0.1 GAMES AGAINST P-INTELLIGENT PLAYERS

0.1.1 Fundamental Concepts

Definition 1. A player behaving with probability p like a normatively intelligent player and with probability $1-p$ like a random mechanism will be called a ***p*-intelligent player**.

Parameter p characterizes the degree of deviation from the rational decision making. If $p = 0$, the player behaves in fact as a random mechanism, if $p = 1$, he is an intelligent player.

It is clear that it is not reasonable to apply the same strategies against the p-intelligent opponents as against intelligent opponents. Consider the case of matrix games.

Definition 2. The optimal strategy for the intelligent player is a row of the matrix A that maximizes the mean value of payoff when the p-intelligent player applies strategy

$$s(p) = py^* + (1-p)r$$

where y^* is a Nash equilibrium strategy of player 2 and r is a uniform probability distribution over columns.

☛ ***Example 1.*** Investigate a matrix game defined by the matrix

$$\begin{pmatrix} 3 & 3 & 3 & 3 \\ 7 & 1 & 7 & 7 \\ 3 & 1 & -1 & 2 \\ 8 & 0 & 8 & 8 \end{pmatrix}$$

Solution

The unique pair of equilibrium strategies is

$$x^* = (1,0,0,0), \qquad y^* = (0,1,0,0).$$

If player 2 is p-intelligent, player 1 expects that player 2 is going to use the strategy

$$s(p) = p(1,0,0,0) + (1-p)(1/4,1/4,1/4,1/4) = (1-p, 1+3p, 1-p, 1-p)/4 \ .$$

We can easily verify that

- the first row is an optimal strategy for player 1 if $p \in \langle 5/9, 1\rangle$
- the second row is an optimal strategy for player 1 if $p \in \langle 1/3, 5/9\rangle$
- the fourth row is an optimal strategy for player 1 if $p \in \langle 0, 1/3\rangle$
- the third row is never an optimal strategy

How much we may lose if we apply a strategy which is optimal against a fully intelligent player when we play against a partly intelligent player?

Definition 3. The function $f(p)$, representing the average additional player's 1 profit due to his deviation from the equilibrium strategy x^* is called **excess function**.

If player 1 uses the optimal strategy against the p-intelligent player, he receives the payoff

$$\max_i a^{(i)} s(p),$$

where $a^{(i)}$ means the i-th row of the matrix A and i goes over all rows of A, that is $i = 1, 2, \ldots, m$. If he mechanically applies the strategy against the fully intelligent opponents, he receives

$$x^{*T} As(p).$$

Therefore,

$$f(p) = \max_i \left[a^{(i)} s(p)\right] - x^{*T} As(p). \tag{1}$$

☛ ***Example 2.*** For the game from example 1 we have

$$f(p) = \begin{cases} \frac{13}{4} - \frac{21}{4}p & \text{for} \quad p \in \langle 0, \frac{1}{3} \rangle \\ \frac{11}{4} - \frac{15}{4}p & \text{for} \quad p \in \langle \frac{1}{3}, \frac{5}{9} \rangle \\ 0 \quad \text{for} \quad p \in \langle \frac{5}{9}, 1 \rangle \end{cases}$$

The following theorem confirms the fact that taking into account the decreased intelligence of the opponent is always an advantage.

Theorem 1. *For any matrix game the excess function is a nonnegative, by parts linear, continuous and nonincreasing function on the interval* $\langle 0, 1 \rangle$.

In other words, the difference between what we get when applying the strategy based on the correct assessment of the opponent's intelligence is always at least so great as when we simply apply the strategy optimal against normatively intelligent player. The difference decreases or remains the same when the intelligence of the opponent increases.