**Laplace Principle**

suggests choosing a strategy which is optimal in a situation where the opponent chooses all strategies with **equal probabilities.**

In other words, according to the Laplace Principle, the best that we can do under uncertainty is to behave as under risk, where all strategies of the opponent might appear with equal probabilities.

In the case of a matrix game given by the matrix $A = (a_{ij})$, the optimal decision according to the Laplace Principle is to choose row $i$ for which

$$\frac{a_{i1} + a_{i2} + \cdots + a_{in}}{n} \quad \text{is maximal.}$$

## Minimax Principle

suggests that under uncertainty the intelligent player should choose a strategy which is optimal in a situation where the opponent applies the worst possible strategy.

In the above notation, the optimal decision according to the Minimax Principle is to choose row $i$ for which

$$\min_j a_{ij} \quad \text{is maximal.}$$

## Principle of Maximin Regret

This principle is based on an observation that in many practical situations the quality of a decision is judged ex post without taking into account that in the time when the decision was made the decision maker had not possessed the information on actions of the opponent. The Principle of Maximin Regret protects the decision maker against these ex post objections. To find a decision optimal according to this principle, we calculate first a **matrix of regrets** by subtracting from each element in $A$ the maximal element in the column in which the element lies. To follow the common intuition that small regret is better than big, we change the signs of the matrix's elements. In each row of this matrix of regrets we find out the maximal regret and as an optimal decision we choose the row in which this maximum is minimal.

In the above notation, the optimal decision according to the Principle of Maximin Regret is to choose row $i$ for which

$$\max_j \left[a_{ij} - (\max_k a_{kj})\right] \quad \text{is minimal.}$$

☛ ***Example***. Chemical Products Ltd. considers a contract to produce AIDS testing sets. They may sign a contract for 2 000, 3 000, 4 000 or 5 000 testing sets or not engage in the business at all. The production costs for the series of tests are 20 000 EUR, 25 000 EUR, 30 000 EUR and 35 000 EUR, respectively. Before the sets are sent to hospitals, they must pass through destructive random sampling tests. If these tests find that less than 2% of the sets give false results, the price of one set is 20 EUR. If the percentage of defective results lies between 2% and 4%, the price of one set is 10 EUR. If there are more than 4% defective sets, the price of one set is 2 EUR. Chemical Products Ltd. never produced AIDS testing sets before, so it is not possible to assess the quality of the product before the series is produced and sampling tests are materialised. What is the best decision?

***Solution.*** The situation can be described by the following matrix game where the elements in the matrix represent the net profit of the firm in thousands of EUR.

| | Defective | | |
|---|---|---|---|
| Series | Less than 2% | 2 – 4% | More than 4% |
| 0 | 0 | 0 | 0 |
| 2 000 | 20 | 0 | -16 |
| 3 000 | 35 | 5 | -19 |
| 4 000 | 50 | 10 | -22 |
| 5 000 | 65 | 15 | -25 |

Using the Laplace Principle, we find the maximum of the row averages for the above matrix, that is

$$\mathbf{max\{0,\ 4/3,\ 7,\ 38/3,\ 55/3\} = 55/3.}$$

The best decision is therefore to produce a series of 5 000 testing sets.

| | Defective | | |
|---|---|---|---|
| Series | Less than 2% | 2 – 4% | More than 4% |
| 0 | 0 | 0 | 0 |
| 2 000 | 20 | 0 | -16 |
| 3 000 | 35 | 5 | -19 |
| 4 000 | 50 | 10 | -22 |
| 5 000 | 65 | 15 | -25 |

Using the Minimax Principle, we find the maximum of the worst possible row profits

$$\mathbf{max\{0,\ -16,\ -19,\ -22,\ -25\} = 0,}$$

that is, the best decision is not to go into the business at all.

| Series | Defective | | |
|---|---|---|---|
| | Less than 2% | 2 – 4% | More than 4% |
| 0 | 0 | 0 | 0 |
| 2 000 | 20 | 0 | -16 |
| 3 000 | 35 | 5 | -19 |
| 4 000 | 50 | 10 | -22 |
| 5 000 | 65 | 15 | -25 |

Using the Principle of Maximin Regret, we need the matrix of regrets

$$\begin{pmatrix} \boxed{65} & \mathbf{15} & \mathbf{0} \\ \mathbf{45} & \mathbf{15} & \mathbf{16} \\ \mathbf{30} & \mathbf{10} & \mathbf{19} \\ \mathbf{15} & \mathbf{5} & \mathbf{22} \\ \mathbf{0} & \mathbf{0} & \mathbf{25} \end{pmatrix}$$

The worst row regrets are

$$\mathbf{65,\ 45,\ 30,\ 22,\ 25.}$$

The minimal regret may be expected when we produce a series of 4 000 testing sets.