3 TWO-PLAYER COOPERATIVE GAMES

1. Prove that the bimatrix game given by

$$\begin{pmatrix} (0,0) & (1,-2) \\ (-1,1) & (1,-1) \end{pmatrix}$$

   is inessential.

2. Find the arbitration pair for the bimatrix game

$$\begin{pmatrix} (-1,-1) & (4,0) \\ (0,4) & (-1,-1) \end{pmatrix}.$$

3. Find the arbitration pair for the bimatrix game

$$\begin{pmatrix} (-1,1) & (0,0) \\ (1,-1) & (0,1) \\ (-1,-1) & (1,1) \end{pmatrix}.$$

4. Find the arbitration pair for the bimatrix game

$$\begin{pmatrix} (-\frac{1}{2},0) & (-\frac{1}{2},-4) \\ (1,2) & (-2,4) \\ (4,-4) & (-\frac{1}{2},0) \end{pmatrix}.$$

5. Find the arbitration pair for the Battle of the Buddies game

$$\begin{pmatrix} (2,1) & (0,0) \\ (0,0) & (1,2) \end{pmatrix}.$$

6. Prove that the noncooperative payoff region for a two-player game is a subset of the cooperative payoff region.

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7. Let P be the probability matrix

$$\begin{pmatrix} \frac{1}{8} & 0 & \frac{1}{3} \\ \frac{1}{4} & \frac{5}{24} & \frac{1}{12} \end{pmatrix}.$$

Do there exist mixed strategies $\boldsymbol{p}$ and $\boldsymbol{q}$ for the two players such that

$$p_i q_j = p_{ij} \qquad \text{for all} \quad i, j?$$

8. Prove that the convex hull of a set is convex.
9. Prove that the symmetric convex hull of a set is symmetric.