

## Dividing a pie:

Marry	John		
1	→		0

## Dividing a pie – 2 rounds:

Marry		John	
	→		1
0	←	$\frac{1}{2}$	$\frac{1}{2}$

## Dividing a pie – 2 rounds:

Marry		John	
$\frac{1}{2}$	→	$\frac{1}{2}$	1
0	←	$\frac{1}{2}$	$\frac{1}{2}$

## Dividing a pie – 3 rounds:

Marry		John	
	→		1
	←		$\frac{2}{3}$
$\frac{1}{3}$	→	0	$\frac{1}{3}$

## Dividing a pie – 3 rounds:

Marry		John	
	→		1
$\frac{1}{3}$	←	$\frac{1}{3}$	$\frac{2}{3}$
$\frac{1}{3}$	→	0	$\frac{1}{3}$

## Dividing a pie – 3 rounds:

Marry		John	
$\frac{2}{3}$	→	$\frac{1}{3}$	1
$\frac{1}{3}$	←	$\frac{1}{3}$	$\frac{2}{3}$
$\frac{1}{3}$	→	0	$\frac{1}{3}$

## Dividing a pie – 4 rounds:

Marry		John	
	→		1
	←		$\frac{3}{4}$
	→		$\frac{2}{4}$
0	←	$\frac{1}{4}$	$\frac{1}{4}$

## Dividing a pie – 4 rounds:

Marry		John	
	→		1
	←		$\frac{3}{4}$
$\frac{1}{4}$	→	$\frac{1}{4}$	$\frac{2}{4}$
0	←	$\frac{1}{4}$	$\frac{1}{4}$



## Dividing a pie – 4 rounds:

Marry		John	
	→		1
$\frac{1}{4}$	←	$\frac{2}{4}$	$\frac{3}{4}$
$\frac{1}{4}$	→	$\frac{1}{4}$	$\frac{2}{4}$
0	←	$\frac{1}{4}$	$\frac{1}{4}$

## Dividing a pie – 4 rounds:

Marry		John	
$\frac{2}{4}$	→	$\frac{2}{4}$	1
$\frac{1}{4}$	←	$\frac{2}{4}$	$\frac{3}{4}$
$\frac{1}{4}$	→	$\frac{1}{4}$	$\frac{2}{4}$
0	←	$\frac{1}{4}$	$\frac{1}{4}$

## Dividing a pie – 4 rounds:

Marry		John	
$\frac{3}{5}$	→	$\frac{2}{5}$	1
$\frac{2}{5}$	←	$\frac{2}{5}$	$\frac{4}{5}$
$\frac{2}{5}$	→	$\frac{1}{5}$	$\frac{3}{5}$
$\frac{1}{5}$	←	$\frac{1}{5}$	$\frac{2}{5}$
$\frac{1}{5}$	→	0	$\frac{1}{5}$

## Dividing a pie – 5 rounds:

Marry		John	
$\frac{3}{5}$	→	$\frac{2}{5}$	1
$\frac{2}{5}$	←	$\frac{2}{5}$	$\frac{4}{5}$
$\frac{2}{5}$	→	$\frac{1}{5}$	$\frac{3}{5}$
$\frac{1}{5}$	←	$\frac{1}{5}$	$\frac{2}{5}$
$\frac{1}{5}$	→	0	$\frac{1}{5}$

## Bargaining of managers and union:

Round	Managers	Union
101	0	← 10

## Bargaining of managers and union:

Round	Managers	Union
100	10	→ 10
101	0	← 10

## Bargaining of managers and union:

Round	Managers		Union
99	10	←	20
100	10	→	10
101	0	←	10

## Bargaining of managers and union:

Round	Managers		Union
98	20	→	20
99	10	←	20
100	10	→	10
101	0	←	10



## Bargaining of managers and union:

Round	Managers		Union
97	20	←	30
98	20	→	20
99	10	←	20
100	10	→	10
101	0	←	10

## Bargaining of managers and union:

Round	Managers		Union
2	500	→	500
	...		...
97	20	←	30
98	20	→	20
99	10	←	20
100	10	→	10
101	0	←	10

## Bargaining of managers and union:

Round	Managers		Union
1	500	←	510
2	500	→	500
	...		...
97	20	←	30
98	20	→	20
99	10	←	20
100	10	→	10
101	0	←	10

## Compound interest

The value of a capital  $K_0$  deposited for  $n$  years with an interest  $i$  p.a.:

$$K_n = K_0(1 + i)^n$$

Year	Capital
0	$K_0$
1	$K_1 = K_0 + iK_0 = K_0(1 + i)$
2	$K_2 = K_1 + iK_1 = K_1(1 + i) = K_0(1 + i)^2$
3	$K_3 = K_2 + iK_2 = K_2(1 + i) = K_0(1 + i)^3$
⋮	.....
$n$	$K_n = K_{n-1} + iK_{n-1} = K_{n-1}(1 + i) = K_0(1 + i)^n$

Today value of a capital  $K_n$ , which we will obtain after  $n$  years:

$$K_0 = \frac{K_n}{(1 + i)^n} = K_n \delta, \quad 0 < \delta < 1$$

$\delta$  is called a **discount factor**

## „Discounting a pie:“

Round	Marry	John
$n$	$K_n$	$\rightarrow 0$

# „Discounting a pie:“

Round	Marry	John
$n - 1$	$K_n \delta$	$\leftarrow K_n(1 - \delta)$
$n$	$K_n$	$\rightarrow 0$

## „Discounting a pie:“

Round	Marry	John
$n - 2$	$K_n(1 - \delta(1 - \delta))$	$\rightarrow \delta K_n(1 - \delta)$
$n - 1$	$K_n\delta$	$\leftarrow K_n(1 - \delta)$
$n$	$K_n$	$\rightarrow 0$

## „Discounting a pie:“

Round	Marry	John
$n - 3$	$K_n \delta(1 - \delta(1 - \delta))$	$\leftarrow K_n(1 - \delta(1 - \delta(1 - \delta)))$
$n - 2$	$K_n(1 - \delta(1 - \delta))$	$\rightarrow \delta K_n(1 - \delta)$
$n - 1$	$K_n \delta$	$\leftarrow K_n(1 - \delta)$
$n$	$K_n$	$\rightarrow 0$



## „Discounting a pie:“

Round	Marry	John
...	...	...
$n - 3$	$K_n \delta(1 - \delta(1 - \delta))$	$\leftarrow K_n(1 - \delta(1 - \delta(1 - \delta)))$
$n - 2$	$K_n(1 - \delta(1 - \delta))$	$\rightarrow \delta K_n(1 - \delta)$
$n - 1$	$K_n \delta$	$\leftarrow K_n(1 - \delta)$
$n$	$K_n$	$\rightarrow 0$

$$\lim_{n \rightarrow \infty} K_n(1 - \dots - \delta(1 - \delta(1 - \delta)) \dots) = \frac{K_n}{\delta + 1}$$

$$\lim_{n \rightarrow \infty} K_n \delta(1 - \dots - \delta(1 - \delta(1 - \delta)) \dots) = \frac{\delta K_n}{\delta + 1}$$