From Parlor Games to Computer Networks

MAGDALENA HYKŠOVÁ, PRAGUE

The paper picks up the threads of author's contribution [5] given in Miesenbach two years ago that discussed several milestones in the history of game theory, including the first known mixed strategy solution related to the card game *le Her*. The aim of this article is to outline the natural connection of the initial game theoretical ideas to the contemporary problems of computer and transportation networking. A typical example is represented by the Internet: here various subjects like network operators, service providers and users interact; moreover, the network itself was not designed by a single person or institution but emerged from the interaction of various entities. Different "players" compete with each other, bargain, build coalitions or learn optimal strategies throughout the time.

1 Non-Cooperative Games

1.1 Historical Overview: from Cournot to Nash

Recall that fundamental solution concept of non-cooperative games, Nash equilibrium (see [5], p. 49), has its roots already in the work [2] published by Antoine Augustin Cournot (1801–1877) in 1838. Besides other topics, Cournot investigates the demand function $D = F(p)$ which he assumes to be downward-sloping and draws it in price-quantity space; this function summarizes the empirical relationship between the price *p* and the quantity *D* sold. The fifth chapter contains a detailed analysis of monopoly. Cournot introduces the concept of a cost function $\phi(D)$ and gives a mathematical solution of the maximization of the profit $pF(p) - \phi(D)$ considered as the function of the price *p*. The next chapter deals with the impact of various taxes and other fees on the income of producers and consumers. The seventh chapter contains today famous Cournot duopoly model: *Let us now imagine two proprietors and two springs of which the qualities are identical, and which, on account of their similar positions, supply the same market in competition.*[1] Cournot denotes by D_1 (D_2) the sales from the first (second) spring and considers the price *p* as the function of the total sales, $p = f(D_1 + D_2)$. First he neglects the cost of production and investigates the situation where the two proprietors – each of them independently – seek to maximize their profits expressed by $D_1 f(D_1 + D_2)$ and $D_2 f(D_1 + D_2)$.

Today we would speak about a *game*: each proprietor controls only a part of the total production; the price he receives for his products to sell them all therefore depends not only on his own decision, but on the decision of the opponent, too, and we would use the best-reply analysis to find the equilibrium point. It is remarkable that Cournot did exactly the same, only without the word "game":

Proprietor (1) can have no direct influence on the determination of D_2: all that he can do, when D_2 has been determined by proprietor (2), is to choose for D_1 the value which is best for him. ... Analytically this is equivalent to saying that D_1 will be determined in terms of D_2 by the condition

$$\frac{d\left[D_1 f(D_1 + D_2)\right]}{dD_1} = 0,$$

[1] [2], English translation, p. 79.

In: *Von der Tontafeln zum Internet. VIII. Österreichisches Symposion zur Geschichte der Mathematik*, TU Wien, Wien, 2006, pp. 184-191.

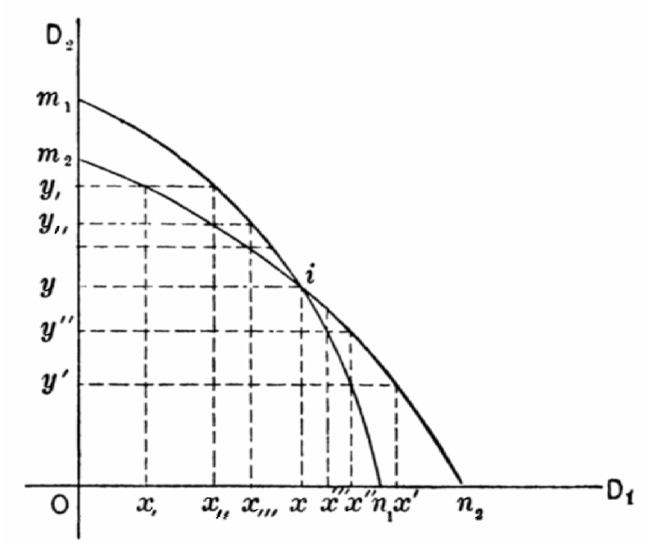
and that D_2 *will be determined in terms of* D_1 *by the analogous condition*

$$\frac{d\left[D_2 f(D_1+D_2)\right]}{dD_2}=0\,,$$ [2]

whence it follows that the final values of D_1 *and* D_2*, and consequently of D and of* p*, will be determined by the system of equations*

$$(1)\quad f(D_1+D_2)+D_1 f'(D_1+D_2)=0, \qquad\qquad (2)\quad f(D_1+D_2)+D_2 f'(D_1+D_2)=0\,.$$

Then Cournot constructs the curves that we today call “reaction curves”, investigates the dynamics of the duopoly “game” and comes to the equilibrium solution that also represents the globally stable steady state:



Let us suppose the curve $m_1 n_1$ *to be the plot of equation (1), and the curve* $m_1 n_1$ *that of equation (2), the variables* D_1 *and* D_2 *being represented by rectangular coordinates. If proprietor (1) should adopt for* D_1 *a value represented by* ox_1*, proprietor (2) would adopt for* D_2 *the value* oy_1*, which, for the supposed value of* D_1*, would give him the greatest profit. But then, for the same reason, producer (1) ought to adopt for* D_1 *the value* ox_{11}*, which gives the maximum profit when* D_2 *has the value* oy_1*. This would bring producer (2) to the value* oy_{11} *for* D_2*, and so forth; from which it is evident that an equilibrium can only be established where the coordinates ox and oy of the point of intersection i represent the values of* D_1 *and* D_2*. The same construction repeated on a point of the figure on the other side of the point i leads to symmetrical results.*

The state of equilibrium corresponding to the system of values ox and oy is therefore stable; i.e. if either of the producers, misled as to his true interest, leaves it temporarily, he will be brought back to it by a series of reactions, constantly declining in amplitude, and of which the dotted lines of the figure give a representation by their arrangement in steps. [3]

In the same chapter Cournot also considers the case of n producers and takes into account also the costs ϕ_i of production, which leads him to the system of equations:

$$f(D)+D_i f'(D)-\phi_i'(D_i)=0, \quad \text{where} \quad i=1,2,\ldots,n, \quad D=D_1+D_2+\cdots+D_n\,.$$

It is not possible to discuss here the whole development of duopoly and oligopoly models. Let us only mention the duopoly model that we will need later in this contribution, namely the model proposed by Heinrich von Stackelberg (1905–1946) in his book [16] published in 1934. Stackelberg considers the situation where the two duopolists act sequentially. One of them – usually called *leader* – moves first, choosing a quantity. The second one – *follower* – observes the leader’s choice and then picks a quantity that repre-

[2] Cournot uses the same notation for the derivative of a one variable function and for partial derivatives.

[3] [2], English translation, pp. 80–81, figure at the end of the book.

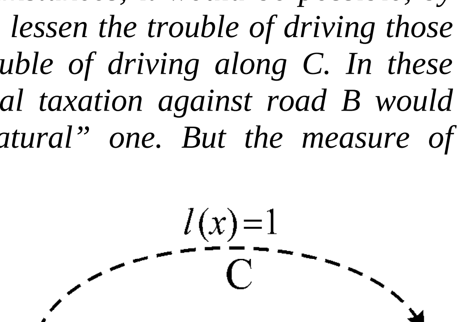
sents his best reply; in his decision, the leader takes into account this follower's reaction. It has become common to speak about a Stackelberg game in the case of any game where one player acts as a leader in the sense that he chooses his strategy first, while the other players act as the followers: they observe the strategy chosen by the leader and then choose their strategies to maximize their payoffs.

1.2 Applications: From Oligopoly to Network Optimization

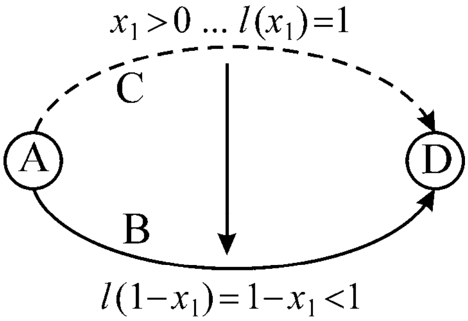
It is not necessary to recall that within the second half of the twentieth century game theory became an inseparable part of the disciplines that deal with the interactions of people or institutions lead by people (economy, political or military sciences). With the fast development of computer as well as transportation networks the game theoretical approach has gradually gained a prominent position also in this optimization domain. Whether we speak about data or cars, the fundamental problem is almost the same: we are given a network and a rate of traffic between the source and destination nodes, and seek an assignment of traffic to source-destination paths. Usually it is assumed that each network user controls a negligible fraction of the overall traffic and the time needed to run through a single link of the network is load-dependent, that is, the common latency suffered by all traffic on the link increases with the increase of the link congestion. When there is a central authority with the competence to control the traffic flow to achieve the best performance of the network, it can prescribe the *optimal assignment* as a feasible assignment minimizing the total latency. Nevertheless, the network users are often free to act according to their own selfish interests; in this case we can expect the network traffic to converge to the *Nash equilibrium*.

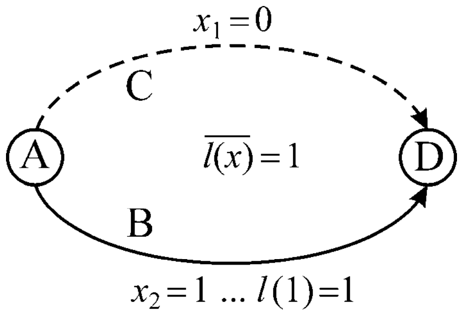
The first example of an equilibrium network flow emerged in the field of economics: it is contained in the first edition of the 1920 book [10] written by Arthur Cecil Pigou (1877 – 1959), the English economist known above all for his advocacy of so-called *Pigouvian taxes* – discouraging taxes on activities that cause negative externalities (pollution, traffic congestion). Although Pigou could not explicitly use the equilibrium concept introduced by John Nash almost 30 years later, he expressed the same:

Suppose there are two roads ABD and ACD both leading from A to D. If left to itself, traffic would be so distributed that the trouble involved in driving a "representative" cart along each of the two roads would be equal [in other words, no driver can improve his travel time by changing the route]. *But, in some circumstances, it would be possible, by shifting a few carts from route B to route C, greatly to lessen the trouble of driving those still left on B, while only slightly increasing the trouble of driving along C. In these circumstances a rightly chosen measure of differential taxation against road B would create an "artificial" situation superior to the "natural" one. But the measure of differentiation must be rightly chosen.*



Today we can imagine *ACD* as a road that is broad enough to accomodate all cars traveling from *A* to *D* without congestion, but with many traffic lights that prolong the travel time which is always equal to one hour. *ABD* can represent a highway that is fast but susceptible to congestion. Denote by x the fraction of the overall traffic from *A* to *D* using the particular road, and $l(x)$ the travel time experienced by drivers on that road. If the fraction of drivers using the road *ACD* was positive, the travel time on the road *ABD* would be lower than one hour and it would be profitable to change the road from *ACD* to *ABD*. In equilibrium, all drivers therefore use the road *ABD* with the travel time equal to one hour.

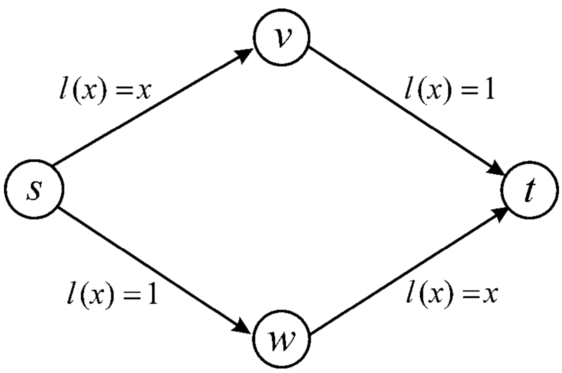


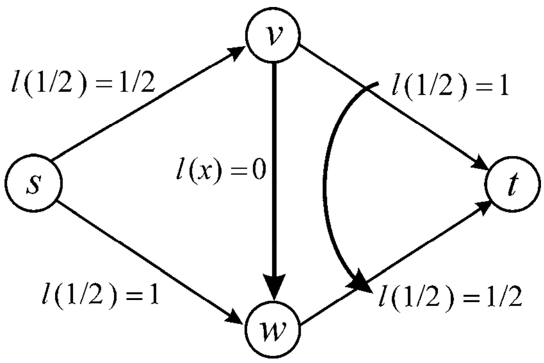


Nevertheless, from the point of view of the whole society whose aim is to minimize the overall travel time $x_1 \cdot 1+(1-x_1)\cdot(1-x_1)=x\cdot(x_1-\frac{1}{2})^2+\frac{3}{4}$, it would be optimal if one half of all drivers use *ACD* and the other half *ABD*, which yields the mean travel time 45 minutes. This is much better than one hour in equilibrium; nevertheless, it does not seem to be fair for those driving on *ACD*.

Pigou introduced this example to illustrate the necessity of discouraging taxes for optimal performance of a network. After the critique by the influential American economist Frank Hyneman Knight (1885–1972) he omitted this example in other editions of the book. Nevertheless, today it is one of the most famous examples used for the illustration of the fact that the selfish behavior by independent, non-cooperative "agents" need not produce a socially desirable outcome.

Another today famous example was independently introduced by Dietrich Braess in his 1968 paper [1]. Using the same notation as above, we can describe it in the following way. Consider a network with two source-destination paths *svt*, *swt* with travel times given by the left figure bellow. In equilibrium, the traffic splits evenly between the two paths and each driver travels one hour and a half.





Now imagine that an additional road from v to w is built of unlimited capacity and zero travel time (right figure). We can easily check that in equilibrium all drivers use the path *svwt* with the travel time equal to two hours. As expressed by Roughgarden: *Breass paradox thus shows that the intuitively helpful (or at least innocuous) action of adding a new zero-latency link may negatively impact all of the traffic.* ([11], p. 6)

The recent literature deals with several interesting questions that have their roots in the above examples:[4] How bad is selfish routing? It means, what is the worst-possible ratio between the total latency of a flow at Nash equilibrium and that of the best coordinated outcome minimizing the total latency? On the other hand, how unfair is the optimal assignment in the comparison with Nash equilibrium? When there is no chance of control, the performance of the system can be positively influenced by the appropriate architecture of the network.

[4] For more details and references see [11], [12].

Still, in many systems there is a mixture of “selfishly” and “centrally” controlled jobs; the congestion due to centrally controlled jobs influences the actions of selfish users. Similarly, there can be a central authority – for example the provider of the network – who can settle the prices for various paths before the selfish users start to act. These situations can be modeled via *Stackelberg games* where the leader is the central authority which searches the strategy that induces the selfish users (followers) to react in a way that minimizes the total latency in the system.

2 Bargaining

2.1 Historical Overview: from Edgeworth to Nash and Rubinstein

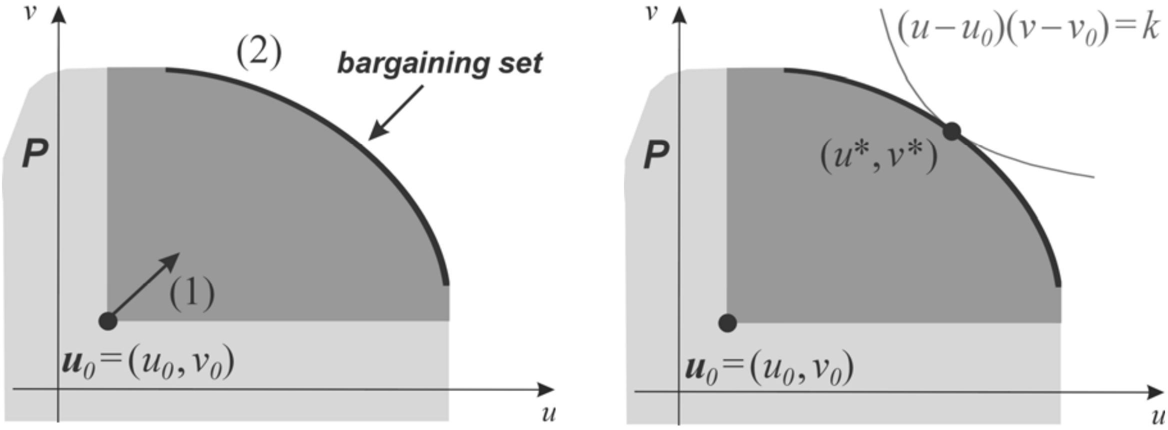
In the domain of non-cooperative games, the players cannot usually take full advantage of the situation. For example, in the case of Cournot or Stackelberg duopoly it would be better for both players to cooperate and act together as a monopolist. Thus we come to *a bargaining problem* that is usually referred to as a problem where two or more individuals have before them several possible contractual agreements. All have interests in reaching an agreement but their interests are not entirely identical. What will be the agreed contract, assuming that all parties behave rationally? This problem was already presented by Francis Ysidro Edgeworth (1845–1926) in his 1881 book [3]. Since then many attempts had been made to get a clear cut solution.

John Forbes Nash and the Axiomatic Approach

One approach to the bargaining problem solution that corresponds to the general inclination to axiomatic foundations of various parts of mathematics and other scientific disciplines was proposed by John Forbes Nash in his paper [9] published in 1953. Nash considered *a two-player bargaining problem* as an ordered pair (P, $\boldsymbol{u_0}$), where P is a cooperative payoff region (the set of all possible payoff pairs), $\boldsymbol{u_0} = (u_0, v_0)$, where u_0, v_0 are the payoffs in the case of a disagreement (”threats”). Let us denote the bargaining solution as $\Psi(P,(u_0,v_0)) = (u^*, v^*)$. Nash called for the satisfaction of the following conditions that correspond to our intuition about a fair bargaining solution:

1. *Individual Rationality*: $u^* \geq u_0,\ \ v^* \geq v_0$.
2. *Pareto Optimality*: there does not exist any other payoff pair $(u,v) \in P$ for which $u \geq u^*$ and $v \geq v^*$, withal at least one inequality is strict.
3. *Feasibility*: $(u^*, v^*) \in P$.
4. *Independence of Irrelevant Alternatives*: if P' is a payoff region contained in P and $(u_0,v_0),(u^*,v^*) \in P'$, then $\Psi(P',(u_0,v_0)) = (u^*,v^*)$.
5. *Independence Under Linear Transformations*: suppose that P' is obtained from P by the linear transformation $u' = au + b,\ v' = cv + d,$ where $a, c > 0$; then
$$\Psi(P',(au_0 + b, cv_0 + d)) = (au^* + b, cv^* + d).$$
6. *Symmetry*: If P is symmetric (i.e. $(u,v) \in P \Leftrightarrow (v,u) \in P$) and $u_0 = v_0$, then $u^* = v^*$.

For a given bargaining problem, Nash constructs the solution which maximizes the product of the utility gains $(u - u_0)(v - v_0)$ on the set of feasible and individually rational payoff pairs, and proves that it is the unique solution satisfying the conditions $1-6$.



More generally, Nash bargaining solution for the case of *n* players and the threats $(d_1, d_2, \ldots, d_n)$ selects the utility vector that maximizes the product of the utility gains $(u_1 - d_1)(u_2 - d_2)\cdots(u_n - d_n)$ of the players over the set of feasible agreements that are individually rational, i.e. $u_1 \geq d_1, \ldots, u_n \geq d_n$.

Later other solutions were proposed that omitted some of the above conditions (mainly the condition 4) and replaced it by other ones.[5]

Rubinstein's Model of Alternating Offers

In 1982 Ariel Rubinstein introduced his well-known strategic bargaining model of alternating offers of two players. Now it is common to consider *n* players who need to reach an agreement on a given issue. It is assumed that the players can take actions in the bargaining only at certain times in the set $T = \{0, 1, 2, \ldots\}$ that are determined in advance and are known to the players. In each period $t \in T$, if the bargaining has not terminated earlier, a player whose turn it is to make an offer at time *t* suggests a possible agreement, and each of the other players may either accept the offer, reject it, or opt out of the negotiation. If an offer is accepted by all the players, the bargaining ends and this offer is implemented. If at least one of the players opts out of the bargaining, it ends with a conflicting outcome. If no player has chosen to opt but at least one of them has rejected the offer, then the bargaining process continues to period $t+1$ when the next player makes a counteroffer, the others respond, and so on.

Strategy of a player now specifies what to do next, for each sequence of offers $s_0, s_1, \ldots, s_t$. It means, in time periods when it is the player's turn to respond to an offer, the strategy specifies whether to accept it, reject it or opt out of the bargaining; when it is the player's turn to make an offer, the strategy specifies which offer to make. *Strategy profile* is a collection of strategies, one for each player.

Nash equilibrium, as it was introduced for one-stage games, denotes the strategy profile such that no player can profitably deviate, given the actions of the other players. Rubinstein highlighted the inadequacy of this concept for multistage bargaining. Following Selten's definition of the perfect equilibrium (see [14], [15]), he based the bargaining analysis on this stronger concept. In simple words, a strategy profile is a *sub-game perfect equilibrium* of a model of alternating offers if the strategy profile *induced in every subgame* is a Nash equilibrium of that subgame.

---

[5] For the detailed survey and references see [17].

2.2 Applications: From Bargaining Politicians to Multi-Agent Systems

One of the greatest challenges for computer science is building computer systems that can work together. Among the examples of such intelligent systems we can find for instance automated agents that monitor electricity transformation networks, teams of robotic systems acting in hostile environments, computational agents that facilitate distributed design and engineering, distributed transportation and planning systems, intelligent agents that negotiate over meeting scheduling options on behalf of people for whom they work or Internet agents that collaborate to provide updated information to their users. In such environments, cooperation may improve the performance of the individual agents or the overall behavior of the system they form (see [6], pp. 79–85).

For example, Kraus [6] uses Rubinstein's model of alternating offers for data allocation problem where the players are servers of a given system and agreement is a distribution of datasets to information servers. Nash conditions or their modifications are usually used for characterization of optimal and fair solutions; one of the application possibilities is the Internet pricing as a game between an Internet service provider who chooses the price, and users who decide on their demand (see [18]).

3 Evolutionary Game Theory

3.1 Historical Overview: from Darwin to Maynard Smith

For a brief historical overview see author's contribution [5], pp. 53–56.

3.2 Applications: From Species Evolution to Internet

Non-cooperative and cooperative games discussed above are based on two essential assumptions: *unbounded rationality* and *global knowledge*. Both of these assumptions cannot be justified when considering large networks like Internet or the transportation system of a large city. The view that the players are not always rational, due to either limited computational capabilities, or limited knowledge of other players' nature is the point of departure for *evolutionary game theory*. As we could see in [5], the evolution process deals with a huge population of players that are "programmed" to adopt some strategies. As time proceeds, many small games are played and the strategies with high payoffs spread within the population. Evolutionary games in large populations thus create a dynamic process where the frequencies of the strategies played by population members change in time because of the learning or selection forces guiding the individuals' strategic choices.

In the field of biology where these ideas were originally formed, the players are the genes in which the instinctive behavior is coded. The development of Internet and other extended networks led to the transmission of the evolutionary theory from biology to the domain of *multi-agent systems*: for the reasons mentioned above, it is more realistic to consider agents as players with only partial information on the nature, possible strategies and payoffs of other players. Agents learn how to perform well in the game by experience; from time to time they revise their strategies on the basis of local observations, giving the strategy that has led to a higher payoff a higher probability of choosing in the next round.

A typical application is the *adaptive network routing* discussed for example by Fischer and Vöcking in their 2005 paper [4]. In the simplest case of the network with one starting point and one terminal point, the system can be described by the following differential equation: $\dot{f}_P = \lambda \cdot f_P \cdot (\overline{l_i} - l_P)$, where f_P denotes the flow on the path P, l_P is the latency of this path, $\overline{l_i}$ is the average latency and $\lambda \geq 0$ is a proportionality factor. Besides investigating stable solutions, evolutionary game theory enables us to study the *dynamics* of routing.

References

[1] Braess, D.: *Über ein Paradoxon aus der Verkehrsplannung*. Unternehmensforschung **12**(1968), pp. 258–268.

[2] Cournot, A. A.: *Recherches sur les principes mathématiques de la théorie des richesses*. Hachette, Paris, 1838. English translation from 1897 by N. T. Bacon: *Mathematical Principles of the Theory of Wealth*, Macmillan, New York, 1927.

[3] Edgeworth, F. Y.: *Mathematical Psychics: An Essay on the Applications of Mathematics to the Moral Sciences*. London, 1881.

[4] Fischer, S.; Vöcking, B.: *On the Evolution of Selfish Routing.* In: Proc. 12th Annual European Symp. on Algorithms, Bergen, 2004, pp. 323–334.

[5] Hykšová, M.: *Several Milestones in the History of Game Theory*. In: *VII. Österreichisches Symposion zur Geschichte der Mathematik*, Wien, 2004, pp. 49–56.

[6] Kraus, S.: *Negotiation and Cooperation in Multi-Agent Environments*. Artificial Intelligence **94**(1997), pp. 79–97.

[7] Kraus, S.: *Strategic Negotiation in Multi-Agent Environments*, MIT, Cambridge, 2001.

[8] Nash, J. F.: *Non-Cooperative Games*. Thesis, Princeton University, 1950, 27 pp.

[9] Nash, J. F.: *Two-person Cooperative Games*. Econometrica **21**(1953), pp. 128–140.

[10] Pigou, A. C.: *The Economics of Welfare*. McMillan, London, 1920.

[11] Roughgarden, T.: *Selfish Routing*. Thesis, Cornell University, 2002.

[12] Roughgarden, T.: *Selfish Routing and the Price of Anarchy*. MIT, Cambridge, 2005.

[13] Rubinstein, A.: *Perfect Equilibrium in a Bargaining Model*. Econometrica **50**(1982), pp. 97–109.

[14] Selten, R.: *Spieltheoretische Behandlung eines Oligopolmodels mit Nachfragetragheit.* Zeitschrift für die Gesamte Staatswiss. **12**(1965), pp. 301–324, 667–689.

[15] Selten, R.: *Re-examination of the Perfectness Concept for Equilibrium Points in Extensive Games*. International Journal of Game Theory **4**(1975), pp. 25–55.

[16] Stackelberg, H. von: *Marktform und Gleichgewicht*. Springer-Verlag, Wien, 1934.

[17] Thomson, W.: *Cooperative Models of Bargaining*. In: *Handbook of Game Theory with Economic Applications*, vol. 2. Elsevier, Amsterdam, 1994, pp. 1237–1284.

[18] Xi-Ren Cao; Hong-Xia Shen; Milito, R.; Wirth, P.: *Internet Pricing with a Game Theoretical Approach: Concepts and Examples*. In: IEEE/ACM Transaction on Networking, vol. 10, No. 2, 2002, pp. 208–216.

Address

RNDr. Magdalena Hykšová, Ph.D.
Faculty of Transportation Sciences
Czech Technical University in Prague
Na Florenci 25, 110 00 Prague 1
Czech Republic
e-mail: *hyksova@fd.cvut.cz*