1 BIMATRIX GAMES

1. Given the bimatrix
$$\begin{pmatrix} (-3,-2) & (-1,-2) & (8,9) \\ (-1,-1) & (4,4) & (-4,-3) \\ (8,9) & (-1,-2) & (-3,-3) \end{pmatrix}.$$
find the dominating and equilibrium strategies for both players.

2. Given the bimatrix
$$\begin{pmatrix} (-1,3) & (1,0) \\ (2,-1) & (0,1) \\ (1,1) & (-2,1) \end{pmatrix},$$
compute the maximin values (the least guaranteed payoffs) for both players.

3. Given the bimatrix
$$\begin{pmatrix} (1,1) & (0,1) & (2,0) \\ (1,2) & (-1,-1) & (1,2) \\ (2,-1) & (1,0) & (-1,-1) \end{pmatrix},$$
compute the equilibrium pairs and the maximin values for both players.

4. Given the bimatrix
$$\begin{pmatrix} (-2,3) & (-1,1) & (1,-2) \\ (0,1) & (-1,-2) & (1,1) \\ (2,2) & (2,-1) & (0,0) \end{pmatrix},$$
compute the maximin values for both players.

5. Given the bimatrix
$$\begin{pmatrix} (2,-3) & (-1,3) \\ (0,1) & (1,-2) \end{pmatrix},$$
compute the equilibrium pairs and the maximin values for both players.

6. Given the bimatrix
$$\begin{pmatrix} (2,-1) & (-1,1) \\ (0,2) & (1,-1) \end{pmatrix},$$
compute the equilibrium pairs and the maximin values for both players.

7. Given the bimatrix
$$\begin{pmatrix} (2,1) & (0,0) \\ (0,0) & (1,5) \end{pmatrix},$$
compute the equilibrium pairs and the maximin values for both players.

Excercises for the course *Game Theory and Optimal Decisions* at FD ČVUT.
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8. Consider the bimatrix

$$\begin{pmatrix} (-3,-2) & (-1,-2) & (8,9) \\ (-1,-1) & (4,4) & (-4,-3) \\ (8,9) & (-1,-2) & (-3,-3) \end{pmatrix}.$$

compute the equilibrium pairs and show that the game has no solution in pure strategies based on the concepts of equilibrium strategies, dominating and interchangebility.

9. Consider the bimatrix game

$$\begin{pmatrix} (2,1) & (0,0) \\ (0,0) & (1,2) \end{pmatrix}.$$

Show that the pairs of strategies

$$\bar{\mathbf{x}} = (1/3, 2/3), \quad \bar{\mathbf{y}} = (2/3, 1/3) \qquad \text{and} \qquad \bar{\bar{\mathbf{x}}} = (2/3, 1/3), \quad \bar{\bar{\mathbf{y}}} = (1/3, 2/3)$$

are equilibrium. Further, show that convex combinations

$$\mathbf{x} = k_1\bar{\mathbf{x}} + k_2\bar{\bar{\mathbf{x}}} \quad \text{a} \quad k_1\bar{\mathbf{y}} + k_2\bar{\bar{\mathbf{y}}}, \quad k_1 \geq 0,\ k_2 \geq 0,\ k_1 + k_2 = 1$$

do not necessarily form equilibrium strategies in the game under study.

10. An investor wants to build two hotels. One of them we will call Big (abbreviated as B); gaining the order for it will bring the profit of 15 million to the building firm. The second hotel will be called Small (abbreviated as S); gaining the order for it will bring the profit of 9 million to the building firm.

    There are two firms competing for the orders, we will denote them 1 and 2. No of them has a potential to build both hotels in full. Each firm can offer the building of one hotel or the cooperation on both of them. The investor has to realize the building by force of these firms and he decides on the base of sealed-bid method. The rules for splitting the orders according to offers are the following:

    1. If only one firm bids for a contract on a hotel, it receives it all.
    2. If two firms bid for a contract on the same hotel and no of them bids for the second one, the investor offers the cooperation to both firms on both hotels, the profits are split fifty-fifty.
    3. If one firms bids for a contract on the whole building of a hotel and the second firm offers a collaboration, than the firm that bids the whole building receives 60% and the second 40% in the case of B, and 80% versus 20% in the case of S. On the other hotel the firms will collaborate fifty-fifty (including splitting the profit).

    In every case the total profit of $15 + 9 = 24$ milion is split between the firms. Find optimal strategies for both firms.