Dynamic Systems Identification
Part 1 - Linear systems

Reference:

Identification of dynamic systems

1. experimental modelling of dynamic systems
2. Basic rule: Do not estimate what you already know!
3. results of research and engineering practice
4. white box model, grey box model, black box model
5. available literature and software
6. black box linear models: linear systems identification (Ljung, Isermann, etc.)
**Static / dynamic model**

- **Static model**
  \[ F[u(t), y(t)] = 0 \]

- **Dynamic model**
  \[ F[t, u(t), u'(t), u''(t), \ldots, u^{(m)}(t), y(t), y'(t), y''(t), \ldots, y^{(n)}(t)] = 0 \]
  \[ F[k, u(k), u(k-1), \ldots, u(k-m), y(k), y(k-1), \ldots, y(k-n)] = 0 \]
Linear regression for dynamic systems

\[ y(k) = -a_1 y(k-1) - a_2 y(k-2) \ldots - a_n y(k-n) + b_1 u(k-1) + b_2 u(k-2) \ldots + b_m u(k-m) \]

\[ y(k) = [-y(k-1) \ -y(k-2) \ldots \ -y(k-n) \ u(k-1) \ u(k-2)\ldots u(k-m)] [a_1 \ a_2 \ldots \ a_n \ b_1 \ b_2 \ldots \ b_m]^T \]

\[ y = \psi \theta \]

\[ \hat{\theta} = \left[ \psi^T \psi \right]^{-1} \psi^T y \]
Autoregressive model with exogenous inputs (ARX)

\[ y(k) + \sum_{j=1}^{n} a_j y(k - j) = \sum_{i=1}^{m} b_i u(k - i - d) + e(k) \]

\[ A(z^{-1}) y(z^{-1}) = B(z^{-1}) u(z^{-1}) + e(z^{-1}) \]

\[ y(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})} u(z^{-1}) + \frac{1}{A(z^{-1})} e(z^{-1}) \]

filtering
Division of identification methods

1. class of mathematical models
   1. nonparametric models
   1. parametric models

2. class of used signals
   1. continuous, discrete
   1. deterministic, random, pseudorandom

3. error between the system and its model
   1. input error
   1. output error
   1. generalised error

4. concurrency
   1. offline
   1. online

5. data processing
   1. nonrecursive
     1. direct
     1. iterative
   1. recursive

6. model structure
   1. linear models
   1. nonlinear models
Nonparametric models

1. mostly linear models

2. I/O characteristics as numeric tables or curves
   1. frequency responses (Bode diagrams)
   2. impulse response, step response
   3. Fourier analysis, analysis of frequency response, correlation analysis, spectral analysis
Parametric models

1. **linear and nonlinear models**
   1. models with explicit parameters
      1. differential equations
      1. difference equations
      1. transfer functions
      1. state-space functions

1. **model structure:**
   1. system order
   1. regressors
On-line model fitting

Input signal

Process

Model

Calculation of residuals

Optimisation

\( y \)

\( \hat{y} \)

\( \varepsilon \)
Off-line model fitting

Data file → Model → Calculation of residuals

Time

Data file

Model

Optimisation

Calculation of residuals

\[ y \]

\[ \hat{y} \]

\[ u \]
Output error model

Input signal

Process

Model

Optimisation

\[ y \]

\[ \hat{y} \]

\[ \varepsilon \]
Input error model

Time

Data file

Inverse model

Optimisation

\[ \hat{u} \]

\[ y \]

\[ u \]
Equation error model
Methods for parametric models identification (System Identification Toolbox)

1. **linear systems**

\[
A(z^{-1})y(z^{-1}) = \frac{B(z^{-1})}{F(z^{-1})} u(z^{-1}) + \frac{C(z^{-1})}{D(z^{-1})} e(z^{-1})
\]

- FIR (A=F=D=1, C=0)
- ARX (F=C=D=1)
- OE (A=C=D=1)
- ARMAX (F=D=1)
- BJ (A=1)

REGRESSORS!
1. Autoregressive model with exogenous inputs (ARX)
\[ y(k) = a_1 y(k-1) + a_2 y(k-2) + b_1 u(k-1) + b_2 u(k-2) + e(k) \]

2. Autoregressive moving average model with exogenous inputs model (ARMAX)
\[ y(k) = a_1 y(k-1) + a_2 y(k-2) + b_1 u(k-1) + b_2 u(k-2) + e(k) + c_1 e(k-1) - c_2 e(k-2) \]
Output error model (OE)

\[ y(k) = a_1[y(k-1) - e(k-1)] + a_2[y(k-2) - e(k-2)] + b_1u(k-1) + b_2u(k-2) + e(k) \]
What are we doing in identification?

1. Example: the first order dynamic system

   \[ y(k) = 0.9512y(k-1) + 0.09754u(k-1) \]

   1. 1st order
   1. Regressors: \(y(k-1), u(k-1)\)
   1. \(y(k)= -a_1y(k-1) + b_1u(k-1)\)

   \[
   \begin{bmatrix}
   y(2) \\
   -y(1) \ u(1) \\
   a_1 \\
   \\
   y(3) \\
   -y(2) \ u(2) \\
   b_1 \\
   \end{bmatrix}
   =
   \begin{bmatrix}
   y \\
   \end{bmatrix}
   = \Psi \Theta
   \]

   \[
   \begin{align*}
   H(z) &= \frac{0.09754z^{-1}}{1-0.9512z^{-1}}
   
   H(z) &= \frac{0.09754}{1-0.9512z^{-1}}
   
   \end{align*}
   \]

   1. Order of rows and columns can be changed!!
   1. Optimal solution by least squares cost function

   \[
   \Theta = (\Psi^\top\Psi)^{-1}\Psi^\top y
   \]

   1. Parameters are optimal for one-step-ahead prediction, validation is done with simulation (multi-step-ahead prediction).
System modeling from data

Input signal

Output signal

Input signal vs. Output signal
Model

Systems modelling from data