What are we doing in identification?

- Example: the first order dynamic system

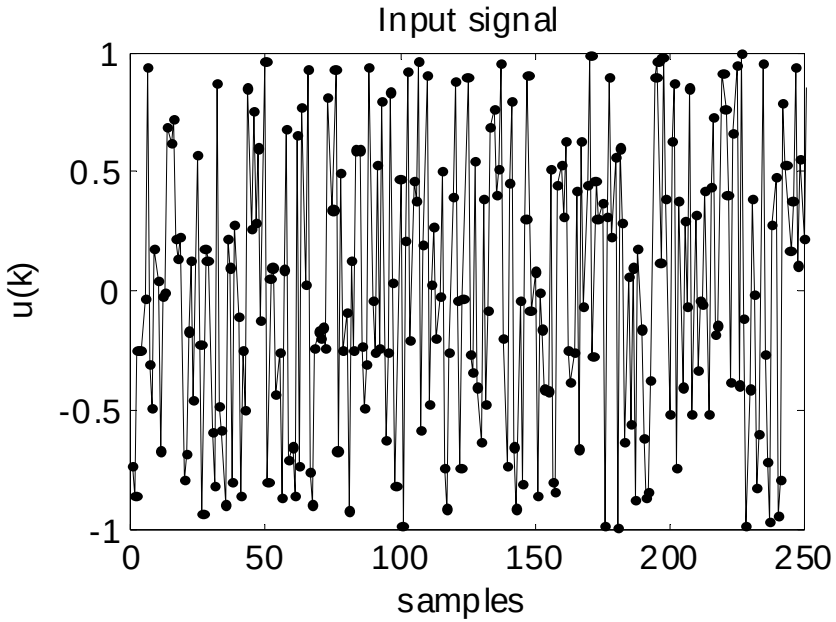
$$y(k) = 0.9512\,y(k-1) + 0.09754\,u(k-1)$$

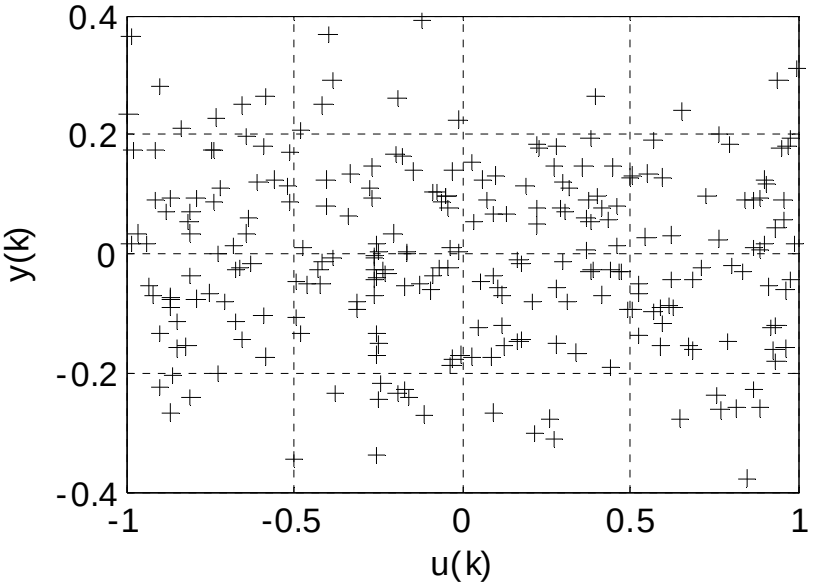
$$H(z) = \frac{0.09754 z^{-1}}{1 - 0.9512 z^{-1}}$$

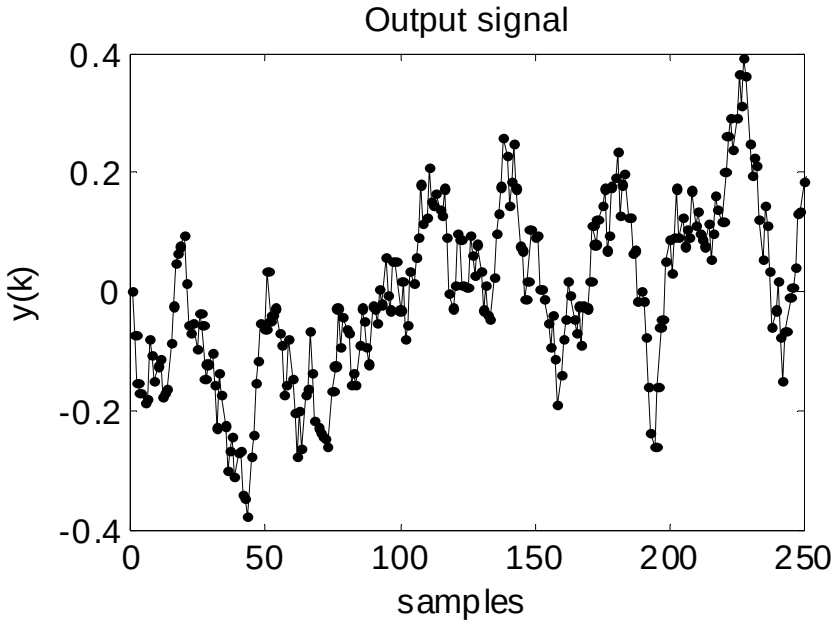
  - *1st order*
  - *Regressors: y(k-1),u(k-1)*
  - $y(k) = -a_1 y(k-1) + b_1 u(k-1)$
  - $\begin{bmatrix} y(2) \\ y(3) \\ \dots \end{bmatrix} = \begin{bmatrix} -y(1) & u(1) \\ -y(2) & u(2) \\ \dots & \dots \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}$ $\quad \mathbf{y=\Psi\theta}$
  - Order of rows and columns can be changed!!
  - Optimal solution by least squares cost function

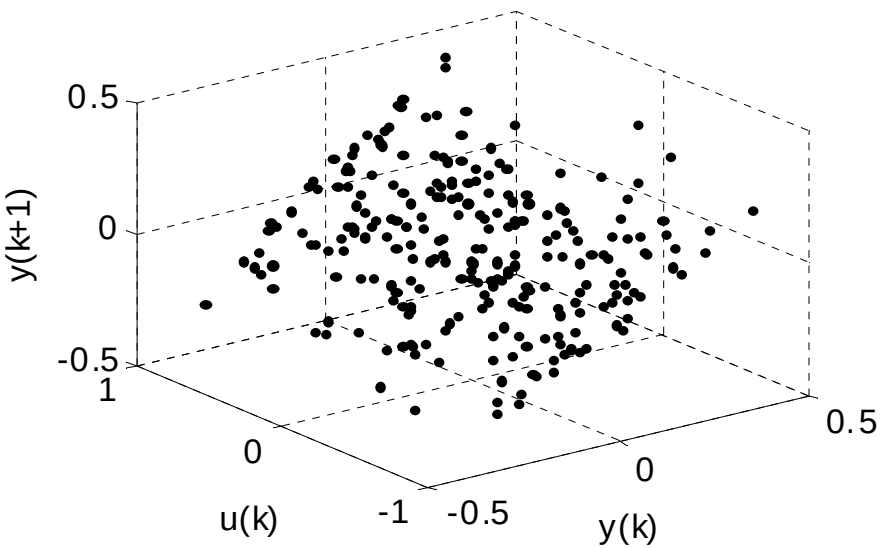
$$\underline{\theta=(\Psi^T\Psi)^{-1}\Psi^T y}$$

  - Parameters are optimal for one-step-ahead prediction, validation is done with simulation (multi-step-ahead prediction).

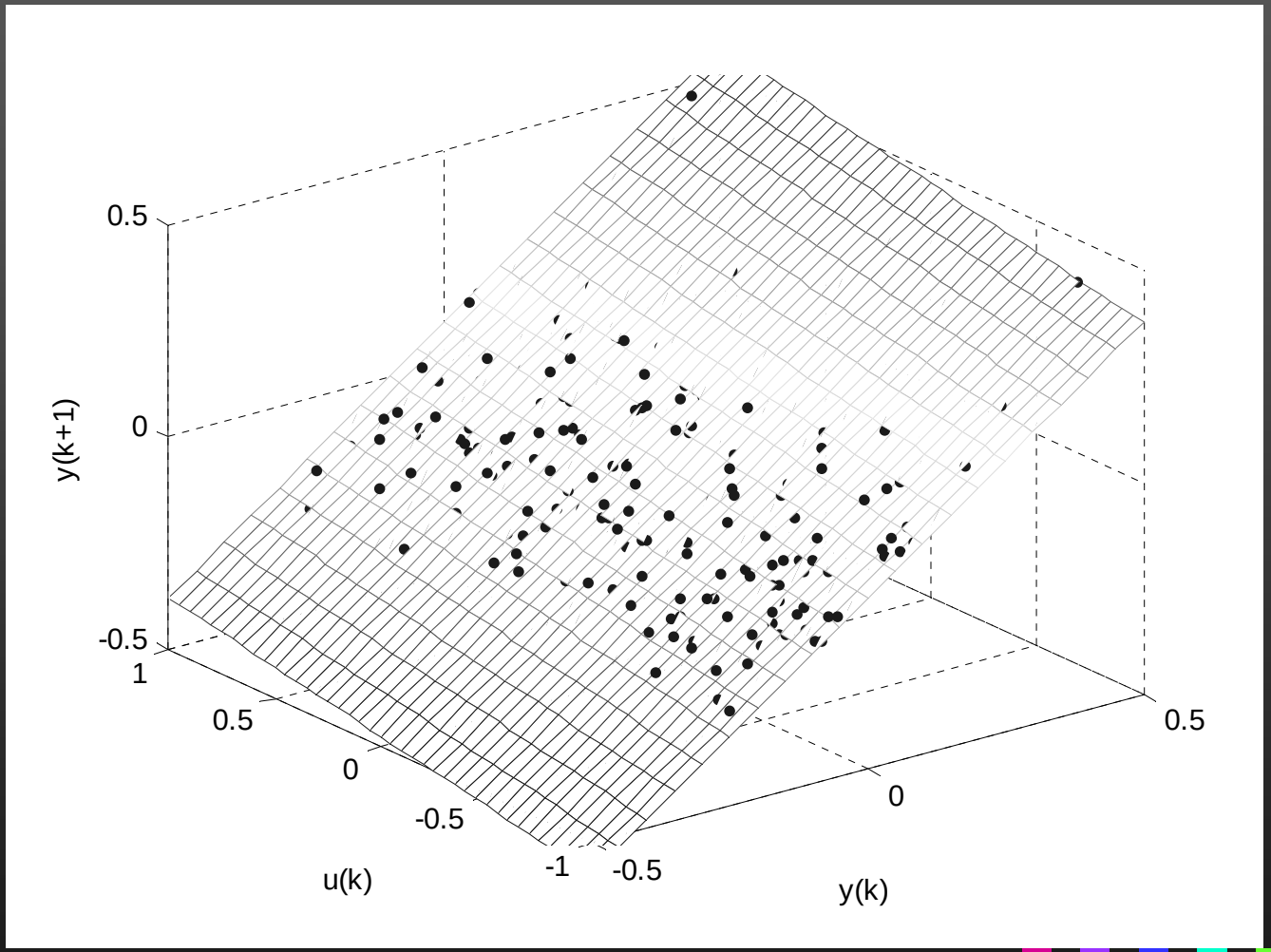

Input signal
u(k)
samples
1
0.5
0
-0.5
-1
0
50
100
150
200
250


y(k)
u(k)
0.4
0.2
0
-0.2
-0.4
-1
-0.5
0
0.5
1


Output signal
y(k)
samples
0.4
0.2
0
-0.2
-0.4
0
50
100
150
200
250

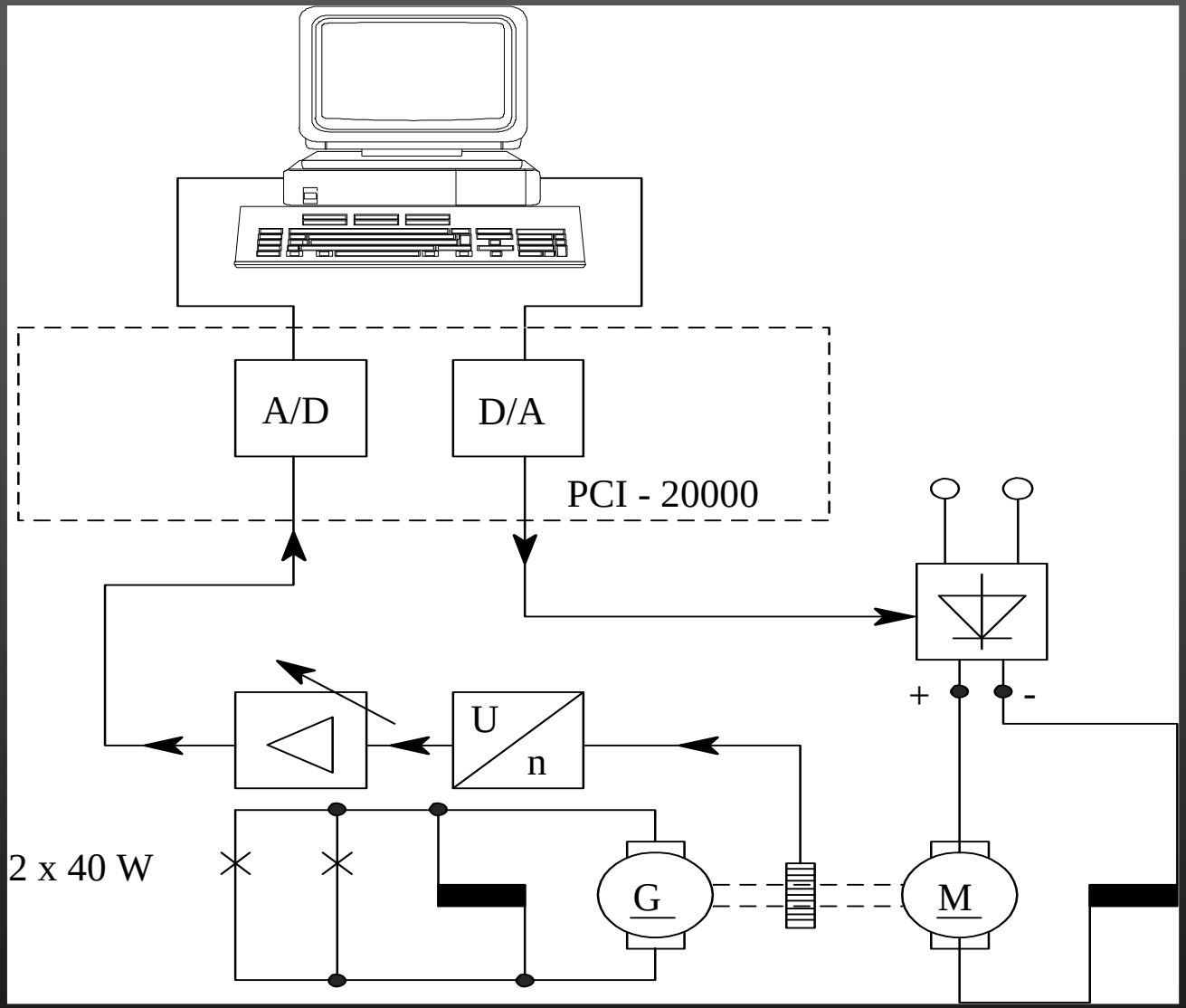

y(k+1)
u(k)
y(k)
0.5
0
-0.5
1
0
-1
-0.5
0
0.5

Model



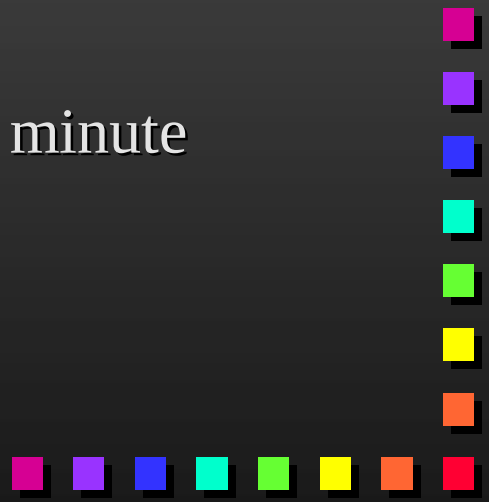
System Identification Case Study

Motor-generator test rig

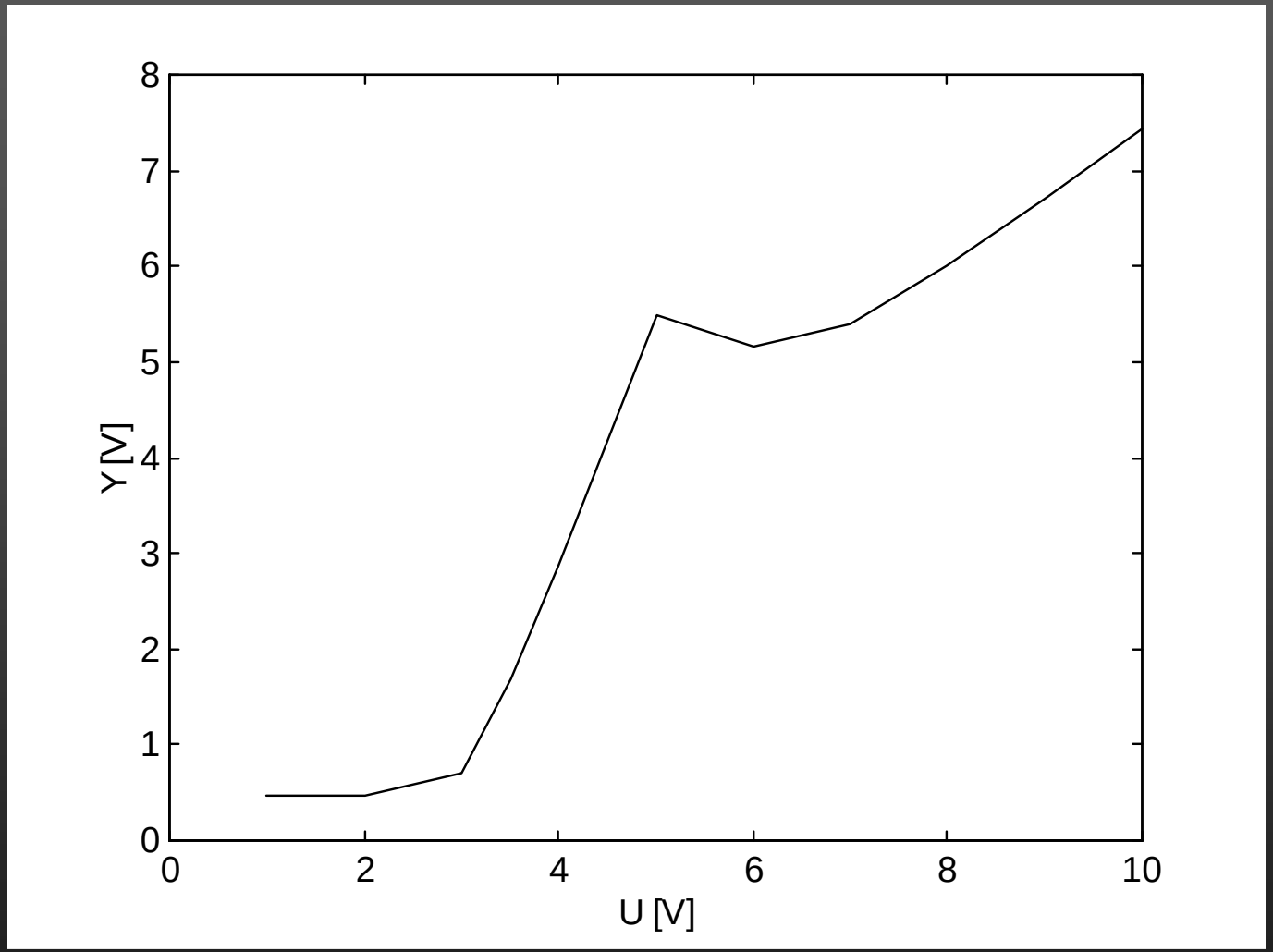


Motor-generator test rig

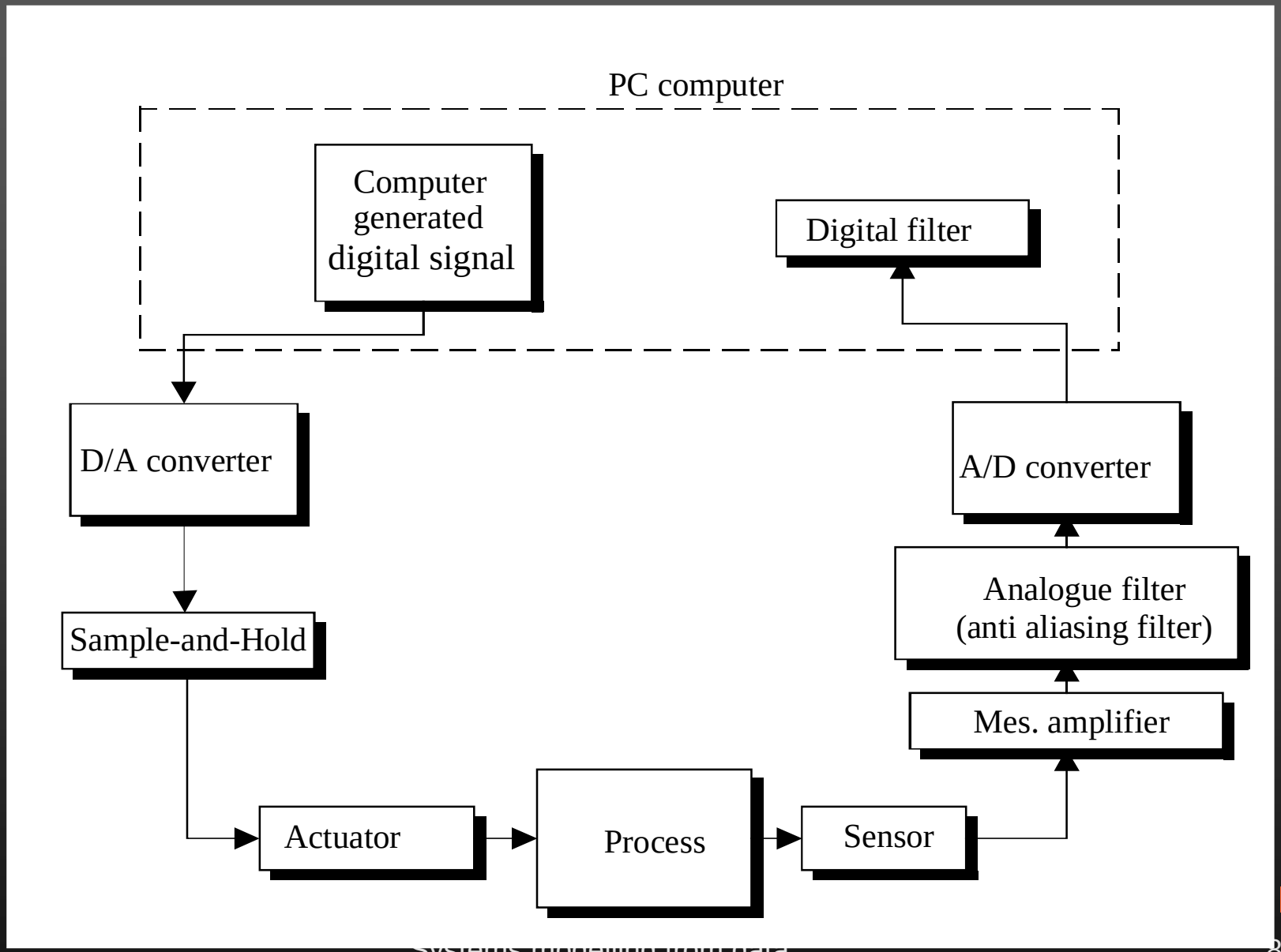
- *100 W DC motor, outside exicited DC generator*
- *load: two 40 W bulbs*
- *data acquisition with A/D converter, input signals via D/A converter and thyristor transmiter*
- *nonlinear static characteristic*
- *process input:* voltage on motor input
- *process output:* motor and generator rotations per minute



Motor-generator test rig

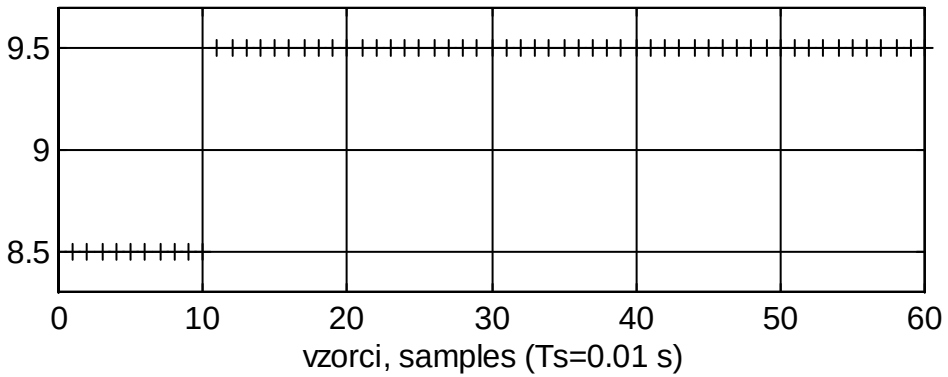


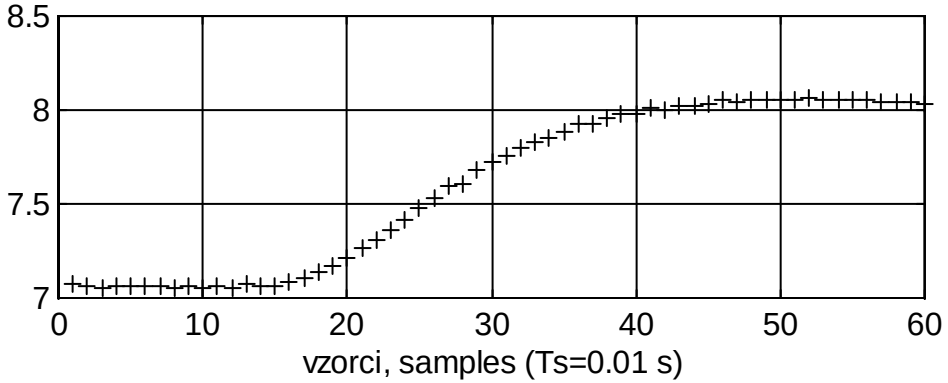
Data acquisition



Sampling time, input signal, pre-processing

- sampling time selection





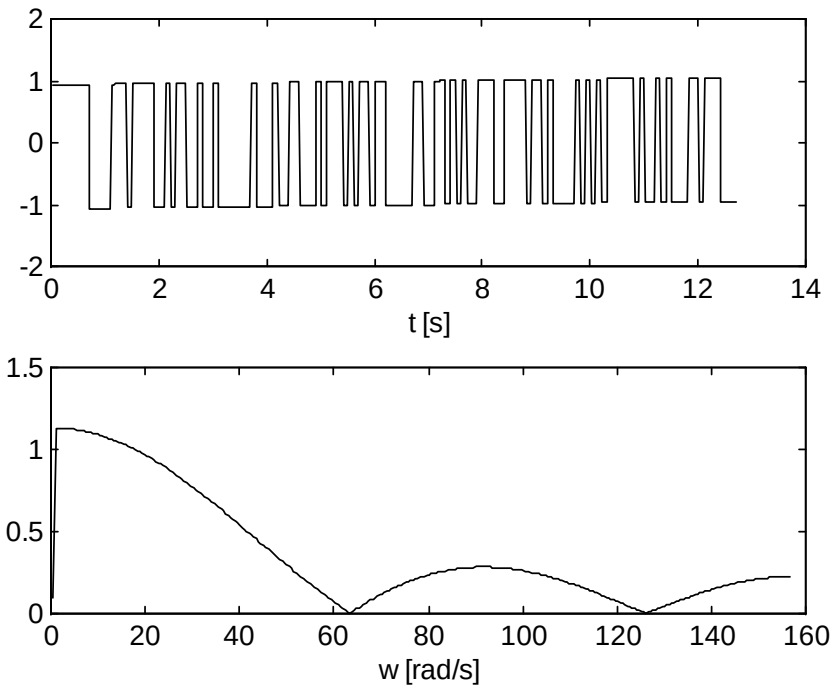
$$T_s = 0.02s$$

$$f_{\max} = \frac{1}{2T_s} = 25Hz$$

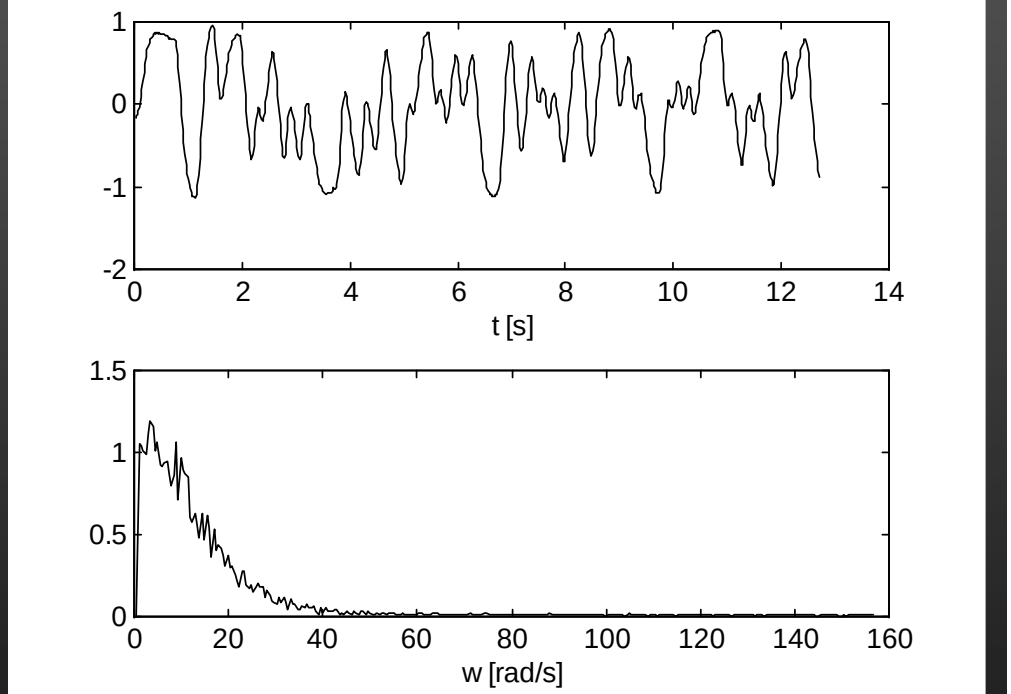
$$\omega_{\max} = 2\pi \cdot f_{\max} = 157rad/s$$

Sampling time, input signal, pre-processing

- *input signal selection*



- *removing offset and filtering*



- PRBS signal, 10-90% settling time

Model structure selection

- *analysis of information matrix*

$$\det\left(\frac{[\boldsymbol{\Psi}^T\boldsymbol{\Psi}]^{-1}}{N}\right)$$

$$G(j\omega)=\frac{\Phi_{uy}(j\omega)}{\Phi_{uu}(j\omega)}$$

$$W_3(j\omega)=\frac{\pi\sin(\omega T_M)}{\omega(\pi^2-\omega^2 T_M)}$$

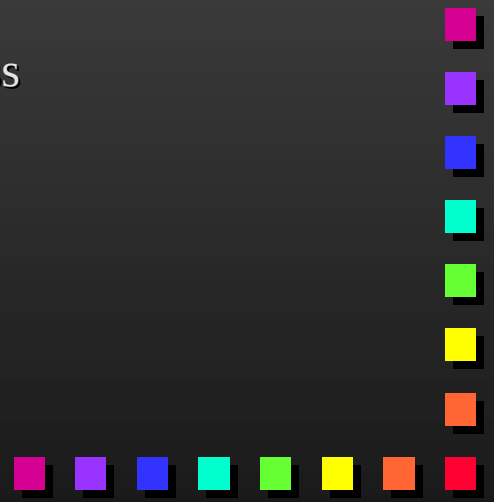
Model structure selection

- *Order and regressors*
  - *2nd order*
  - *Regressors: y(k-1),y(k-2),u(k-1),u(k-2)*
  - $y(k)=-a_1y(k-1)-a_2y(k-2)+b_1u(k-1)+b_2u(k-2)$
  - $\begin{bmatrix} y(3) \\ y(4) \\ \dots \end{bmatrix} = \begin{bmatrix} -y(2) & -y(1) & u(2) & u(1) \\ -y(3) & -y(2) & u(3) & u(2) \\ \dots & \dots & \dots & \dots \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$
  - Least-squares cost function - ARX

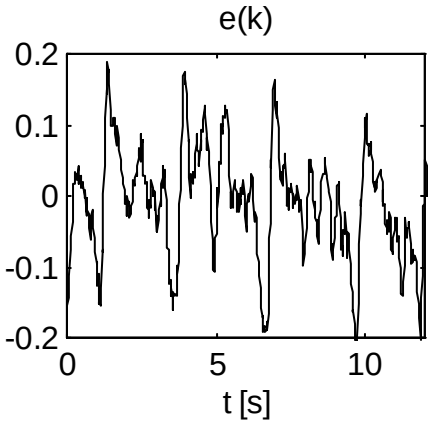
Model validation

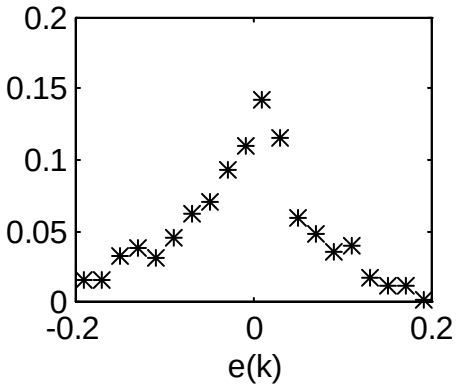
- same input signal as for identification
- different input signal for validation – cross-validation

- test of residuals
- frequency response consistency
- input-output consistency
- consistency of parameters, covariance matrix, parameter variances, model reduction, model purposiveness



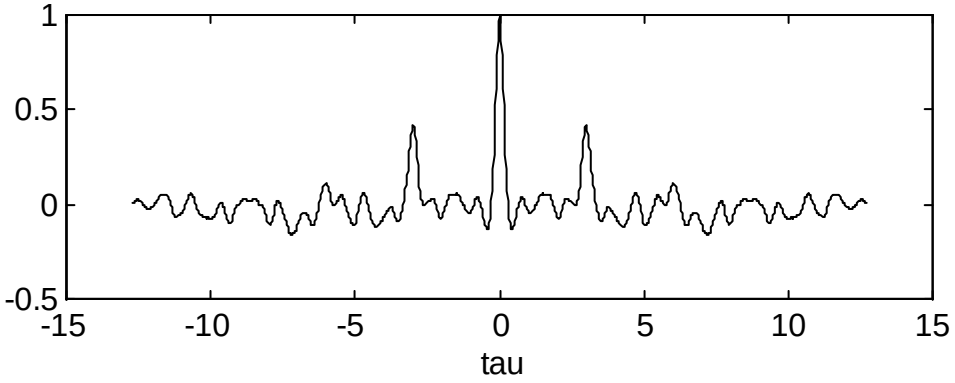
Test of residuals

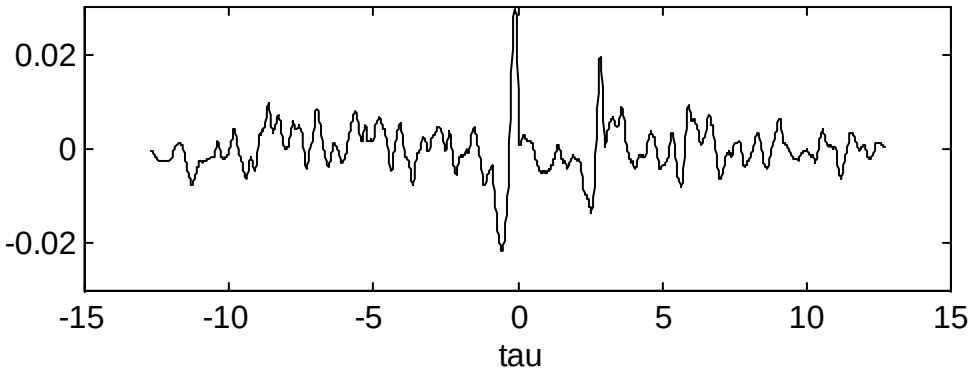




R_{ee}

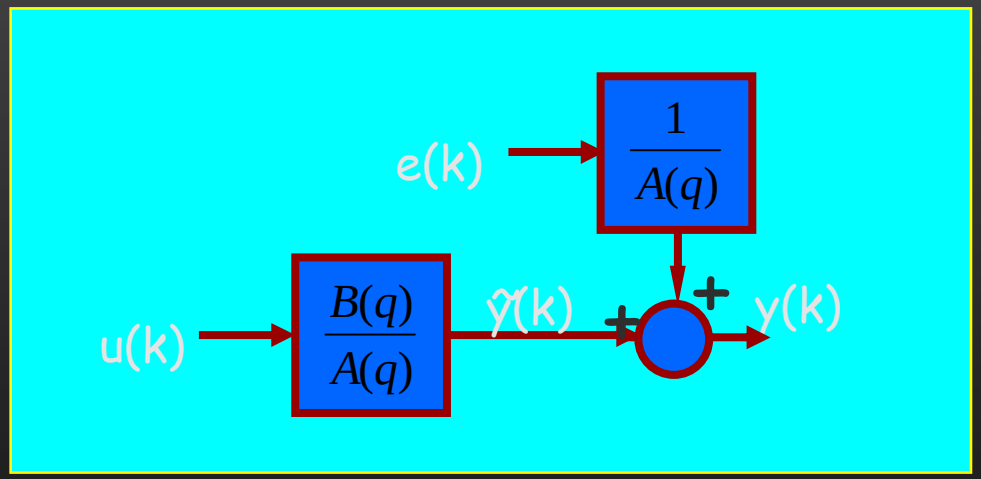
ϕ_{eu}



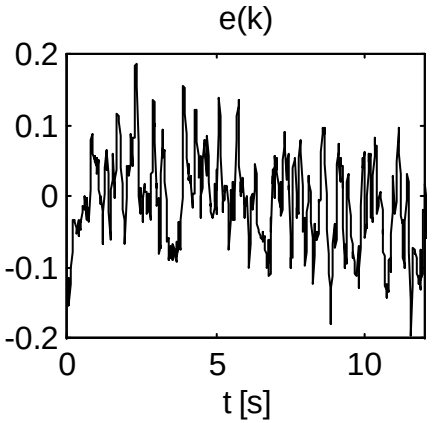


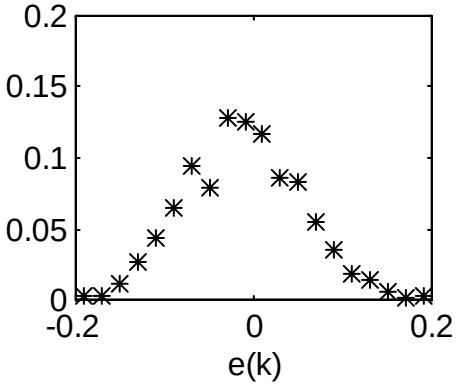
- Autoregressive model with exogenous inputs (ARX)

$$y(k)=a_1y(k-1)+a_2y(k-2)+b_1u(k-1)+b_2u(k-2)+e(k)$$



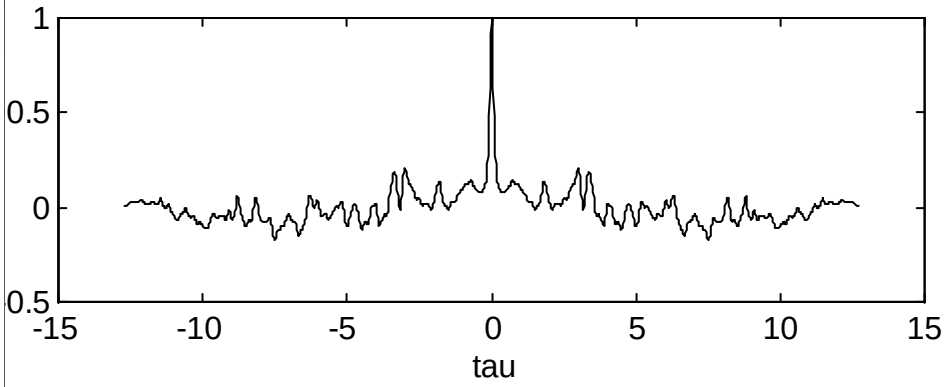
Test of residuals

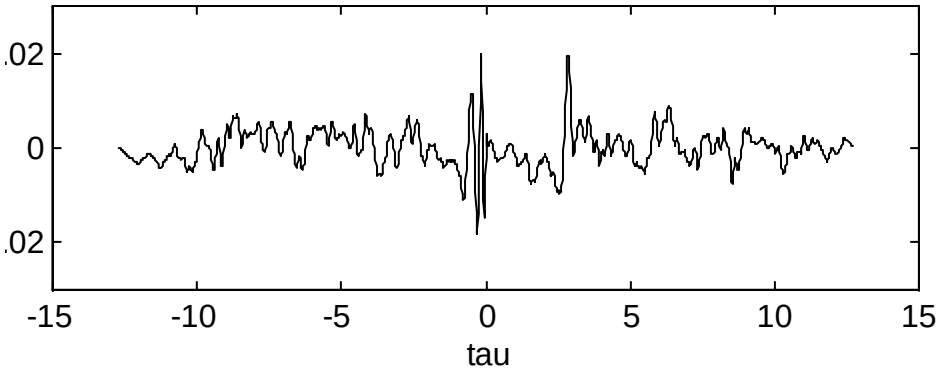




R_{ee}

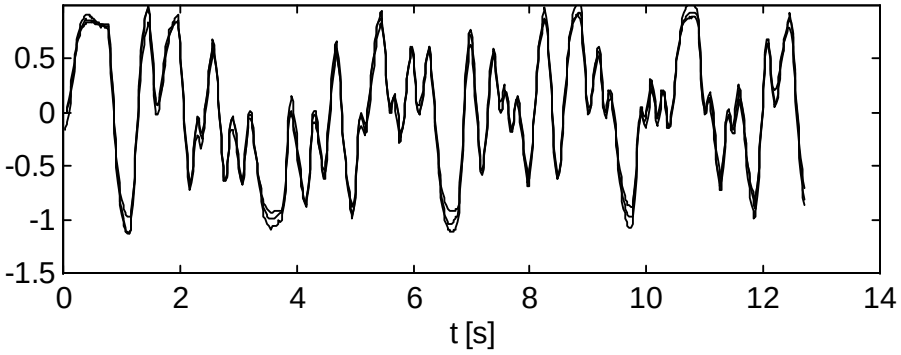
ϕ_{eu}

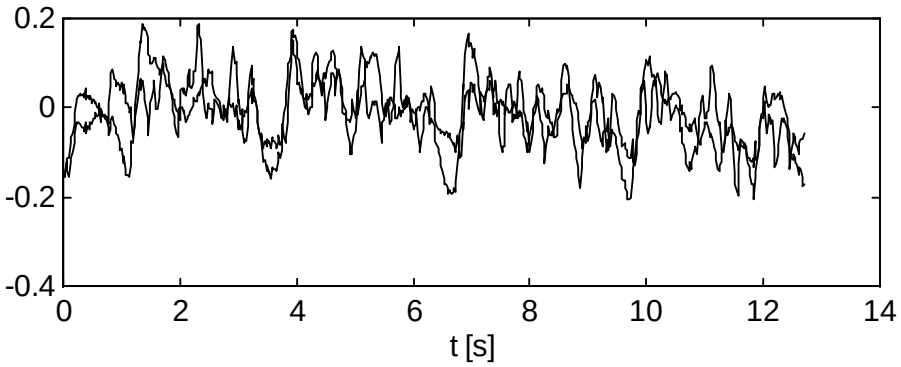




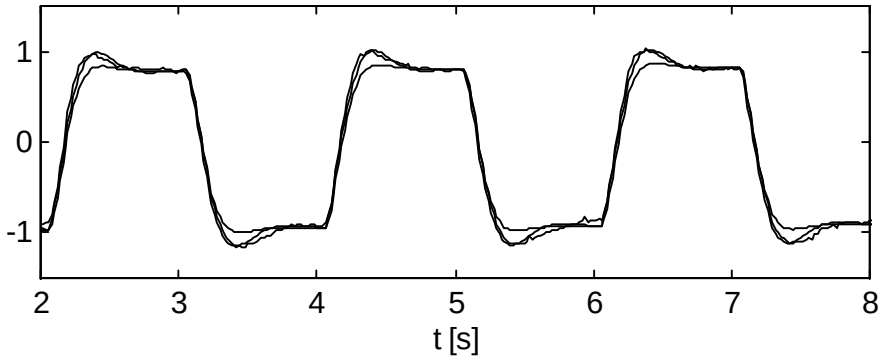
Input-output consistency

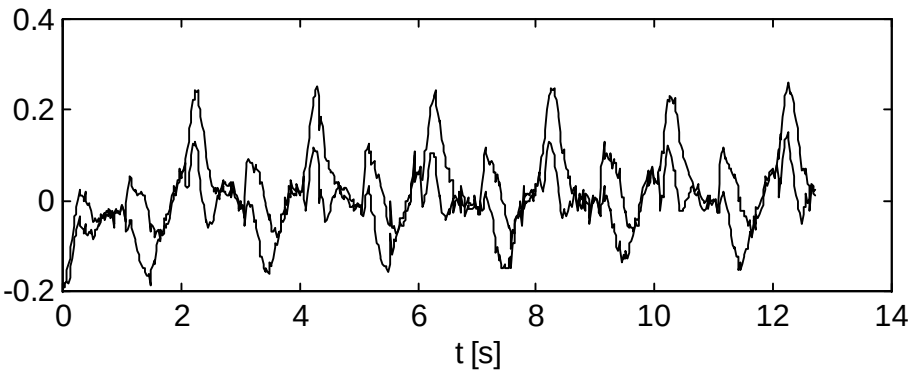
Identification signal





Validation signal





Consistency of parameters, model reduction, covariance matrix, model purposiveness

$$\mathrm{cov}[\hat{\boldsymbol{\Theta}} - \boldsymbol{\Theta}] = \sigma_e^2 E\{[\boldsymbol{\Psi}^T \boldsymbol{\Psi}]^{-1}\}$$

$\hat{G}_{pn}$:

$a_1 = -1.6379; \sigma = 0.0117;$

$a_2 = 0.6890; \sigma = 0.0112;$

$b_1 = 0.0281; \sigma = 0.0013;$

$b_2 = 0.0165; \sigma = 0.0017;$

$\hat{G}_{pf}$:

$a_1 = -1.7558; \sigma = 0.0040;$

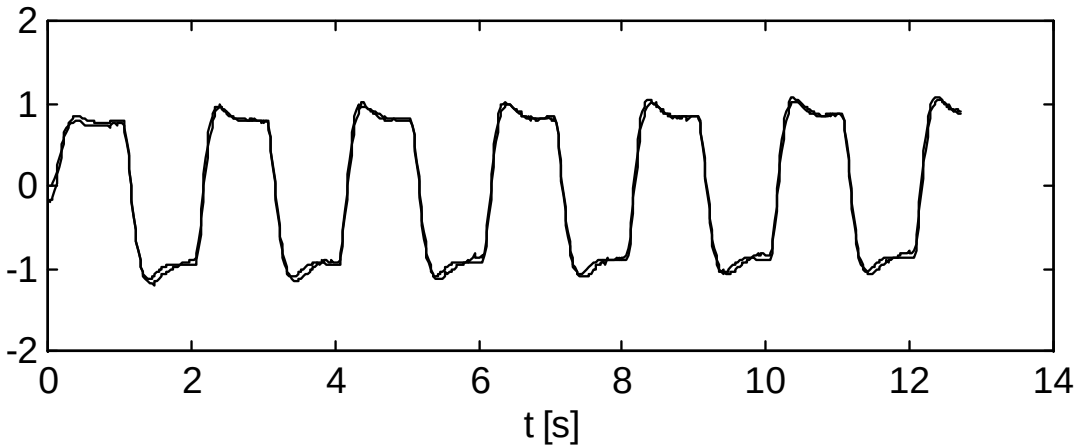
$a_2 = 0.7923; \sigma = 0.0035;$

$b_1 = 0.0590; \sigma = 0.0014;$

$b_2 = -0.0272; \sigma = 0.0019;$

$d = 2 samples$

Results of identification



$\hat{G}_{final}$:

$a_1 = -1.7144;$

$a_2 = 0.7589;$

$b_1 = 0;$

$b_2 = 0.0390;$

$d = 2 samples;$

- Dynamic systems identification is a routine operation in practice whenever we deal with linear and close-to-linear systems.
- Importance of input signal selection, sampling time selection, signals pre-processing, selection of regressors and model validation.
- System identification is highly ITERATIVE, but can never be automatic.
- The result of modelling is always a model, never a copy.
- The quality of model depends on the entire identification process.

MATLAB System Identification Toolbox

- ARX, ARMAX, OE, BJ, ARARX, ARARMAX, PEM, IV
- Recursive models: ARMAX, ARX, BJ, OE, PEM
- Time-series identification: AR, ARMA, IVAR
- Segmentation of data
- Structure search
- Other tools