MEASURING THE POWER IN POLITICS

Lloyd Shapley (*1923), Martin Shubik (*1926)

A Method for Evaluating the Distribution of Power in a Committee System, 1954

The model of a voting situation: cooperative characteristic function form game where a coalition that can pass a bill (winning coalition) is assigned the value 1, the coalition that can not pass a bill (loosing coalition) is assigned the value 0.

How the power of particular voters in the voting game can be measured?

There is a group of individuals all willing to vote for some bill. They vote in order. As soon as enough members have voted for it, it is declared passed, and the member who voted last is given credit for having passed it. Let us choose the voting order of members randomly. Then we may compute how often a given individual is pivotal. This latter number serves to give us our index. (Shapley, Shubik, 1954)

In other words, the **Shapley-Shubik index** of voter i is

$$\varphi_i = \frac{\text{the number of voting orders, in which } i \text{ is pivotal}}{n!}$$

The combinatorial formula for ”S-S” index:

$$\varphi_i = \sum_{i \text{ swings for } S} \frac{(s-1)!(n-s)!}{n!}, \qquad s = |S|$$

where a *swing voter* for coalition S means that the coalition S is winning, but the coalition $S \setminus \{i\}$ is not winning.

Shapley value:

$$H_i = \sum_{S \subset Q,\ i \in S} \frac{(N-s)!(s-1)!}{n!} \left(v(K) - v(K \setminus \{i\})\right)$$

John F. Banzhaf III.

Weighted Voting doesn't work: a Mathematical Analysis, 1965

The appropriate measure of a legislator's power is simply the number of different situations in which he is able to determine the outcome. More explicitly, in a situations in which there are n legislators, each acting independently and each capable of influencing the outcome only by means of his votes, the ratio of the power of legislator X to the power of legislator Y is the same as the ratio of the number of possible voting combinations of the entire legislature in which X can alter the outcome by changing his vote, to the number of combinations in which Y can alter the outcome by changing his vote. (Bahzhaf, 1965)

In other words:

The voter i's power should be proportional to the number of coalitions for which i is a swing voter. It is convenient to divide this number by the total number of coalitions containing voter i.

Unnormalized Banzhaf index:

$$\beta'_i = \frac{\text{number of swings for voter } i}{2^{n-1}}$$

Normalized Banzhaf index:

$$\beta_i = \frac{\beta'_i}{\sum_i \beta'_i}$$

One Man, 3,312 Votes: A Mathematical Analysis of the Electoral College, 1968