Exercise 6 – differential equations

Solve the initial value problem $x' + \dfrac{x}{t+1} = -t^2, \quad x(0) = 2.$

$$\left[x(t) = \frac{2}{t+1} - \frac{t^3(3t+4)}{12(t+1)}\,, \quad t > -1\,.\right]$$

Solve the initial value problem $x' = -\dfrac{3}{t}\,x + \dfrac{2}{t^3}, \quad x(2) = 3.$

$$\left[x(t) = \frac{20}{t^3} + \frac{2}{t^2}\,, \quad t > 0\,.\right]$$

Solve the initial value problem $x' = -\dfrac{4tx}{t^2+1} + \dfrac{1}{t^2+1}, \quad x(0) = 0.$

$$\left[x(t) = \frac{t(t^2+3)}{3(t^2+1)^2}\,.\right]$$

Solve the initial value problem $x' = x\,\mathrm{tg}\,t + \dfrac{1}{\cos t}, \quad x(0) = 1.$

$$\left[x(t) = \frac{1+t}{\cos t}\,, \quad t \in \left(-\tfrac{1}{2}\,\pi, \tfrac{1}{2}, \pi\right)\,.\right]$$

Solve the initial value problem $x' = -2tx + 2t\mathrm{e}^{-t^2}, \quad x(0) = 4.$

$$\left[x(t) = (t^2+4)\mathrm{e}^{-t^2}\,.\right]$$

Solve the initial value problem $x' = x\,\mathrm{cotg}\,t + \sin 2t, \quad x(\tfrac{1}{2}\,\pi) = 1.$

$$\left[x(t) = 2\sin^2 t - \sin t\,.\right]$$

Solve the initial value problem $x' = \dfrac{2x}{1+t} + (1+t)^3, \quad x(0) = -3.$

$$\left[x(t) = -\tfrac{7}{2}\,(1+t)^2 + \tfrac{1}{2}\,(1+t)^4\,, \quad t > -1\,.\right]$$

Najděte řešení Cauchyho úlohy $x' = x\sin t + \sin t\cos t, \quad x(\tfrac{1}{2}\,\pi) = 2.$

$$\left[x(t) = \mathrm{e}^{-\cos t} + 1 - \cos t\,.\right]$$

Solve the initial value problem $x' = \dfrac{2tx}{1+t^2} + t^2 + 1, \quad x(1) = 4.$

$$\left[x(t) = (1+t)(1+t^2)\,.\right]$$

Solve the initial value problem $x' = -\dfrac{2tx}{1+t^2} + \dfrac{3t^2}{1+t^2}, \quad x(0) = 2.$

$$\left[x(t) = \frac{2+t^3}{1+t^2}\,.\right]$$

Solve the initial value problem $x' = -\dfrac{x}{t} + t, \quad x(2) = 0.$

$$\left[x(t) = \frac{t^3 - 8}{3t}, \quad t > 0\,.\right]$$

Solve the initial value problem $x' - \dfrac{3x}{t} = t^3 \sin t, \quad x\left(-\tfrac{1}{2}\,\pi\right) = -\tfrac{1}{8}\,\pi^3.$

$$\left[x(t) = t^3(1 - \cos t)\,.\right]$$

Solve the initial value problem $x' = \dfrac{tx}{t^2 - 1} + \arcsin t, \quad x(0) = 0.$

$$\left[x(t) = \tfrac{1}{2}\sqrt{1 - t^2}\,\arcsin^2 t\,, \quad -1 < t < 1\,.\right]$$

Solve the initial value problem $x' - x \operatorname{cotg} t = 2t \sin t, \quad x\left(\tfrac{1}{2}\,\pi\right) = 1 + \tfrac{1}{4}\,\pi^2.$

$$\left[x(t) = (1 + t^2)\sin t\,, \quad 0 < t < \pi\,.\right]$$

Solve the initial value problem $x' = \dfrac{tx}{t^2 - 1} - \arccos t, \quad x(0) = 0.$

$$\left[x(t) = \left(\tfrac{1}{2}\,\arccos^2 t - \tfrac{1}{8}\,\pi^2\right)\sqrt{1 - t^2}\,, \quad t \in (-1, 1)\,.\right]$$

Solve the initial value problem $x' - x \operatorname{cotg} t = \sin^2 t, \quad x\left(\tfrac{1}{2}\,\pi\right) = -2.$

$$\left[x(t) = -(2 + \cos t)\sin t\,, \quad 0 < t < \pi\,.\right]$$

Solve the initial value problem $x' + \dfrac{2}{t}\,x = 3 \ln t, \quad x(1) = 0.$

$$\left[x(t) = \frac{1}{3t^2} - \frac{t}{3} + t \ln t\,, \quad t > 0\right]$$

Solve the initial value problem $x' = -2tx + 2t\mathrm{e}^{-t^2}, \quad x(0) = 1.$

$$\left[x(t) = (1 + t^2)\mathrm{e}^{-t^2}\,.\right]$$

Solve the initial value problem $x' = -\dfrac{3}{t}\,x + \dfrac{2}{t^3}, \quad x(1) = 1.$

$$\left[x(t) = \frac{2t - 1}{t^3}, \quad t > 0\,.\right]$$

Solve the initial value problem $x' + 2x = \dfrac{\mathrm{e}^{-2t}}{2 - t}, \quad x(1) = 1.$

$$\left[x(t) = \mathrm{e}^{-2t}\left(\mathrm{e}^2 - \ln(2 - t)\right), \quad t < 2\,.\right]$$

Solve the initial value problem $x' - 3x = \mathrm{e}^{3t} \operatorname{tg} t, \quad x(0) = 2.$

$$\left[x(t) = \left(2 - \ln(\cos t)\right)\mathrm{e}^{3t}\,, \quad t \in \left(-\tfrac{1}{2}\,\pi, \tfrac{1}{2}\,\pi\right)\,.\right]$$

Solve the initial value problem $x' + \dfrac{2x}{t} = \dfrac{1}{t - 2}, \quad x(1) = 3.$

$$\left[x(t) = \frac{t^2 + 4t + 1}{2t^2} + \frac{4\ln(2 - t)}{t^2}, \quad t \in (0, 2)\,.\right]$$

Solve the initial value problem $x' - \dfrac{x}{t} = 2\ln t, \quad x(1) = 0.$ $\left[x(t) = t\ln^2 t\,, \quad t > 0\,.\right]$

Solve the initial value problem $x' + x\,\mathrm{tg}\,t = (t-1)\mathrm{e}^{-t}\cos t, \quad x(0) = 1.$

$\left[x(t) = \left(1 - t\mathrm{e}^{-t}\right)\cos t\,.\right]$

Solve the initial value problem $x' + \dfrac{x}{t} = \sin t, \quad x(\pi) = 0.$ $\left[x(t) = \dfrac{\sin t - \pi}{t} - \cos t\,.\right]$

Solve the initial value problem $x' + x = \dfrac{1}{\mathrm{e}^t(t^2+1)}, \quad x(0) = 0.$ $\left[x(t) = \mathrm{e}^{-t}\,\mathrm{arctg}\,t\,.\right]$

Solve the initial value problem $x' + 2tx = t + t^3, \quad x(0) = 4.$ $\left[x(t) = 4\mathrm{e}^{-t^2} + \tfrac{1}{2}t^2\,.\right]$

Solve the initial value problem $x' + x\sin t = \sin t\cos t, \quad x(0) = 2.$

$\left[x(t) = 1 + \cos t\,.\right]$

Solve the initial value problem $x' + x\cos t = \sin t\cos t, \quad x(0) = 1.$

$\left[x(t) = 2\mathrm{e}^{-\sin t} - 1 + \sin t\,.\right]$

Solve the initial value problem $x' + \dfrac{2x}{t} = \dfrac{4\ln t}{t}, \quad x(1) = 2.$

$\left[x(t) = \dfrac{3}{t^2} + 2\ln t - 1\,, \quad t > 0\,.\right]$

Solve the initial value problem $x' + 2x = 5\mathrm{e}^{-t}\cos 2t, \quad x(0) = 2.$

$\left[x(t) = \mathrm{e}^{-2t} + \mathrm{e}^{-t}\cos 2t + 2\mathrm{e}^{-t}\sin 2t\,.\right]$

Solve the initial value problem $x' + 3x = \mathrm{e}^{-t}\big(2\sin t - \cos t\big), \quad x(0) = 1.$

$\left[x(t) = \tfrac{9}{5}\,\mathrm{e}^{-3t} + \tfrac{1}{5}\,\mathrm{e}^{-t}(3\sin t - 4\cos t)\,.\right]$

Solve the initial value problem $x' + 2x = 4\mathrm{e}^{-t}\cos 2t, \quad x(0) = 1.$

$\left[x(t) = \tfrac{1}{5}\,\mathrm{e}^{-2t} + \tfrac{4}{5}\,\mathrm{e}^{-t}(\cos 2t + 2\sin 2t)\,.\right]$

Solve the initial value problem $x' + x = 3\sin 4t, \quad x(0) = -1.$

$\left[x(t) = -\tfrac{5}{17}\,\mathrm{e}^{-t} - \tfrac{12}{17}\,\cos 4t + \tfrac{3}{17}\,\sin 4t\,.\right]$

Solve the initial value problem $x' + 3x = 2\cos 2t, \quad x(0) = 0.$

$\left[x(t) = \tfrac{1}{13}\left(-6\mathrm{e}^{-3t} + 6\cos 2t + 4\sin 2t\right).\right]$

Solve the initial value problem $x' + 2x = 3\mathrm{e}^{t}\cos 2t + \mathrm{e}^{t}\sin 2t, \quad x(0) = 1.$

$\left[x(t) = \tfrac{6}{13}\,\mathrm{e}^{-2t} + \tfrac{1}{13}\,\mathrm{e}^{t}(7\cos 2t + 9\sin 2t)\,.\right]$

Solve the initial value problem $x' - 3x = 2\mathrm{e}^{-t}\cos t + 3\mathrm{e}^{-t}\sin t, \quad x(0) = 2.$

$$\left[x(t) = \tfrac{45}{17}\,\mathrm{e}^{3t} - \tfrac{1}{17}\,\mathrm{e}^{-t}(11\cos t + 10\sin t)\,.\right]$$

Solve the initial value problem $x' + x = \mathrm{e}^{-t}\cos t + 2\mathrm{e}^{-t}\sin t, \quad x(0) = -1.$

$$\left[x(t) = \mathrm{e}^{-t}(1 - 2\cos t + \sin t)\,.\right]$$

Solve the initial value problem $x' - 2x = (t+1)\mathrm{e}^{t} + 3\mathrm{e}^{2t}, \quad x(0) = 3.$

$$\left[x(t) = (3t+5)\mathrm{e}^{2t} - (t+2)\mathrm{e}^{t}\,.\right]$$

Solve the initial value problem $x' + 3x = (6t+1)\mathrm{e}^{3t} - \mathrm{e}^{-3t}, \quad x(0) = 2.$

$$\left[x(t) = (2-t)\mathrm{e}^{-3t} + t\mathrm{e}^{3t}\,.\right]$$

Solve the initial value problem $x' + 2x = \mathrm{e}^{-t}\sin 3t, \quad x(0) = 0.$

$$\left[x(t) = \tfrac{3}{10}\,\mathrm{e}^{-2t} - \tfrac{3}{10}\,\mathrm{e}^{-t}\cos 3t + \tfrac{1}{10}\,\mathrm{e}^{-t}\sin 3t\,.\right]$$

Solve the initial value problem $x' - 3x = \mathrm{e}^{2t}\cos t, \quad x(0) = 1.$

$$\left[x(t) = \tfrac{3}{2}\,\mathrm{e}^{3t} + \tfrac{1}{2}\,\mathrm{e}^{2t}(\sin t - \cos t)\,.\right]$$

Solve the initial value problem $x' + x = \sin 2t - \cos 2t, \quad x(0) = 0.$

$$\left[x(t) = \tfrac{3}{5}\,\mathrm{e}^{-t} - \tfrac{3}{5}\,\cos 2t - \tfrac{1}{5}\,\sin 2t\,.\right]$$

Solve the initial value problem $x' - x = \mathrm{e}^{-t} - \cos 2t, \quad x(0) = 0.$

$$\left[x(t) = \tfrac{3}{10}\,\mathrm{e}^{t} - \tfrac{1}{2}\,\mathrm{e}^{-t} + \tfrac{1}{5}\,\cos 2t - \tfrac{2}{5}\,\sin 2t\,.\right]$$

Solve the initial value problem $x' - 2x = \mathrm{e}^{t}(1 + \mathrm{e}^{t}), \quad x(0) = 0.$

$$\left[x(t) = (1+t)\mathrm{e}^{2t} - \mathrm{e}^{t}\,.\right]$$

Solve the initial value problem $txx' - \sqrt{x^2+1} = 0, \quad x(1) = -2.$

$$\left[x(t) = -\sqrt{(\ln t + \sqrt{5})^2 - 1}\,, \quad t > \mathrm{e}^{1-\sqrt{5}}\right]$$

Solve the initial value problem $x' - x^2 = 1, \quad x\left(\tfrac{5}{4}\,\pi\right) = 1.$

$$\left[x(t) = \operatorname{tg} t\,, \quad t \in \left(\tfrac{1}{2}\,\pi, \tfrac{3}{2}\,\pi\right).\right]$$

Solve the initial value problem $tx' + x = x\ln x, \quad x(1) = 1.$

$$\left[x(t) = \mathrm{e}^{1-t}\,, \quad \text{radši} \;\; t > 0\,.\right]$$

Solve the initial value problem $(1 + \mathrm{e}^{t})xx' = \mathrm{e}^{t}, \quad x(0) = -1.$

$\left[x(t) = -\sqrt{1 + 2\ln\left(e^t + 1\right) - \ln 4}\,.\right]$

Solve the initial value problem $x' = \dfrac{1 - t^2}{tx}$, $\quad x(1) = 2$. $\qquad \left[x(t) = \sqrt{5 + 2\ln t - t^2}\,.\right]$

Solve the initial value problem $x' = -\dfrac{2tx^2}{t^2 - 1}$, $\quad x(0) = 1$.

$\left[x(t) = \dfrac{1}{1 + \ln(1 - t^2)}\,, \quad t \in \left(-\sqrt{1 - e^{-1}}, \sqrt{1 - e^{-1}}\right).\right]$

Solve the initial value problem $x' = -\dfrac{x^2 + 1}{tx}$, $\quad x(-1) = -2$.

$\left[x(t) = \dfrac{\sqrt{5 - t^2}}{t}, \quad t \in \left(-\sqrt{5}, 0\right)\right]$

Solve the initial value problem $x' = \dfrac{2t\sqrt{1 - x^2}}{t^2 - 4}$, $\quad x(0) = 0$.

$\left[x(t) = \sin\left(\ln(1 - \tfrac{1}{4}t^2)\right), \quad -2 < t < 2\,.\right]$

Solve the initial value problem $x' = tx^2 - t$, $\quad x(0) = 0$. $\qquad \left[x(t) = \dfrac{1 - e^{t^2}}{1 + e^{t^2}}\,.\right]$

Solve the initial value problem $x' + x^2 = 2x$, $\quad x(1) = 1$. $\qquad \left[x(t) = \dfrac{2e^{2(t-1)}}{1 + e^{2(t-1)}}\,.\right]$

Solve the initial value problem $x' + x = \dfrac{1}{x}$, $\quad x(0) = 2$. $\qquad \left[x(t) = \sqrt{3e^{-2t} + 1}\,.\right]$

Solve the initial value problem $x' + tx^2 + t = 0$, $\quad x(0) = 1$.

$\left[x(t) = \operatorname{tg}\left(\tfrac{1}{4}\pi - \tfrac{1}{2}t^2\right), \quad -\sqrt{\tfrac{3}{2}\pi} < t < \sqrt{\tfrac{3}{2}\pi}\,.\right]$

Solve the initial value problem $x' + x^2 \sin t + \sin t = 0$, $\quad x(0) = 0$. $\qquad \left[x(t) = \operatorname{tg}(\cos t - 1)\,.\right]$

Solve the initial value problem $x' + tx^2 = t$, $\quad x(0) = 0$. $\qquad \left[x(t) = \dfrac{e^{t^2} - 1}{e^{t^2} + 1}\,.\right]$

Solve the initial value problem $x' + \dfrac{x^2}{t} = \dfrac{1}{t}$, $\quad x(1) = 0$.

$\left[x(t) = \dfrac{t^2 - 1}{t^2 + 1}\,, \quad \text{radši} \quad t > 0\,.\right]$

Solve the initial value problem $x' - x = x^2$, $\quad x(0) = 2$.

$\left[x(t) = \dfrac{2e^t}{3 - 2e^t}\,, \quad t \in \left(-\infty, \ln\tfrac{3}{2}\right).\right]$

Solve the initial value problem $\left(1 + e^{-t}\right)x' = \dfrac{1}{x}$, $\quad x(0) = 1$.

$\left[x(t) = \sqrt{1 + 2\ln\left(1 + e^t\right) - \ln 4}\,.\right]$

Solve the initial value problem $tx' = \operatorname{tg} x, \quad x(1) = \frac{1}{6}\pi$.

$$\left[x(t) = \arcsin\left(\tfrac{1}{2}\,t\right), \quad \text{radši} \;\; t \in (0,2)\,.\right]$$

Solve the initial value problem $txx' + x^2 + 1 = 0, \quad x(1) = -2$.

$$\left[x(t) = -\frac{\sqrt{5-t^2}}{t}\,, \quad 0 < t < \sqrt{5}\,.\right]$$

Solve the initial value problem $x' + x = x \ln x, \quad x(0) = \mathrm{e}^2$. $\left[x(t) = \mathrm{e}^{\mathrm{e}^t+1}\,.\right]$

Find the general solution of the differential equation $x'' - 3x' + 2x = 2t^2 - 3 + 5\cos t$.

$$\left[x(t) = c_1\mathrm{e}^t + c_2\mathrm{e}^{2t} + t^2 + 3t + 2 + \tfrac{1}{2}\,\cos t - \tfrac{3}{2}\,\sin t\,.\right]$$

Find the general solution of the differential equation $x'' - 4x' + 4x = 2\mathrm{e}^{2t} + 4\sin 2t$.

$$\left[x(t) = c_1\mathrm{e}^{2t} + c_2 t\mathrm{e}^{2t} + t^2\mathrm{e}^{2t} + \tfrac{1}{2}\,\cos 2t\,.\right]$$

Find the general solution of the differential equation $x'' + x = 2t^2 - t + 2 - 2t\mathrm{e}^{-t}$.

$$\left[x(t) = c_1\cos t + c_2\sin t + 2t^2 - t - 2 - \mathrm{e}^{-t}(t+1)\,.\right]$$

Find the general solution of the differential equation $x'' - 4x = -4\mathrm{e}^{-2t} + 17\mathrm{e}^{2t}\cos t$.

$$\left[x(t) = c_1\mathrm{e}^{2t} + c_2\mathrm{e}^{-2t} + t\mathrm{e}^{-2t} + \mathrm{e}^{2t}(-\cos t + 4\sin t)\,.\right]$$

Find the general solution of the differential equation $x'' + x = \mathrm{e}^{-t} + 2t\mathrm{e}^t$.

$$\left[x(t) = c_1\cos t + c_2\sin t + \tfrac{1}{2}\,\mathrm{e}^{-t} + (t-1)\mathrm{e}^t\,.\right]$$

Find the general solution of the differential equation $x'' + x = t + \sin t$.

$$\left[x(t) = c_1\cos t + c_2\sin t + t - \tfrac{1}{2}\,t\cos t\,.\right]$$

Find the general solution of the differential equation $x'' - 4x = 2\mathrm{e}^{-2t} + 6\sin 2t$.

$$\left[x(t) = c_1\mathrm{e}^{2t} + c_2\mathrm{e}^{-2t} - \tfrac{1}{2}\,t\mathrm{e}^{-2t} - \tfrac{3}{4}\,\sin 2t\,.\right]$$

Find the general solution of the differential equation $x'' + x' - 2x = 5\cos 2t + 6\mathrm{e}^{-2t}$.

$$\left[x(t) = c_1\mathrm{e}^{t} + c_2\mathrm{e}^{-2t} - 2t\mathrm{e}^{-2t} + \tfrac{1}{4}\,(\sin 2t - 3\cos 2t)\,.\right]$$

Find the general solution of the differential equation $x'' + 4x = 8\sin^2 t$.

$$\left[x(t) = c_1\cos 2t + c_2\sin 2t + 1 - t\sin 2t\,.\right]$$

Find the general solution of the differential equation $x'' + x = 2 - t^2 - 2t\mathrm{e}^{-t}$.

$$\left[x(t) = c_1\cos t + c_2\sin t - t^2 + 4 - \mathrm{e}^{-t}(t+1)\,.\right]$$

Find the general solution of the differential equation $x'' + x' - 2x = -6\mathrm{e}^{-2t} + 5\sin t$.

$$\left[x(t) = c_1 e^{t} + c_2 e^{-2t} + 2t e^{-2t} - \tfrac{1}{2}(\cos t + 3\sin t)\,.\right]$$

Find the general solution of the differential equation $x'' - x' - 2x = 3e^{-t} - 10\cos 2t$.

$$\left[x(t) = c_1 e^{-t} + c_2 e^{2t} - t e^{-t} + \tfrac{3}{2}\cos 2t + \tfrac{1}{2}\sin 2t\,.\right]$$

Find the general solution of the differential equation $x'' + 4x' + 4x = e^{-2t} - 8t$.

$$\left[x(t) = c_1 e^{-2t} + c_2 t e^{-2t} + \tfrac{1}{2} t^2 e^{-2t} + 2(1-t)\,.\right]$$

Find the general solution of the differential equation $x'' + 4x' + 13x = 3e^{-2t} + 20\sin t$.

$$\left[x(t) = c_1 e^{-2t}\cos 3t + c_2 e^{-2t}\sin 3t + \tfrac{1}{3} e^{-2t} - \tfrac{1}{2}\cos t + \tfrac{3}{2}\sin t\,.\right]$$

Find the general solution of the differential equation $x'' + 4x = 2e^{-2t} + 5\cos 3t$.

$$\left[x(t) = c_1\cos 2t + c_2\sin 2t + \tfrac{1}{4} e^{-2t} - \cos 3t\,.\right]$$

Find the general solution of the differential equation $x'' + 2x' - 3x = 2e^{-t} - 15\cos 3t$.

$$\left[x(t) = c_1 e^{t} + c_2 e^{-3t} - \tfrac{1}{2} e^{-t} + \cos 3t - \tfrac{1}{2}\sin 3t\,.\right]$$

Solve the initial value problem

$$\begin{aligned} x_1' + x_2' + 4x_1 + 2x_2 &= 0\,, \qquad & x_1(0) &= 2\,,\\ 3x_1' + x_2' + 10x_1 &= 0\,, \qquad & x_2(0) &= 1\,. \end{aligned}$$

$$\left[x_1(t) = e^{-3t}(2\cos t + \sin t)\,, \quad x_2(t) = e^{-3t}(\cos t - 2\sin t)\,.\right]$$

Solve the initial value problem

$$\begin{aligned} x_1' - x_2' + 2x_1 + x_2 &= 0\,, \qquad & x_1(0) &= 2\,,\\ 5x_1' + x_2' + 3x_2 &= 0\,, \qquad & x_2(0) &= 5\,. \end{aligned}$$

$$\left[x_1(t) = 2\cos t - 4\sin t\,, \quad x_2(t) = 5\cos t + 5\sin t\,.\right]$$

Solve the initial value problem

$$\begin{aligned} x_1' - 4x_2' + 5x_1 &= 0\,, \qquad & x_1(0) &= 2\,,\\ 2x_1' - 3x_2' + 5x_2 &= 0\,, \qquad & x_2(0) &= 4\,. \end{aligned}$$

$$\left[x_1(t) = e^{t}(2\cos 2t - 6\sin 2t)\,, \quad x_2(t) = e^{t}(4\cos 2t - 2\sin 2t)\,.\right]$$

Solve the initial value problem

$$\begin{aligned} x_1' + x_2' + 5x_2 &= 0\,, \qquad & x_1(0) &= 0\,,\\ 3x_1' - 2x_2' + 5x_1 &= 0\,, \qquad & x_2(0) &= 5\,. \end{aligned}$$

$$\left[x_1(t) = -10e^{-2t}\sin t\,, \quad x_2(t) = 5e^{-2t}(\cos t - \sin t)\,.\right]$$

Solve the initial value problem

$$\begin{aligned} x_1' + x_2' - 4x_1 &= 0\,, \qquad & x_1(0) &= 1\,, \\ x_1' - x_2' - 2x_1 + 2x_2 &= 0\,, & x_2(0) &= -1\,. \end{aligned}$$

$$\left[x_1(t) = (2t+1)e^{2t}\,, \quad x_2(t) = (2t-1)e^{2t}\,.\right]$$

Solve the initial value problem

$$\begin{aligned} x_1' - \ \ x_2' + 3x_1 + x_2 &= 0\,, \qquad & x_1(0) &= 8\,, \\ 2x_1' - 3x_2' + 3x_1 + 6x_2 &= 0\,, & x_2(0) &= 0\,. \end{aligned}$$

$$\left[x_1(t) = 9e^{-5t} - e^{3t}\,, \quad x_2(t) = 3e^{-5t} - 3e^{3t}\,.\right]$$

Solve the initial value problem

$$\begin{aligned} 2x_1' + \ \ x_2' + 2x_1 - 2x_2 &= 0\,, \qquad & x_1(0) &= 1\,, \\ x_1' + 2x_2' + 10x_1 + 5x_2 &= 0\,, & x_2(0) &= 2\,. \end{aligned}$$

$$\left[x_1(t) = e^{-t}(\cos 3t + 3\sin 3t)\,, \quad x_2(t) = e^{-t}(2\cos 3t - 4\sin 3t)\,.\right]$$

Solve the initial value problem

$$\begin{aligned} 2x_1' + x_2' - 8x_1 &= 0\,, \qquad & x_1(0) &= 1\,, \\ 2x_1' - x_2' + 4x_2 &= 0 & x_2(0) &= 4\,. \end{aligned}$$

$$\left[x_1(t) = e^{2t}\big(\cos 2t - 2\sin 2t\big)\,, \quad x_2(t) = 2e^{2t}\big(2\cos 2t + \sin 2t\big)\,.\right]$$

Find the general solution of the system of differential equations

$$x_1' = x_1 - 3x_2 + 3t + 1\,, \qquad x_2' = 4x_1 - 6x_2 + 2e^t\,.$$

$$\left[x_1(t) = c_1e^{-2t} + 3c_2e^{-3t} + 3t - 1 - \tfrac{1}{2}\,e^t\,, \quad x_2(t) = c_1e^{-2t} + 4c_2e^{-3t} + 2t - 1\,.\right]$$

Find the general solution of the system of differential equations

$$x_1' = 2x_2\,, \qquad x_2' = -5x_1 + 7x_2 + (2t+1)e^{3t}\,.$$

$$\left[x_1(t) = c_1e^{2t} + 2c_2e^{5t} - 2te^{3t}\,, \quad x_2(t) = c_1e^{2t} + 5c_2e^{5t} - (3t+1)e^{3t}\,.\right]$$

Find the general solution of the system of differential equations

$$x_1' = 7x_1 - 18x_2 + e^{-t}\,, \qquad x_2' = 3x_1 - 8x_2 + te^{-t}\,.$$

$$\left[x_1(t) = 3c_1\mathrm{e}^{t} + 2c_2\mathrm{e}^{-2t} + (9t-8)\mathrm{e}^{-t} \quad x_2(t) = c_2\mathrm{e}^{t} + c_2\mathrm{e}^{-2t} + (4t-4)\mathrm{e}^{-t}\,.\right]$$

Find the general solution of the system of differential equations

$$x_1' = x_2 + (4t+3)\mathrm{e}^{t}\,, \qquad x_2' = -x_1 - 2x_2 - \mathrm{e}^{t}\,.$$

$$\left[x_1(t) = c_1\mathrm{e}^{-t} + c_2(t+1)\mathrm{e}^{-t} + 3t\mathrm{e}^{t}\,, \quad x_2(t) = -c_1\mathrm{e}^{-t} - c_2t\mathrm{e}^{-t} - t\mathrm{e}^{t}\,.\right]$$

Find the general solution of the system of differential equations

$$x_1' = x_1 + x_2 + 1\,, \qquad x_2' = -x_1 + x_2 + 2t\,.$$

$$\left[x_1(t) = c_1\mathrm{e}^{t}\cos t + c_2\mathrm{e}^{t}\sin t + t + \tfrac{1}{2}\,, \quad x_2(t) = -c_1\mathrm{e}^{t}\sin t + c_2\mathrm{e}^{t}\cos t - t - \tfrac{1}{2}\,.\right]$$

Find the general solution of the system of differential equations

$$x_1' = 2x_1 - 3x_2 + 10t\mathrm{e}^{t}\,, \qquad x_2' = 3x_1 + 2x_2 - 2\mathrm{e}^{t}\,.$$

$$\left[\begin{array}{l} x_1(t) = c_1\mathrm{e}^{2t}\cos 3t + c_2\mathrm{e}^{2t}\sin 3t + \left(\tfrac{7}{5} - t\right)\mathrm{e}^{t}\,, \\ x_2(t) = c_1\mathrm{e}^{2t}\sin 3t - c_2\mathrm{e}^{2t}\cos 3t + \left(\tfrac{4}{5} + 3t\right)\mathrm{e}^{t}\,. \end{array}\right]$$

Find the general solution of the system of differential equations

$$x_1' = x_1 - 5x_2 + 8t\,, \qquad x_2' = 5x_1 - 9x_2\,.$$

$$\left[x_1(t) = c_1\mathrm{e}^{-4t} + c_2(5t+1)\mathrm{e}^{-4t} + \tfrac{9}{2}\,t - \tfrac{7}{4}\,, \quad x_2(t) = c_1\mathrm{e}^{-4t} + 5c_2t\mathrm{e}^{-4t} + \tfrac{5}{2}\,t - \tfrac{5}{4}\,.\right]$$

Find the general solution of the system of differential equations

$$x_1' = 2x_1 - x_2 - 4t\mathrm{e}^{t}\,, \qquad x_2' = x_1 + 4x_2\,.$$

$$\left[x_1(t) = c_1\mathrm{e}^{3t} + c_2(t-1)\mathrm{e}^{3t} + (3t+2)\mathrm{e}^{t}\,, \quad x_2(t) = -c_1\mathrm{e}^{3t} - c_2t\mathrm{e}^{-3t} - (t+1)\mathrm{e}^{t}\,.\right]$$

Find the general solution of the system of differential equations

$$x_1' = x_1 + 3x_2 + 5\mathrm{e}^{-t}\,, \qquad x_2' = 3x_1 + x_2 - 3\mathrm{e}^{t}\,.$$

$$\left[x_1(t) = c_1\mathrm{e}^{-2t} + c_2\mathrm{e}^{4t} + 2\mathrm{e}^{-t} + \mathrm{e}^{t}\,, \quad x_2(t) = -c_1\mathrm{e}^{-2t} + c_2\mathrm{e}^{4t} - 3\mathrm{e}^{-t}\,.\right]$$

Find the general solution of the system of differential equations

$$x_1' = -3x_1 - 8x_2 + \mathrm{e}^{2t}\,, \qquad x_2' = 2x_1 + 5x_2 + 3\mathrm{e}^{2t}\,.$$

$$\left[x_1(t) = 2c_1\mathrm{e}^{t} + c_2(4t-1)\mathrm{e}^{t} - 27\mathrm{e}^{2t}\,, \quad x_2(t) = -c_1\mathrm{e}^{t} - 2c_2t\mathrm{e}^{t} + 17\mathrm{e}^{2t}\,.\right]$$

Find the general solution of the system of differential equations

$$x_1' = 3x_1 - 2x_2 + 5t\mathrm{e}^{3t}\,, \qquad x_2' = 5x_1 + x_2 - \mathrm{e}^{3t}\,.$$

$$\left[\begin{aligned} x_1(t) &= 2c_1\mathrm{e}^{2t}\cos 3t + 2c_2\mathrm{e}^{2t}\sin 3t + \tfrac{1}{2}\,(2t+1)\mathrm{e}^{3t}\,, \\ x_2(t) &= c_1\mathrm{e}^{2t}(\cos 3t + 3\sin 3t) + c_2\mathrm{e}^{2t}(\sin 3t - 3\cos 3t) + \tfrac{1}{2}\,(5t-1)\mathrm{e}^{3t}\,. \end{aligned}\right]$$

Find the general solution of the system of differential equations

$$x_1' = -4x_1 - 2x_2 + 3\mathrm{e}^{-t}\,, \qquad x_2' = 3x_1 + x_2 - \mathrm{e}^{-t}\,.$$

$$\left[x_1(t) = 2c_1\mathrm{e}^{-t} + c_2\mathrm{e}^{-2t} + (1-4t)\mathrm{e}^{-t}\,, \quad x_2(t) = -3c_1\mathrm{e}^{-t} - c_2\mathrm{e}^{-2t} + (2+6t)\mathrm{e}^{-t}\,.\right]$$

Find the general solution of the system of differential equations

$$x_1' = 3x_1 - x_2 - 2\mathrm{e}^{t}\,, \qquad x_2' = x_1 + x_2 + 8\mathrm{e}^{-2t}\,.$$

$$\left[x_1(t) = c_1\mathrm{e}^{2t} + c_2(t+1)\mathrm{e}^{2t} - \tfrac{1}{2}\,\mathrm{e}^{-2t}\,, \quad x_2(t) = c_1\mathrm{e}^{2t} + c_2t\mathrm{e}^{2t} - 2\mathrm{e}^{t} - \tfrac{5}{2}\,\mathrm{e}^{-2t}\,.\right]$$

Find the general solution of the system of differential equations

$$x_1' = 3x_1 - 2x_2 + (2t+1)\mathrm{e}^{t}\,, \qquad x_2' = 4x_1 - x_2 - \mathrm{e}^{t}\,.$$

$$\left[\begin{aligned} x_1(t) &= c_1\mathrm{e}^{t}\cos 2t + c_2\mathrm{e}^{t}\sin 2t + \big(t+\tfrac{3}{2}\big)\mathrm{e}^{t}\,, \\ x_2(t) &= c_1\mathrm{e}^{t}(\cos 2t + \sin 2t) + c_2\mathrm{e}^{t}(\sin 2t - \cos 2t) + \big(2t+\tfrac{3}{2}\big)\mathrm{e}^{t}\,. \end{aligned}\right]$$

Find the general solution of the system of differential equations

$$x_1' = 7x_1 - 4x_2 + 4\mathrm{e}^{t}\,, \qquad x_2' = 4x_1 - 3x_2 - 5\,.$$

$$\left[x_1(t) = c_1\mathrm{e}^{-t} + 2c_2\mathrm{e}^{5t} - 2\mathrm{e}^{t} - 4\,, \quad x_2(t) = 2c_1\mathrm{e}^{-t} + c_2\mathrm{e}^{5t} - 2\mathrm{e}^{t} - 7\,.\right]$$

Find the general solution of the system of differential equations

$$x_1' = 2x_1 - 3x_2 - 4\mathrm{e}^{t}\,, \qquad x_2' = 3x_1 - 4x_2 + 2\mathrm{e}^{2t}\,.$$

$$\left[x_1(t) = c_1\mathrm{e}^{-t} + c_2(3t+1)\mathrm{e}^{-t} - 5\mathrm{e}^{t} - \tfrac{2}{3}\,\mathrm{e}^{2t}\,, \quad x_2(t) = c_1\mathrm{e}^{-t} + 3c_2t\mathrm{e}^{-t} - 3\mathrm{e}^{t}\,.\right]$$
