4 N-PLAYER COOPERATIVE GAMES

1. Find the core of the three-player cooperative game with the characteristic function

$$\begin{aligned} &v(\{1,2,3\} = 3/2, \\ &v(\{1,2\}) = v(\{1,3\}) = v(\{2,3\}) = 1, \\ &v(\{1\}) = v(\{2\}) = v(\{3\}) = 1/2. \end{aligned}$$

2. Find the core of the three-player cooperative game with the characteristic function

$$\begin{aligned} &v(\{1,2,3\} = 2, \\ &v(\{1,2\}) = v(\{1,3\}) = v(\{2,3\}) = 1, \\ &v(\{1\}) = v(\{2\}) = v(\{3\}) = 0. \end{aligned}$$

3. Find the core of the three-player cooperative game with the characteristic function

$$\begin{aligned} &v(\{1,2,3\} = 0, \\ &v(\{1,2\}) = v(\{1,3\}) = v(\{2,3\}) = 1, \\ &v(\{1\}) = v(\{2\}) = v(\{3\}) = -1. \end{aligned}$$

4. Find the core of the three-player cooperative game with the characteristic function

$$\begin{aligned} &v(\{1,2,3\} = 6/5, \\ &v(\{1,2\}) = v(\{1,3\}) = v(\{2,3\}) = 1, \\ &v(\{1\}) = v(\{2\}) = v(\{3\}) = 1/2. \end{aligned}$$

5. Find the core of the three-player cooperative game with the characteristic function

$$\begin{aligned} &v(\{1,2,3\} = 1, \\ &v(\{1,2\}) = 1/4,\ v(\{1,3\}) = 0,\ v(\{2,3\}) = 1/2, \\ &v(\{1\}) = -1/2,\ v(\{2\}) = 0,\ v(\{3\}) = -1/2. \end{aligned}$$

6. Consider the four-player cooperative game in with the characteristic function

$$\begin{aligned} &v(\{1,2,3,4\} = 2, \\ &v(\{1,2,3\}) = 1,\ v(\{1,2,4\}) = 2,\ v(\{1,3,4\}) = 0,\ v(\{2,3,4\}) = 1, \\ &v(\{1,2\}) = 0,\ v(\{1,3\}) = -1,\ v(\{1,4\}) = 1, \\ &v(\{2,3\}) = 0,\ v(\{2,4\}) = 1,\ v(\{3,4\}) = 0 \\ &v(\{1\}) = -1,\ v(\{2\}) = 0,\ v(\{3\}) = -1,\ v(\{4\}) = 0. \end{aligned}$$

   Verify that v is a superaditive function.

7. Find the Shapley value of the three-player cooperative game with the characteristic function

$$v(\{1,2,3\} = 0,$$
$$v(\{1,2\}) = v(\{1,3\}) = v(\{2,3\}) = 1,$$
$$v(\{1\}) = v(\{2\}) = v(\{3\}) = -1.$$

8. Find the Shapley value of the three-player cooperative game with the characteristic function

$$v(\{1,2,3\} = 200,$$
$$v(\{1,2\}) = 150,\ v(\{1,3\}) = 110,\ v(\{2,3\}) = 20,$$
$$v(\{1\}) = 100,\ v(\{2\}) = 10,\ v(\{3\}) = 0.$$

9. Solve the game with the set of players $Q = \{1,2,3,4\}$, in which the values of the characteristic function $v(K)$ are equal to the sum of the numbers of players forming the coalition K; for example $v(\{1,3,4\} = 8$, $v(Q) = 10$.

10. Find the Shapley value of the N-player game in the form with characteristic function which is given by the

$$v(K) = 5|K| \qquad \text{for all} \qquad K \in Q,$$

that is, each coalition obtains the amount equal to the number of its members.

11. Find optimal strategies for both players in the game with the characteristic function

$$v(\{1,2,3\} = 8,75,$$
$$v(\{1,2\}) = 1,\ v(\{1,3\}) = 5,\ v(\{2,3\}) = -1,$$
$$v(\{1\}) = 0,\ v(\{2\}) = -3,\ v(\{3\}) = -0,75,$$

and determine the optimal division of the total payoff.

12. Consider a three-player normal form game with strategy sets

$$S_1 = \{0,1,2,3,4,5,6\},\ S_2 = \{0,1,2,3\},\ S_3 = \{0,1,2,3,4\}$$

and payoff functions

$$u_1(x) = -(x_1^2 + x_1x_2 + x_1x_3 - 11x_1 + 4)/2,$$
$$u_2(x) = -(x_1x_2 + x_2^2 + x_2x_3 - 11x_2 + 6)/2,$$
$$u_1(x) = -(x_1x_3 + x_2x_3 + x_3^2 - 21x_3/2 + 2)/2.$$

Find the characteristic function of this game.