin^2\tfrac{t}{2}\big)\,.$$

$\left[16\right]$

Find the weight of a curve $\mathcal{C}$ given by parametric equations

$$x = t\cos t\,, \quad y = t\sin t\,, \quad z = t\,, \qquad 0 \le t \le \pi\,,$$

provided its linear density is $\rho(x,y,z)=\sqrt{x^2+y^2}$. $\left[\frac{1}{3}\left((2+\pi)\sqrt{2+\pi}-2\sqrt{2}\right)\right]$

Find the weight of a curve $\mathcal{C}$ given by parametric equations

$$x=\mathrm{e}^{-t}\cos t\,,\quad y=\mathrm{e}^{-t}\sin t\,,\quad z=\mathrm{e}^{-t}\,,\qquad 0<t<\infty\,,$$

provided its linear density is $\rho(x,y,z)=\sqrt{x^2+y^2+z^2}$. $\left[\sqrt{\frac{3}{2}}\right]$

Find the weight of a curve $\mathcal{C}$ given by parametric equations

$$x=\cos t+t\sin t\,,\quad y=\sin t-t\cos t\,,\quad z=t\,,\qquad 0\le t\le 1\,,$$

provided its linear density is $\rho(x,y,z)=z$. $\left[\frac{1}{3}\left(2\sqrt{2}-1\right)\right]$

Find the weight of a curve $\mathcal{C}$ given by parametric equations

$$x=t-\sin t\,,\quad y=1-\cos t\,,\qquad 0\le t\le 2\pi\,,$$

provided its linear density is $\rho(x,y)=\sqrt{y}$. $\left[2\sqrt{2}\,\pi\right]$

Find the coordinate z_{cg} of a centre of gravity of a homogenous curve $\mathcal{C}\subset\mathbb{R}^2$ given by the equations

$$2x^2+z^2=2\,,\quad y=x\,,\quad z\ge 0\,,$$

provided you know that its length is $\sqrt{2}\,\pi$. $\left[z_{cg}=\frac{2\sqrt{2}}{\pi}\right]$

Find the coordinate x_{cg} of a centre of gravity of a homogenous curve $\mathcal{C}\subset\mathbb{R}^2$ given by parametric equations

$$x=\cos t\,,\quad y=\sin t\,,\quad z=1-\ln\cos t\,,\qquad 0\le t\le\tfrac{1}{3}\,\pi\,,$$

provided you know that its length is $\ln\left(2+\sqrt{3}\right)$. $\left[x_{cg}=\frac{\pi}{3\ln\left(2+\sqrt{3}\right)}\right]$

Find the coordinate y_{cg} of a centre of gravity of a homogenous curve $\mathcal{C}\subset\mathbb{R}^2$ given by parametric equations

$$x=\cos t\,,\quad y=\sin t\,,\quad z=1-\ln\cos t\,,\qquad 0\le t\le\tfrac{1}{3}\,\pi\,,$$

provided you know that its length is $\ln\left(2+\sqrt{3}\right)$. $\left[y_{cg}=\frac{\ln 2}{\ln\left(2+\sqrt{3}\right)}\right]$

Find the coordinate y_{cg} of a centre of gravity of a homogenous curve $\mathcal{C}\subset\mathbb{R}^2$ given by the equation

$$x^2+y^2=4x\,,\qquad y\le x\,,$$

provided you know that its length is 3π. $\left[y_{cg}=-\frac{4}{3\pi}\right]$

Find the coordinate x_{cg} of a centre of gravity of a homogenous curve $\mathcal{C} \subset \mathbb{R}^2$ given by the equation

$$x^2 + y^2 = 4x\,, \qquad x \le y\,,$$

provided you know that its length is π. $\left[x_{cg} = \frac{2\pi-4}{\pi}\right]$

Find the coordinate x_{cg} of a centre of gravity of a homogenous curve $\mathcal{C} \subset \mathbb{R}^2$ given by parametric equations

$$x = \cos t + t\sin t\,, \quad y = \sin t - t\cos t\,, \quad z = t^2\,, \qquad 0 \le t \le 2\pi\,,$$

provided you know that its length is $2\sqrt{5}\,\pi^2$. $\left[x_{cg} = -2\right]$

Find the coordinate y_{cg} of a centre of gravity of a homogenous curve $\mathcal{C} \subset \mathbb{R}^2$ given by parametric equations

$$x = \cos t + t\sin t\,, \quad y = \sin t - t\cos t\,, \quad z = t^2\,, \qquad 0 \le t \le 2\pi\,,$$

provided you know that its length is $2\sqrt{5}\,\pi^2$. $\left[y_{cg} = -\frac{3}{\pi}\right]$

Find the coordinate x_{cg} of a centre of gravity of a homogenous curve $\mathcal{C} \subset \mathbb{R}^2$ given by parametric equations

$$x = \mathrm{e}^{-t}\cos t\,, \quad y = \mathrm{e}^{-t}\sin t\,, \quad z = \mathrm{e}^{-t}\,, \qquad 0 < t < \infty\,,$$

provided you know that its length is $\sqrt{3}$. $\left[x_{cg} = \frac{2}{5}\right]$

Find the coordinate y_{cg} of a centre of gravity of a homogenous curve $\mathcal{C} \subset \mathbb{R}^2$ given by parametric equations

$$x = \mathrm{e}^{-t}\cos t\,, \quad y = \mathrm{e}^{-t}\sin t\,, \quad z = \mathrm{e}^{-t}\,, \qquad 0 < t < \infty\,,$$

provided you know that its length is $\sqrt{3}$. $\left[y_{cg} = -\frac{1}{5}\right]$

Find the coordinate x_{cg} of a centre of gravity of a homogenous curve $\mathcal{C} \subset \mathbb{R}^2$, which is the boundary of a planar region

$$0 \le y \le x\,, \qquad x^2 + y^2 \le 4\,,$$

provided you know that its length is $4 + \frac{1}{2}\,\pi$. $\left[x_{cg} = \frac{4+6\sqrt{2}}{8+\pi}\right]$

A curve $\mathcal{C} \subset \mathbb{R}^2$ is the boundary of a planar region

$$0 \le y \le x\,, \qquad x^2 + y^2 \le 4\,.$$

Find the coordinate y_{cg} of its centre of gravity, provided you know that its length is $4 + \frac{1}{2}\,\pi$. $\left[y_{cg} = \frac{8+2\sqrt{2}}{8+\pi}\right]$

Find the moment of inertia with respect to the axis z of the piece of a straight line $\mathcal{C}$ from the point $A = [-1, 0, -2]$ the point $B = [1, 1, 0]$, the density of which is equal to one, i.e., the integral

$$J_z = \int_{\mathcal{C}} (x^2 + y^2)\, \mathrm{d}s\,.$$

$\left[2\right]$

Find the moment of inertia with respect to the axis z of the curve $\mathcal{C}$ given by equations

$$x = \cos t + t \sin t\,, \quad y = \sin t - t\cos t\,, \quad z = 4\,, \qquad 0 \le t \le 2\pi\,,$$

provided its density is equal to one, i.e., the integral

$$J_z = \int_{\mathcal{C}} (x^2 + y^2)\, \mathrm{d}s\,.$$

$\left[2\pi^2\left(1 + 2\pi^2\right)\right]$

Find the moment of inertia with respect to the axis z of the curve $\mathcal{C}$ given by equations

$$x = \sin t \cos t\,, \quad y = \sin^2 t\,, \quad z = t\,, \qquad 0 \le t \le \pi\,,$$

provided its density is equal to one, i.e., the integral

$$J_z = \int_{\mathcal{C}} (x^2 + y^2)\, \mathrm{d}s\,.$$

$\left[\frac{\pi}{\sqrt{2}}\right]$

Find the moment of inertia with respect to the axis z of the curve $\mathcal{C}$ which is the boundary of the intersection of the set

$$x + y \ge 0\,, \qquad x - y \ge 0\,, \qquad x^2 + y^2 \le 1$$

with the plane $z = 0$, provided its density is equal to 1, i.e., the integral

$$J_z = \int_{\mathcal{C}} (x^2 + y^2)\, \mathrm{d}s\,.$$

$\left[\frac{1}{6}\,(4 + 3\pi)\right]$

Find the line integral

$$\int_{\mathcal{C}} \frac{\mathrm{d}s}{\sqrt{y}}\,,$$

where $\mathcal{C}$ is a piece of a straight line from the point $A = [0, 4]$ do bodu $B = [3, 0]$. $\left[\frac{5}{2}\right]$

Find the line integral

$$\int_{\mathcal{C}} \frac{\mathrm{d}s}{\sqrt{x^2+y^2+z^2}}\,,$$

where $\mathcal{C}$ is a curve given by parametric equations

$$x = t\sin t + \cos t\,,\quad y = t\cos t - \sin t\,,\quad z = 2\,,\qquad 0 \le t \le 2\,.$$

$\left[3-\sqrt{5}\right]$

Find the line integral

$$\int_{\mathcal{C}} (x+y)\mathrm{e}^{x+y+z}\,\mathrm{d}s\,,$$

where $\mathcal{C}$ is a piece of a straight line from the point $A = [1,2,-3]$ to the point $B = [3,1,-1]$.
$\left[\frac{1}{3}\left(11\mathrm{e}^3-8\right)\right]$

Find a line integral $\oint_{\mathcal{C}} (x\,\mathrm{d}y - y\,\mathrm{d}x)$, where $\mathcal{C}$ is a positively oriented boundary of an ellipse $x^2+4y^2 \le 4$. $\left[4\pi\right]$

Find a line integral $\int_{\mathcal{C}} (y\,\mathrm{d}x - x\,\mathrm{d}y)$, where $\mathcal{C}$ is a curve given by parametric equations

$$x = t - \sin t\,,\quad y = 1-\cos t\,,\qquad 0 \le t \le 2\pi\,,$$

oriented in the direction of an increasing parameter. $\left[6\pi\right]$

Find a line integral $\oint_{\mathcal{C}} \big((x-y)\,\mathrm{d}x + (x+y)\,\mathrm{d}y\big)$, where $\mathcal{C}$ is a positively oriented boundary of the region $\Omega \subset \mathbb{R}^2$ given by the inequalities

$$x+y \ge 0\,,\qquad x-y \ge 0\,,\qquad x^2+y^2 \le 1\,.$$

$\left[\frac{1}{2}\,\pi\right]$

Find a line integral $\int_{\mathcal{C}} (y\,\mathrm{d}x + x\,\mathrm{d}y + z\,\mathrm{d}z)$, where $\mathcal{C}$ is a curve given by parametric equations

$$x = \mathrm{e}^{-\varphi}\cos\varphi\,,\quad y = \mathrm{e}^{-\varphi}\sin\varphi\,,\quad z = \varphi\mathrm{e}^{-\varphi}\,,\qquad \varphi \in (0,\infty)\,,$$

oriented in the direction of an increasing parameter φ. $\left[0\right]$

Find a line integral $\int_{\mathcal{C}} (y\,\mathrm{d}x + x\,\mathrm{d}z)$, where $\mathcal{C}$ is a curve given by parametric equations

$$x = t + \cos t\,,\quad y = \cos t\,,\quad z = t\,,\qquad t \in \langle 0, 2\pi\rangle\,,$$

oriented in the direction of an increasing parameter t. $\left[2\pi^2\right]$

Find a line integral $\int_{\mathcal{C}} (z\,\mathrm{d}x - y\,\mathrm{d}z)$, where $\mathcal{C}$ is a curve given by the equations

$$x^2 + y^2 = z^2\,, \quad x^2 + y^2 = 2x\,, \quad z \geq 0,$$

oriented such that the second component of its tangent vector in the point $[2, 0, 2]$ is positive. $\left[\frac{8}{3}\right]$

Find a line integral $\int_{\mathcal{C}} (x\,\mathrm{d}x + z\,\mathrm{d}y - 2y\,\mathrm{d}z)$, where $\mathcal{C}$ is a curve given by the equations

$$x^2 + y^2 + z^2 = 1\,, \quad x^2 + y^2 = y\,,$$

which lies in the first octant, i.e., x, y, $z \geq 0$, and starts in the point $[0, 0, 1]$. $\left[2\right]$

Find a line integral $\int_{\mathcal{C}} (y\,\mathrm{d}x - x\,\mathrm{d}y + z\,\mathrm{d}z)$, where $\mathcal{C}$ is a curve given by the equations

$$x^2 + y^2 = 1\,, \quad x + z = 2\,,$$

oriented such that the first component of its tangent vector in the point $[0, 1, 2]$ is positive. $\left[\pi\right]$

Find a line integral

$$\int_{\mathcal{C}} \left(\frac{\mathrm{d}x}{z - y} + \frac{\mathrm{d}y}{x - z} + \frac{\mathrm{d}z}{y - x} \right),$$

where $\mathcal{C}$ is a straight line segment from the point $A = [1, -5, 4]$ the point $B = [-1, -2, 5]$. $\left[\ln \frac{7}{2} - \frac{6}{5} \ln 3\right]$

Find a line integral

$$\oint_{\mathcal{C}} (z\,\mathrm{d}x + x\,\mathrm{d}y + y\,\mathrm{d}z)\,,$$

where $\mathcal{C}$ is a triangle with the vertices $A = [1, 0, 0]$, $B = [0, 2, 0]$ and $C = [0, 0, 3]$, oriented in the direction $A \to B \to C \to A$. $\left[\frac{11}{2}\right]$

Let $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$ be a unit vector in the direction of an axis x, y and z, respectively. Find the work of a force field $\mathbf{f} = z\mathbf{e}_1 + x\mathbf{e}_2 + y\mathbf{e}_3$ over a straight line segment with the starting point $A = [-1, 0, 1]$ and the endpoint $B = [3, 1, -1]$. $\left[0\right]$

Let $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ be a unit vector in the direction of an axis x, y and z, respectively. Find the work of a force field $\mathbf{f} = xz\mathbf{i} - y\mathbf{k}$ over a curve $\mathcal{C}$ given by parametric equations

$$\mathbf{x}(t) = 2\mathbf{i} + \mathrm{e}^t\mathbf{j} + t^2\mathbf{k}\,, \qquad 0 \leq t \leq 1\,,$$

oriented in the direction of an increasing parameter t. $\left[-2\right]$

Let $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$ be a unit vector in the direction of an axis x, y and z, respectively. Find the work of a force field $\mathbf{f} = yz\mathbf{e}_1 - xz\mathbf{e}_2 + (x^2 + y^2)\mathbf{e}_3$ over a curve $\mathcal{C}$ given by parametric equations

$$\mathbf{x}(t) = t\cos t\,\mathbf{e}_1 + t\sin t\,\mathbf{e}_2 + t^2\,\mathbf{e}_3\,, \qquad 0 \le t \le \pi\,,$$

oriented in the direction of an increasing parameter. $\left[\frac{1}{10}\,\pi^4(5 - 2\pi)\right]$

Let $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ be a unit vector in the direction of an axis x, y and z, respectively. Find the work of a force field $\mathbf{f} = z\mathbf{i} + x\mathbf{j} + 2y\mathbf{k}$ over a curve $\mathcal{C}$ given by parametric equations

$$x = \ln t\,, \quad y = t\,, \quad z = \frac{1}{t}\,, \qquad 1 \le t \le 2\,,$$

oriented in the direction of an increasing parameter t. $\left[-\frac{1}{2}\right]$

Let $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ be a unit vector in the direction of an axis x, y and z, respectively. Find the work of a force field $\mathbf{f} = z\mathbf{i} + y\mathbf{k}$ over a curve $\mathcal{C}$ given by parametric equations

$$x = \cos t\,, \quad y = t - \sin t\,, \quad z = t\,, \qquad 0 \le t \le 2\pi\,,$$

oriented in the direction of an increasing parameter t. $\left[2\pi(\pi + 1)\right]$

Let $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ be a unit vector in the direction of an axis x, y and z, respectively. Find the work of a force field $\mathbf{f} = y\mathbf{i} - x\mathbf{j} + (x + y)\mathbf{k}$ over a curve $\mathcal{C}$ given by parametric equations

$$x = t + \cos t\,, \quad y = t + \sin t\,, \quad z = \tfrac{1}{2}\,t^2\,, \qquad 0 \le t \le \pi\,,$$

oriented in the direction of an increasing parameter t. $\left[\frac{1}{3}\,\pi\big(2\pi^2 - 3\big)\right]$

Let $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ be a unit vector in the direction of an axis x, y and z, respectively. Find the work of a force field $\mathbf{f} = (y + z)\mathbf{i} - (x + z)\mathbf{j}$ over a curve $\mathcal{C}$ given by parametric equations

$$\mathbf{x}(t) = \mathrm{e}^{-t}\cos t\,\mathbf{i} + \mathrm{e}^{-t}\sin t\,\mathbf{j} + \mathrm{e}^{-t}\,\mathbf{k}\,, \qquad 0 \le t < \infty\,,$$

oriented in the direction of an increasing parameter. $\left[\frac{3}{10}\right]$

Let $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ be a unit vector in the direction of an axis x, y and z, respectively. Find the work of a force field $\mathbf{f} = y\mathbf{i} + x\mathbf{j} + z\mathbf{k}$ over the curve given by the equations

$$x^2 + 2y^2 + 3z^2 = 3\,, \qquad x = y\,,$$

which starts in the point $A = [1, 1, 0]$, ends in the point $B = [0, 0, 1]$ and lies in the first octant, i.e., x, y, $z \ge 0$. $\left[-\frac{1}{2}\right]$

Let $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ be a unit vector in the direction of an axis x, y and z, respectively. Find the work of a force field $\mathbf{f} = y\mathbf{i} - x\mathbf{j} + z\mathbf{k}$ over the curve $\mathcal{C}$ given by the equations

$$x^2 + y^2 = 1\,, \qquad xy = z\,,$$

which starts in the point $A = [1, 0, 0]$, ends in the point $B = [0, 1, 0]$ and lies in the first octant, i.e., $x,\ y,\ z \geq 0$. $\left[-\frac{1}{2}\,\pi\right]$

Let $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$ be a unit vector in the direction of an axis x, y and z, respectively. Find the work of a force field $\mathbf{f} = -y\mathbf{e}_1 + x\mathbf{e}_2 + z\mathbf{e}_3$ over the curve $\mathcal{C}$ given by the equations

$$x^2 + y^2 = 1\,, \qquad x^2 - y^2 = z\,, \qquad x,\ y \geq 0$$

from the point $A = [1, 0, 1]$ to the point $B = [0, 1, -1]$. $\left[\frac{1}{2}\,\pi\right]$

Let $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ be a unit vector in the direction of an axis x, y and z, respectively. Find the work of a force field $\mathbf{f} = z\mathbf{i} + y\mathbf{k}$ over the curve given by the equations

$$x^2 + 4y^2 = 4\,, \qquad z = xy\,,$$

oriented such that the first component of the tangent vector in the point $A = [0, 1, 0]$ is negative.

$\left[0\right]$

Let $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ be a unit vector in the direction of an axis x, y and z, respectively. Find the work of a force field $\mathbf{f} = y\mathbf{i} - z\mathbf{j} + y\mathbf{k}$ over the curve given by the equations

$$x^2 + y^2 + z^2 = 25\,, \qquad x + 2y = 0\,,$$

oriented such that the first component of the tangent vector in the point $A = [0, 0, 5]$ is positive.

$\left[-10\sqrt{5}\,\pi\right]$

Let $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ be a unit vector in the direction of an axis x, y and z, respectively. Find the work of a force field $\mathbf{f} = (2x - y - z)\mathbf{i} + (2y - x - z)\mathbf{j} + (2z - x - y)\mathbf{k}$ over the triangle $A = [1, 0, 0]$, $B = [0, 2, 0]$, $C = [0, 0, 3]$, oriented in the direction $A \to B \to C \to A$.

$\left[0\right]$

Let $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ be a unit vector in the direction of an axis x, y and z, respectively. Find the work of a force field $\mathbf{f} = y\mathrm{e}^z\,\mathbf{i} + z\mathrm{e}^x\,\mathbf{j} + x\mathrm{e}^y\,\mathbf{k}$ over a straight line segment with the starting point $A = [-1, 2, 1]$ and the end point $B = [2, 3, -1]$. $\left[2\mathrm{e}^3 - \frac{73}{9}\,\mathrm{e}^2 + \frac{15}{4}\,\mathrm{e} - \frac{209}{36}\,\mathrm{e}^{-1}\right]$

Show that the line integral

$$\int_{\mathcal{C}} \Big(x(3x + 2y)\,\mathrm{d}x + (x^2 - 2y + 3z)\,\mathrm{d}y + (3y - 2z + 1)\,\mathrm{d}z\Big)$$

does not depend on the path of integration, and calculate it over the curve $\mathcal{C}$ starting in the point $A = [1, 1, 1]$ end ending in the point $B = [-1, 2, 0]$. $\left[-7\right]$

Show that the line integral

$$\int_{\mathcal{C}} \Big((2x + y - z)(y + z)\,\mathrm{d}x + (x + 2y + z)(x - z)\,\mathrm{d}y + (x - y - 2z)(x + y)\,\mathrm{d}z\Big)$$

does not depend on the path of integration, and calculate it over the curve $\mathcal{C}$ starting in the point $A = [1, 2, 3]$ end ending in the point $B = [3, 1, 2]$. $\left[42\right]$

Show that the line integral

$$\int_{\mathcal{C}} \Big((\sin y - z \sin x) \, \mathrm{d}x + (\sin z + x \cos y) \, \mathrm{d}y + (\cos x + y \cos z) \, \mathrm{d}z \Big)$$

does not depend on the path of integration, and calculate it over the curve $\mathcal{C}$ starting in the point $A = [0, \pi, 0]$ and ending in the point $B = [\pi, 0, \pi]$. $\left[-\pi\right]$

Find a potential of a vector field

$$\mathbf{f}(x, y, z) = \left(2x + yz, \; xz + \frac{2y}{\sqrt{y^2 - z}} \, , \; xy - \frac{1}{\sqrt{y^2 - z}} \right)$$

in the region $y^2 > z$. Using this result, calculate the work $\mathbf{f}$ along a curve $\mathcal{C}$, which starts in the point $A = [2, 1, 0]$, ends in the point $B = [1, 3, 5]$ and lies in the region $y^2 > z$. $\left[U = x^2 + xyz + 2\sqrt{y^2 - z} \, ; \quad 14\right]$

Find a potential of a vector field

$$\mathbf{f}(x, y, z) = \left(\frac{y + x \ln z}{x}, \; \frac{z + y \ln x}{y}, \; \frac{x + z \ln y}{z} \right)$$

in the first octant, i.e., for $x, \, y, \, z > 0$. Using this result, calculate the work $\mathbf{f}$ along a curve $\mathcal{C}$ whith the starting point $A = [1, 2, 1]$ and the end point $B = [4, 1, 2]$. $\left[U = x \ln z + y \ln x + z \ln y \, ; \quad 5 \ln 2\right]$

Find a potential of a vector field

$$\mathbf{f}(x, y, z) = \left(\frac{x}{(x^2 + y^2 + z^2)^{3/2}}, \; \frac{y}{(x^2 + y^2 + z^2)^{3/2}}, \; \frac{z}{(x^2 + y^2 + z^2)^{3/2}} \right).$$

Using this result, calculate the work $\mathbf{f}$ along a curve $\mathcal{C}$, which starts in the point $A = [2, -1, 2]$, ends in the point $B = [4, 0, -3]$ and does not pass through the origin. $\Bigg[U = -\frac{1}{\sqrt{x^2 + y^2 + z^2}} \, ;$
