4 N-PLAYER COOPERATIVE GAMES

1. Find the core of the three-player cooperative game with the characteristic function
   \[ v(\{1, 2, 3\}) = \frac{3}{2}, \]
   \[ v(\{1, 2\}) = v(\{1, 3\}) = v(\{2, 3\}) = 1, \]
   \[ v(\{1\}) = v(\{2\}) = v(\{3\}) = \frac{1}{2}. \]

2. Find the core of the three-player cooperative game with the characteristic function
   \[ v(\{1, 2, 3\}) = 2, \]
   \[ v(\{1, 2\}) = v(\{1, 3\}) = v(\{2, 3\}) = 1, \]
   \[ v(\{1\}) = v(\{2\}) = v(\{3\}) = 0. \]

3. Find the core of the three-player cooperative game with the characteristic function
   \[ v(\{1, 2, 3\}) = 0, \]
   \[ v(\{1, 2\}) = v(\{1, 3\}) = v(\{2, 3\}) = 1, \]
   \[ v(\{1\}) = v(\{2\}) = v(\{3\}) = -1. \]

4. Find the core of the three-player cooperative game with the characteristic function
   \[ v(\{1, 2, 3\}) = \frac{6}{5}, \]
   \[ v(\{1, 2\}) = v(\{1, 3\}) = v(\{2, 3\}) = 1, \]
   \[ v(\{1\}) = v(\{2\}) = v(\{3\}) = \frac{1}{2}. \]

5. Find the core of the three-player cooperative game with the characteristic function
   \[ v(\{1, 2, 3\}) = 1, \]
   \[ v(\{1, 2\}) = \frac{1}{4}, v(\{1, 3\}) = 0, v(\{2, 3\}) = \frac{1}{2}, \]
   \[ v(\{1\}) = -\frac{1}{2}, v(\{2\}) = 0, v(\{3\}) = -\frac{1}{2}. \]

6. Consider the four-player cooperative game in with the characteristic function
   \[ v(\{1, 2, 3, 4\}) = 2, \]
   \[ v(\{1, 2, 3\}) = 1, v(\{1, 2, 4\}) = 2, v(\{1, 3, 4\}) = 0, v(\{2, 3, 4\}) = 1, \]
   \[ v(\{1, 2\}) = 0, v(\{1, 3\}) = -1, v(\{1, 4\}) = 1, \]
   \[ v(\{2, 3\}) = 0, v(\{2, 4\}) = 1, v(\{3, 4\}) = 0 \]
   \[ v(\{1\}) = -1, v(\{2\}) = 0, v(\{3\}) = -1, v(\{4\}) = 0. \]

Verify that \(v\) is a superaditive function.
7. Find the Shapley value of the three-player cooperative game with the characteristic function
\[ v(\{1, 2, 3\}) = 0, \]
\[ v(\{1, 2\}) = v(\{1, 3\}) = v(\{2, 3\}) = 1, \]
\[ v(\{1\}) = v(\{2\}) = v(\{3\}) = -1. \]

8. Find the Shapley value of the three-player cooperative game with the characteristic function
\[ v(\{1, 2, 3\}) = 200, \]
\[ v(\{1, 2\}) = 150, \quad v(\{1, 3\}) = 110, \quad v(\{2, 3\}) = 20, \]
\[ v(\{1\}) = 100, \quad v(\{2\}) = 10, \quad v(\{3\}) = 0. \]

9. Solve the game with the set of players \( Q = \{1, 2, 3, 4\} \), in which the values of the characteristic function \( v(K) \) are equal to the sum of the numbers of players forming the coalition \( K \); for example \( v(\{1, 3, 4\}) = 8, \quad v(Q) = 10. \)

10. Find the Shapley value of the \( N \)-player game in the form with characteristic function which is given by the
\[ v(K) = 5|K| \quad \text{for all} \quad K \in Q, \]
that is, each coalition obtains the amount equal to the number of its members.

11. Find optimal strategies for both players in the game with the characteristic function
\[ v(\{1, 2, 3\}) = 8, 75, \]
\[ v(\{1, 2\}) = 1, \quad v(\{1, 3\}) = 5, \quad v(\{2, 3\}) = -1, \]
\[ v(\{1\}) = 0, \quad v(\{2\}) = -3, \quad v(\{3\}) = -0, 75, \]
and determine the optimal division of the total payoff.

12. Consider a three-player normal form game with strategy sets
\[ S_1 = \{0, 1, 2, 3, 4, 5, 6\}, \quad S_2 = \{0, 1, 2, 3\}, \quad S_3 = \{0, 1, 2, 3, 4\} \]
and payoff functions
\[ u_1(x) = -(x_1^2 + x_1x_2 + x_1x_3 - 11x_1 + 4)/2, \]
\[ u_2(x) = -(x_1x_2 + x_2^2 + x_2x_3 - 11x_2 + 6)/2, \]
\[ u_1(x) = -(x_1x_3 + x_2x_3 + x_3^2 - 21x_3/2 + 2)/2. \]

Find the characteristic function of this game.