Stationary and non-stationary signals

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1. Stationary and non-stationary

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Stationary and **non-stationary**

<table>
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<tr>
<th>Continuous system</th>
<th>Discrete system</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u(t) ) ... input (control) vector</td>
<td>( u(n) ) ... input (control) vector</td>
</tr>
<tr>
<td>( x(t) ) ... state vector</td>
<td>( x(n) ) ... state vector</td>
</tr>
<tr>
<td>( y(t) ) ... output vector</td>
<td>( y(n) ) ... output vector</td>
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Linear state variable system

Continuous:
\[
\begin{align*}
\dot{x}(t) &= A(t) x(t) + B(t) u(t) \\
y(t) &= C(t) x(t) + D(t) u(t)
\end{align*}
\]

Discrete:
\[
\begin{align*}
x(n+1) &= M(n) x(n) + N(n) u(n) \\
y(n) &= C(n) x(n) + D(n) u(n)
\end{align*}
\]

- \( A(t) \) system matrix \((n \times n)\)
- \( B(t) \) matrix of inputs \((n \times r)\)
- \( C(t) \) matrix of outputs \((m \times n)\)
- \( D(t) \) matrix of outputs \((m \times r)\)
- \( M(n) \) system matrix
- \( N(n) \) matrix of inputs
- \( C(n) \) matrix of outputs
- \( D(n) \) matrix of outputs
### Stationary and non-stationary

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**Linear state variable system**

- Continuous system:
  - $\dot{x}(t) = A x(t) + B u(t)$
  - $y(t) = C x(t) + D u(t)$
- Discrete system:
  - $x(n + 1) = M x(n) + N u(n)$
  - $y(n) = C x(n) + D u(n)$

<table>
<thead>
<tr>
<th>$A$ system matrix $(n \times n)$</th>
<th>$M$ system matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$ matrix of inputs $(n \times r)$</td>
<td>$N$ matrix of inputs</td>
</tr>
<tr>
<td>$C$ matrix of outputs $(m \times n)$</td>
<td>$C$ matrix of outputs</td>
</tr>
<tr>
<td>$D$ matrix of outputs $(m \times r)$</td>
<td>$D$ matrix of outputs</td>
</tr>
</tbody>
</table>
• Non-stationary signals ⇔ differential/difference equations with time-varying coefficients

\[ \ddot{y}(t) - t \dot{y}(t) = 0 \]

• Airy’s functions

\[
Ai(t) = \frac{1}{3} \sqrt{t} \left[ I_{-1/3} \left( \frac{2}{3} t^{3/2} \right) - I_{1/3} \left( \frac{2}{3} t^{3/2} \right) \right]
\]

\[
Bi(t) = \frac{1}{3} \sqrt{t} \left[ I_{-1/3} \left( \frac{2}{3} t^{3/2} \right) + I_{1/3} \left( \frac{2}{3} t^{3/2} \right) \right]
\]
Stationary and non-stationary

Conclusion...

\[ \Re(B_i(t)) \quad \Re(A_i(t)) \]

\[ t \rightarrow \]

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lecture 3. 12. 2009
Stationary and non-stationary

• Stationary signals $\Leftrightarrow$ differential/difference equations with constant coefficients

$$\ddot{y}(t) + \omega_0^2 y(t) = 0$$

• Harmonic wave (periodic functions)

$$\cos(\omega_0 t) \quad \sin(\omega_0 t)$$
Stationary and non-stationary

\[
sin(\pi t) \quad \cos(\pi t)
\]

\[ t \rightarrow \]

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About stationarity

A deterministic signal is said to be stationary if it can be written as a discrete sum of cosine waves or exponentials:

\[ x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi k f_0 t + \Phi_k) \]  \hspace{1cm} (1)

\[ x(t) = \sum_{k=-N}^{N} C_k \exp(2\pi j k f_0 t + \Phi_k) \]  \hspace{1cm} (2)

i.e. as a sum of elements which have constant instantaneous amplitude and instantaneous frequency.
About stationarity

In the random case, a signal \( \{ x(n) \} \) is said to be wide-sense stationary (or stationary up to the second order) if its variance is independent of time

\[
\sigma^2 = E[(x - \mu)^2] = \frac{1}{N} \sum_{n=0}^{N-1} (x - \mu)^2(n)
\]
The autocorrelation function for a discrete process of length $N \{x(n)\}$ with known mean $\mu$ and variance $\sigma$, 

$$\varrho_{xx}(n, n + m) = \frac{1}{N\sigma^2} \sum_{n=1}^{N} (x(n) - \mu)(x(n + m) - \mu)$$

depends only on the time difference $m$. 

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A signal is said to be non-stationary if one of these fundamental assumptions is no longer valid. For example, a finite duration signal, and in particular a transient signal (for which the length is short compared to the observation duration), is non-stationary.
Shor time Fourier transform of a non-stationarity signal

1. removing the mean of a signal
2. moving average filtering
3. segmentation of a signal using window functions
4. Fourier transform
Moving average filtering

- Assume we have a EEG signal corrupted with noise
- Set the mean to zero
- Apply moving average filter to the noisy signal (use filter order=3 and 5)
- The higher filter order will remove more noise, but it will also distort the signal more (i.e. remove the signal parts also)
- So, a compromise has to be found (normally by trial and error)
Moving average filtering - MATLAB file

% moving average filtering
% December 3, 2009
load('zdroj.mat');
y=EEG(3).Data(:,1);
N=length(y);
% length of average window is 3
for i=1:N-2,
signal3(i)=(y(i)+y(i+1)+y(i+2))/3;
end
signal3(N-1)=(y(N-1)+y(N))/2;
signal3(N)=y(256);
% length of average window is 5
for i=1:N-4,
signal5(i)=(y(i)+y(i+1)+y(i+2)+y(i+3)+y(i+4))/5;
end
Moving average filtering - MATLAB file

```matlab
signal5(N-3) = (y(N-3)+y(N-2)+y(N-1)+y(N))/4;
signal5(N-2) = (y(N-2)+y(N-1)+y(N))/3;
signal5(N-1) = (y(N-1)+y(N))/2;
signal5(N) = y(N);
subplot(3,1,1), plot(y, 'g'); title('original EEG')
subplot(3,1,2), plot(signal3,'r'); title('EEG signal with averaging of length 3')
subplot(3,1,3), plot(signal5,'b'); title('EEG signal with averaging of length 5')
print -depsc figureEEG
```
Moving average filtering

- Original EEG
- EEG signal with averaging of length 3
- EEG signal with averaging of length 5
## STFT, Wavelets, Huang Transform

<table>
<thead>
<tr>
<th></th>
<th>STFT</th>
<th>Wavelets</th>
<th>Huang</th>
</tr>
</thead>
<tbody>
<tr>
<td>inversion</td>
<td>yes</td>
<td>yes, but</td>
<td>no inversion</td>
</tr>
<tr>
<td>resolution in time</td>
<td>limited</td>
<td>good</td>
<td>good</td>
</tr>
<tr>
<td>resolution in frequency</td>
<td>good</td>
<td>bad</td>
<td>floating frequency</td>
</tr>
</tbody>
</table>
Thank you for your attention