<u>Exercise 5</u> – surface integrals

Find the surface area of a surface $\mathcal{S}$ given by parametric equations

$$x = u\cos v\,, \quad y = u\sin v\,, \quad z = v\,, \qquad 0 < v < u < 1\,.$$

$\left[\frac{1}{3}\left(2\sqrt{2}-1\right)\right]$

Find the surface area of a surface $\mathcal{S}$ given by parametric equations

$$x = uv\,, \quad y = \tfrac{1}{2}\,(u^2 - v^2)\,, \quad z = \tfrac{1}{2}(u^2 + v^2)\,, \qquad u^2 + v^2 \le 2\,, \quad u,\, v \ge 0\,.$$

$\left[\frac{1}{2}\sqrt{2}\,\pi\right]$

Find the surface area of a surface $\mathcal{S}$ given by parametric equations

$$x = 2r\cos^2\varphi\,, \quad y = r\sin^2\varphi\,, \quad z = r\,, \qquad 0 \le \varphi \le \tfrac{1}{2}\,\pi\,, \quad 0 < r < 1\,.$$

$\left[\frac{3}{2}\right]$

Find the surface area of a surface $\mathcal{S}$ given by parametric equations

$$x = \mathrm{e}^{-u}\cos v\,, \quad y = \mathrm{e}^{-u}\sin v\,, \quad z = \mathrm{e}^{-u}\,, \qquad 0 \le v \le \pi\,, \quad 0 < u < \infty\,.$$

$\left[\frac{1}{2}\sqrt{2}\,\pi\right]$

Find the surface area of a part of a sphere

$$x^2 + y^2 + z^2 = 25\,, \qquad z \ge 4\,.$$

$\left[8\pi\right]$

Find the surface area of a part of a surface $\mathcal{S}$ given by the equation $z = 4 + x^2 + xy - y^2$ and lying inside a cylinder $x^2 + y^2 \le 2$. $\left[\frac{2}{15}\,\pi\left(11\sqrt{11}-1\right)\right]$

Find the surface area of a part of a surface $\mathcal{S}$ given by the equation $z = x^2 + 2y^2$ and lying inside an elliptic cylinder $x^2 + 4y^2 \le 1$. $\left[\frac{1}{12}\,\pi\left(5\sqrt{5}-1\right)\right]$

Find the surface area of a part of a surface $\mathcal{S}$ given by the equation

$$z^2 = 4(x^2 + y^2)\,, \qquad \text{where} \ \ 0 \le z \le 4\,.$$

$\left[2\sqrt{5}\,\pi\right]$

Find the surface area of a surface $\mathcal{S}$ given by the equation

$$2x - 2y + z = 1\,, \qquad \text{where} \ \ x^2 + y^2 + z \le 1\,.$$

$\left[6\pi\right]$

Find the weight of a cylindric surface $\mathcal{S}$ given by the equation

$$x^2 + y^2 = z^2\,, \qquad z > 0\,,$$

and having the density $\rho(x, y, z) = \mathrm{e}^{-z^2}$. $\left[\sqrt{2}\,\pi\right]$

Find the weight of a surface $\mathcal{S}$ given by parametric equations

$$x = r\cos\varphi\,, \quad y = r\sin\varphi\,, \quad z = \cosh r\,, \qquad 0 \le \varphi \le 2\pi\,, \quad 0 \le r \le 1$$

and having the density $\rho(x, y, z) = \dfrac{1}{z}$. $\left[\pi\right]$

Find the weight of a surface $\mathcal{S}$ given by parametric equations

$$x = r\cos\varphi\,, \quad y = r\sin\varphi\,, \quad z = \varphi\,, \qquad 0 \le \varphi \le 2\pi\,, \quad 0 \le r \le 1$$

and having the density $\rho(x, y, z) = \sqrt{x^2 + y^2}$. $\left[\tfrac{2}{3}\,\pi\big(2\sqrt{2} - 1\big)\right]$

Find the weight of a surface $\mathcal{S}$ given by the equation

$$z = \tfrac{1}{2}\,(x^2 + y^2)\,, \qquad z \le 2$$

and having the density $\rho(x, y, z) = z$. $\left[\tfrac{2}{15}\,\pi\big(25\sqrt{5} + 1\big)\right]$

Find the weight of a surface $\mathcal{S}$ given by the equation

$$z = \operatorname{arctg}\frac{y}{x}\,, \qquad x^2 + y^2 \le 1\,, \qquad x \ge 0$$

and having the density $\rho(x, y, z) = \sqrt{x^2 + y^2}$. $\left[\tfrac{1}{3}\,\pi\big(2\sqrt{2} - 1\big)\right]$

Find the weight of a surface $\mathcal{S}$ given by parametric equations

$$x = r^2\cos\varphi\,, \quad y = r^2\sin\varphi\,, \quad z = r\,, \qquad 0 \le \varphi \le \tfrac{1}{2}\,\pi\,, \quad 1 \le r \le 2$$

and having the density $\rho(x, y, z) = \dfrac{1}{z}$. $\left[\tfrac{1}{24}\,\pi\big(17\sqrt{17} - 5\sqrt{5}\big)\right]$

Find the weight of a surface $\mathcal{S}$ given by the equation

$$z^2 = x^2 - y^2\,, \quad y^2 + z^2 \le 2y\,, \qquad x,\, z \ge 0$$

and having the density $\rho(x, y, z) = z$. $\left[\tfrac{2}{3}\,\sqrt{2}\right]$

Find the coordinate z_{cg} of a centre of gravity of a half of a homogenous sphere

$$x^2 + y^2 + z^2 = 1\,, \qquad z \ge 0\,,$$

provided you know that its surface area is equal to 2π. $\left[z_{cg} = \frac{1}{2}\right]$

Find the coordinate y_{cg} of a centre of gravity of a homogenous surface $\mathcal{S}$ given by the equation

$$z = \sqrt{x^2 + y^2}\,, \qquad \text{where } x^2 + y^2 \le 2x\,, \qquad y \ge 0\,,$$

provided you know that its surface area is equal to $\frac{1}{2}\sqrt{2}\,\pi$. $\left[y_{cg} = \frac{4}{3\pi}\right]$

Find the coordinate z_{cg} of a centre of gravity of a homogenous surface $\mathcal{S}$ given by the equation

$$z = x^2 + y^2\,, \qquad 0 \le z \le 2\,,$$

provided you know that its surface area is equal to $\frac{13}{3}\,\pi$. $\left[z_{cg} = \frac{74}{65}\right]$

Find the coordinate x_{cg} of a centre of gravity of a homogenous surface

$$z = \operatorname{arctg}\frac{y}{x}\,, \qquad x^2 + y^2 \le 1\,, \qquad x > 0\,,$$

provided you know that its surface area is equal to $\frac{1}{2}\,\pi\big(\sqrt{2}+\ln(1+\sqrt{2})\big)$. $\left[x_{cg} = \dfrac{4\big(2\sqrt{2}-1\big)}{3\pi\big(\sqrt{2}+\ln(1+\sqrt{2})\big)}\right]$

Find the coordinate z_{cg} of a centre of gravity of a homogenous triangle with the vertices $A = [2,0,0]$, $B = [0,3,0]$ and $C = [0,0,6]$, provided you know that its surface area is equal to $3\sqrt{14}$. $\left[z_{cg} = 2\right]$

Find the coordinate x_{cg} of a centre of gravity of a homogenous triangle with the vertices $A = [4,0,0]$, $B = [0,3,0]$ and $C = [0,0,2]$, provided you know that its surface area is equal to $\sqrt{61}$. $\left[x_{cg} = \frac{4}{3}\right]$

Find the coordinate x_{cg} of a centre of gravity of a homogenous surface $\mathcal{S}$ given by parametric equations

$$x = 2r\cos^2\varphi\,, \quad y = r\sin^2\varphi\,, \quad z = r\,, \qquad 0 \le \varphi \le \tfrac{1}{2}\pi\,, \quad 0 < r < 2\,,$$

provided you know that its surface area is equal to 6. $\left[x_{cg} = \frac{4}{3}\right]$

Find the coordinate z_{cg} of a centre of gravity of a homogenous surface $\mathcal{S}$ given by the equation

$$z^2 = x^2 - y^2\,, \quad y^2 + z^2 \le 2y\,, \qquad x,\, z \ge 0,$$

provided you know that its surface area is equal to $\frac{\sqrt{2}}{2}\,\pi$. $\left[z_{cg} = \frac{4}{3\pi}\right]$

For the surface given by the relations

$$z = xy\,, \qquad x^2 + y^2 \le 1\,, \quad x,\, y \ge 0\,,$$

find the moment of inertia with respect to the axis z, i.e., the integral

$$J_z = \iint_S (x^2 + y^2)\,\mathrm{d}S\,.$$

$\left[J_z = \frac{1}{15}\,\pi\big(\sqrt{2}+1\big)\right]$

For the surface given by the relations

$$x^2 + y^2 + z^2 = 1\,, \quad z \geq 0\,,$$

find the moment of inertia with respect to the axis z, i.e., the integral

$$J_z = \iint_S (x^2 + y^2)\,\mathrm{d}S\,.$$

$\left[J_z = \frac{4}{3}\,\pi\right]$

For the surface given by the relations

$$4z^2 = x^2 + y^2\,, \quad 0 \leq z \leq 2\,,$$

find the moment of inertia with respect to the axis z, i.e., the integral

$$J_z = \iint_S (x^2 + y^2)\,\mathrm{d}S\,.$$

$\left[J_z = 16\sqrt{5}\,\pi\right]$

For the surface given by the relations

$$x + 2y - 2z = 4\,, \qquad x \geq 0\,, \quad y \geq 0\,, \quad z \leq 0\,,$$

find the moment of inertia with respect to the axis z, i.e., the integral

$$J_z = \iint_S (x^2 + y^2)\,\mathrm{d}S\,.$$

$\left[J_z = 20\right]$

For the surface given by the relations

$$x^2 + y^2 + z^2 = 25\,, \qquad z \geq 4\,,$$

find the moment of inertia with respect to the axis z, i.e., the integral

$$J_z = \iint_S (x^2 + y^2)\,\mathrm{d}S\,.$$

$\left[J_z = \frac{28}{3}\,\pi\right]$

Find the surface integral $\iint_{\mathcal{S}} (x\,\mathrm{d}y\,\mathrm{d}z + y\,\mathrm{d}z\,\mathrm{d}x + z\,\mathrm{d}x\,\mathrm{d}y)$, where $\mathcal{S}$ is a surface given by parametric equations

$$x = r\cos\varphi\,, \quad y = r\sin\varphi\,, \quad z = \varphi\,, \qquad 0 \le \varphi \le 2\pi\,, \quad 1 \le r \le 2\,,$$

oriented such that the third component of its normal vector is positive. $\left[3\pi\right]$

Find the surface integral $\iint_{\mathcal{S}} (x\,\mathrm{d}y\,\mathrm{d}z + y\,\mathrm{d}z\,\mathrm{d}x + z\,\mathrm{d}x\,\mathrm{d}y)$, where $\mathcal{S}$ is a surface given by parametric equations

$$x = 2uv\,, \quad y = u^2 - v^2\,, \quad z = u^2 + v^2\,, \qquad u^2 + v^2 \le 1\,, \quad v \ge 0\,,$$

oriented such that the third component of its normal vector is positive. $\left[0\right]$

Find the surface integral $\iint_{\mathcal{S}} \mathbf{f}\,\mathrm{d}\mathbf{S}$, where $\mathbf{f} = (z, 0, x)$ and the surface $\mathcal{S}$ is given by parametric equations

$$x = \frac{u^2}{v}\,, \quad y = \frac{v^2}{u}\,, \quad z = uv\,, \qquad 1 \le u \le 2\,, \quad 1 \le v \le 2,$$

oriented such that the third component of its normal vector is negative. $\left[\frac{45}{4} - 7\ln 2\right]$

Find the surface integral $\iint_{\mathcal{S}} (z\,\mathrm{d}y\,\mathrm{d}z + y\,\mathrm{d}z\,\mathrm{d}x + \ \mathrm{d}x\,\mathrm{d}y)$, where $\mathcal{S}$ is a surface given by parametric equations

$$x = \tfrac{1}{2}\,(u^2 + v^2)\,, \quad y = uv\,, \quad z = u + v\,, \qquad 0 \le u \le v \le 2\,,$$

oriented such that the third component of its normal vector is positive. $\left[-\frac{16}{15}\right]$

Find the surface integral $\iint_{\mathcal{S}} (z\,\mathrm{d}y\,\mathrm{d}z + y\,\mathrm{d}z\,\mathrm{d}x + \sqrt{x^2 + y^2}\,\mathrm{d}x\,\mathrm{d}y)$, where $\mathcal{S}$ is a surface given by parametric equations

$$x = \frac{u}{v}\,, \quad y = \frac{v}{u}\,, \quad z = uv\,, \qquad 1 < u < 4\,, \quad 1 < v < 2\,,$$

oriented such that the third component of its normal vector is positive. $\left[20\right]$

Find the surface integral $\iint_{\mathcal{S}} (x\,\mathrm{d}y\,\mathrm{d}z + y\,\mathrm{d}z\,\mathrm{d}x)$, where $\mathcal{S}$ is the surface

$$z = xy \qquad x,\, y > 0\,, \quad x + y \le 1\,,$$

oriented such that the third component of its normal vector is positive. $\left[-\frac{1}{12}\right]$

Find the surface integral $\iint_{\mathcal{S}} (xy\,\mathrm{d}y\,\mathrm{d}z + yz\,\mathrm{d}z\,\mathrm{d}x + xz\,\mathrm{d}x\,\mathrm{d}y)$, where $\mathcal{S}$ is a triangle with the vertices $A = [3,0,0]$, $B = [0,2,0]$ and $C = [0,0,6]$, oriented such that the third component of its normal vector is positive. $\left[18\right]$

Find the surface integral $\iint_{\mathcal{S}} (y\,\mathrm{d}y\,\mathrm{d}z + z\,\mathrm{d}x\,\mathrm{d}y)$, where $\mathcal{S}$ is a surface

$$z = x^2 + y^2 \qquad 0 \le z \le 4\,,$$

oriented such that the third component of its normal vector is positive. $\left[8\pi\right]$

Find the surface integral $\iint_{\mathcal{S}} (x\,\mathrm{d}y\,\mathrm{d}z + y\,\mathrm{d}z\,\mathrm{d}x)$, where $\mathcal{S}$ is the surface

$$x^2 + y^2 + z^2 = 1 \qquad z \ge 0\,,$$

$\mathcal{S}$ is oriented such that the third component of its normal vector is positive. $\left[\frac{4}{3}\pi\right]$

Find the surface integral $\iint_{\mathcal{S}} \Big(x\,\mathrm{d}z\,\mathrm{d}x + (z^2-1)\,\mathrm{d}x\,\mathrm{d}y\Big)$, where $\mathcal{S}$ is the surface

$$x^2 + y^2 = 1 \qquad -1 \le z \le 2\,,$$

oriented such that the first component of a normal vector in the point $A = [1,0,0]$ is positive. $\left[0\right]$

Find the surface integral $\iint_{\mathcal{S}} (x\,\mathrm{d}y\,\mathrm{d}z + y\,\mathrm{d}z\,\mathrm{d}x)$, where $\mathcal{S}$ is the surface

$$x^2 + y^2 = 1 \qquad -1 \le z \le 2\,,$$

oriented such that the first component of a normal vector in the point $A = [1,0,0]$ is negative. $\left[-6\pi\right]$

Find the surface integral $\iint_{\mathcal{S}} \Big((x-y)\,\mathrm{d}y\,\mathrm{d}z + (x+y)\,\mathrm{d}z\,\mathrm{d}x + z\,\mathrm{d}x\,\mathrm{d}y\Big)$, where $\mathcal{S}$ is the surface

$$z = x^2 + y^2 \qquad 0 \le z \le 1\,,$$

$\mathcal{S}$ is oriented such that the third component of its normal vector is positive. $\left[-\frac{1}{2}\pi\right]$

Find the surface integral $\iint_{\mathcal{S}} \mathbf{f}\,\mathrm{d}\mathbf{S}$, where $\mathbf{f} = (y, x, z)$ and the surface $\mathcal{S}$ is described by the equation

$$z = 4 - x^2 - y^2\,, \qquad z \ge 0$$

$\mathcal{S}$ is oriented such that the third component of its normal vector is negative. $\left[-8\pi\right]$

Find the surface integral $\iint_{\mathcal{S}} \mathbf{f}\, \mathrm{d}\mathbf{S}$, kde $\mathbf{f} = (yz, xz, xy)$ and the surface $\mathcal{S}$ is described by the equation

$$x^2 + y^2 = 4\,, \qquad x^2 + y^2 + z^2 \leq 13\,, \quad x,\, y,\, z \geq 0$$

$\mathcal{S}$ is oriented such that the first component of its normal vector is negative. $\left[-6\right]$

Find the surface integral $\iint_{\mathcal{S}} \mathbf{f}\, \mathrm{d}\mathbf{S}$, kde $\mathbf{f} = (x, y, z)$ and the surface $\mathcal{S}$ is described by the equation

$$4(x^2 + y^2) = (1 - z)^2\,, \qquad 0 \leq z \leq 1$$

$\mathcal{S}$ is oriented such that the third component of its normal vector is positive. $\left[\tfrac{1}{4}\,\pi\right]$

Find the surface integral $\iint_{\mathcal{S}} \mathbf{f}\, \mathrm{d}\mathbf{S}$, kde $\mathbf{f} = (x, y, z)$ and the surface $\mathcal{S}$ is described by the equation

$$z(x^2 + y^2) = 1\,, \qquad 1 \leq z \leq 4.$$

$\mathcal{S}$ is oriented such that the third component of its normal vector is positive. $\left[6\pi \ln 2\right]$

Let $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ be a unit vector in the direction of an axis x, y and z, respectively. Find the flux of the vector $\mathbf{v} = y\mathbf{i} - x\mathbf{j} + z^2\mathbf{k}$ through the surface $\mathcal{S}$ given by parametric equations

$$\mathbf{x} = r\cos t\,\mathbf{i} + r\sin t\,\mathbf{j} + r^2\,\mathbf{k}\,, \qquad 0 < t < \pi\,, \quad 0 < r < 1\,,$$

and oriented such that the third component of its normal vector is positive. $\left[\tfrac{1}{3}\,\pi\right]$

Let $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ be a unit vector in the direction of an axis x, y and z, respectively. Find the flux of the vector $\mathbf{v} = y\mathbf{i} + x\mathbf{j} + z\mathbf{k}$ through the surface $\mathcal{S}$ given by parametric equations

$$x = r\cos^2 t\,, \quad y = r\sin^2 t\,, \quad z = r^2\,, \qquad 0 \leq r \leq 1\,, \quad 0 \leq t \leq \tfrac{1}{2}\,\pi$$

and oriented such that the third component of its normal vector is positive. $\left[-\tfrac{1}{4}\right]$

Let $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ be a unit vector in the direction of an axis x, y and z, respectively. Find the flux of the vector $\mathbf{v} = y\mathbf{i} - x\mathbf{k}$ through the surface $\mathcal{S}$ given by the equation

$$x^2 + y^2 = 4\,, \qquad 0 \leq z \leq 3\,, \quad x,\, y > 0\,,$$

and oriented such that the third component of its normal vector is negative. $\left[-\tfrac{3}{2}\right]$

Let $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ be a unit vector in the direction of an axis x, y and z, respectively. Find the flux of the vector $\mathbf{v} = y\mathbf{i} - x\mathbf{j} + z\mathbf{k}$ through the surface $\mathcal{S}$ given by the equation

$$z = x^2 + y^2\,, \qquad z \leq 4$$

and oriented such that the third component of its normal vector is positive. $\left[8\pi\right]$

Let $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ be a unit vector in the direction of an axis x, y and z, respectively. Find the flux of the vector $\mathbf{v} = y\mathbf{i} - x\mathbf{j} + z\mathbf{k}$ through the surface $\mathcal{S}$ given by the equation

$$z = 1 - \sqrt{x^2 + y^2}\,, \qquad x,\, y,\, z \geq 0$$

and oriented such that the third component of its normal vector is positive. $\left[\frac{1}{12}\,\pi\right]$

Let $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ be a unit vector in the direction of an axis x, y and z, respectively. Find the flux of the vector $\mathbf{v} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ through the surface $\mathcal{S}$ given by the equation

$$x^2 + 4y^2 = z^2\,, \qquad 0 \leq z \leq 2$$

and oriented such that the third component of its normal vector is negative. $\left[0\right]$

Let $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ be a unit vector in the direction of an axis x, y and z, respectively. Find the flux of the vector $\mathbf{v} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ through the surface $\mathcal{S}$ given by the equation

$$z = 1 - \sqrt{x^2 + y^2}\,, \qquad 0 \leq z \leq 1$$

and oriented such that the third component of its normal vector is positive. $\left[\pi\right]$

Let $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ be a unit vector in the direction of an axis x, y and z, respectively. Find the flux of the vector $\mathbf{v} = x\mathbf{i} + y\mathbf{j}$ through the surface $\mathcal{S}$ given by the equation

$$z^2 = 4x^2 + y^2\,, \qquad 0 \leq z \leq 2$$

and oriented such that the third component of its normal vector is negative. $\left[\frac{8}{3}\,\pi\right]$

Let $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ be a unit vector in the direction of an axis x, y and z, respectively. Find the flux of the vector $\mathbf{v} = y\mathbf{i} + x\mathbf{j} + z\mathbf{k}$ through the surface $\mathcal{S}$ given by the equation

$$x + y = z^2\,, \qquad y,\, z \geq 0\,, \qquad y + z \leq 2$$

and oriented such that the third component of its normal vector is positive. $\left[\frac{4}{3}\right]$

Let $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ be a unit vector in the direction of an axis x, y and z, respectively. Find the flux of the vector $\mathbf{v} = x\mathbf{j} + z\mathbf{k}$ through the surface $\mathcal{S}$ given by the equation

$$z = 1 - x^2 - y^2\,, \qquad z \geq 0$$

and oriented such that the third component of its normal vector is positive. $\left[\frac{1}{2}\,\pi\right]$

Let $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ be a unit vector in the direction of an axis x, y and z, respectively. Find the flux of the vector $\mathbf{v} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ through a part of a plane which contains the points $A = [1, 0, 0]$, $B = [0, 2, 0]$ and $C = [0, 0, 3]$, lies in the first octant, i.e., $x,\, y,\, z \geq 0$, and oriented such that the third component of its normal vector is positive. $\left[3\right]$

Find the surface integral $\iint_{\mathcal{S}} \mathbf{f}\, \mathrm{d}\mathbf{S}$, where

$$\mathbf{f} = \left(x^2 + xy + y^2, y^2 + yz + z^2, x^2 + xz + z^2\right)$$

and the surface $\mathcal{S}$ is a positively oriented boundary of a tetrahedron

$$x \geq 0\,, \qquad y \geq 0\,, \qquad z \geq 0\,, \qquad x + y + z \leq 1\,.$$

$\left[\frac{3}{8}\right]$

Find the surface integral

$$\iint_{\mathcal{S}} \Big((x^2 + yz)\,\mathrm{d}y\,\mathrm{d}z + (y^2 + xz)\,\mathrm{d}z\,\mathrm{d}x + (z^2 + xy)\,\mathrm{d}x\,\mathrm{d}y\Big),$$

where $\mathcal{S}$ is a half of a positively oriented sphere

$$x^2 + y^2 + z^2 \leq 1\,, \qquad z \geq 0\,.$$

$\left[\frac{1}{2}\,\pi\right]$

Let $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ be a unit vector in the direction of an axis x, y and z, respectively. Find the flux of the vector $\mathbf{v} = xy\mathbf{i} + yz\mathbf{j} + xz\mathbf{k}$ through a positively oriented surface of a cylinder

$$x^2 + y^2 \leq 4\,, \qquad 0 \leq z \leq 1\,.$$

$\left[2\pi\right]$

Let $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ be a unit vector in the direction of an axis x, y and z, respectively. Find the flux of the vector $\mathbf{v} = x^2\mathbf{i} + yz\mathbf{j} + xy\mathbf{k}$ through the surface of a solid

$$x^2 + y^2 \leq z^2\,, \qquad 0 \leq z \leq 1\,,$$

oriented such that the third component of its normal vector on the plane $z = 1$ is positive.

$\left[\frac{1}{4}\,\pi\right]$
