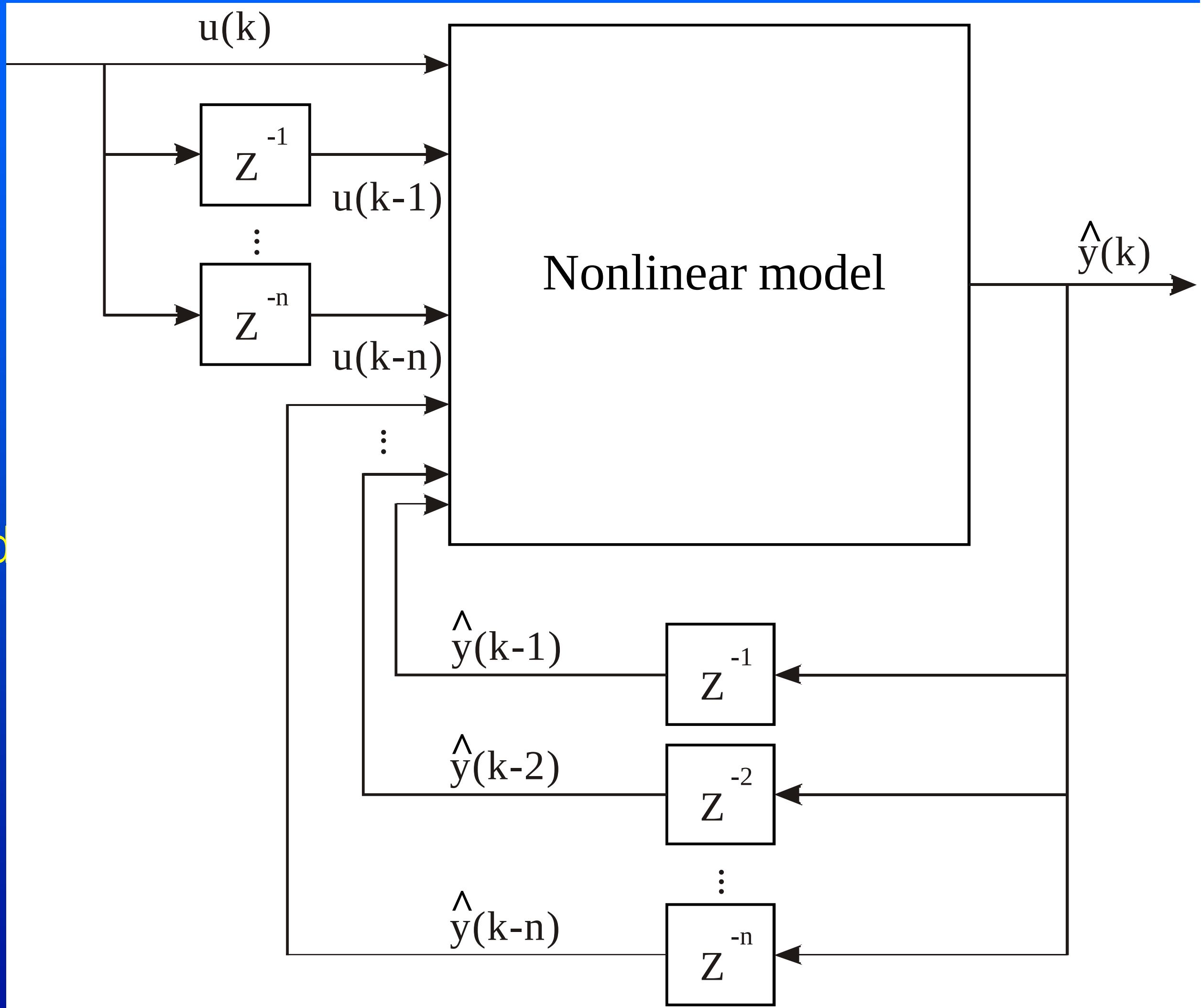
Dynamic Systems Identification
Part 2 - Nonlinear systems

Reference:

J. Sjöberg et al. (1995):Non-linear Black-Box Modeling in System Identification: a Unified Overview, Automatica, Vol. 31, 12, str. 1691 - 1724.

Nonlinear dynamic systems

- Static models
  - Input/output data
  - Interpolation or extrapolation
- Dynamic model
  - Input/output data expanded with delayed input and output data
  - Learning for one-step-ahead prediction
  - Validation for multi-step-ahead prediction

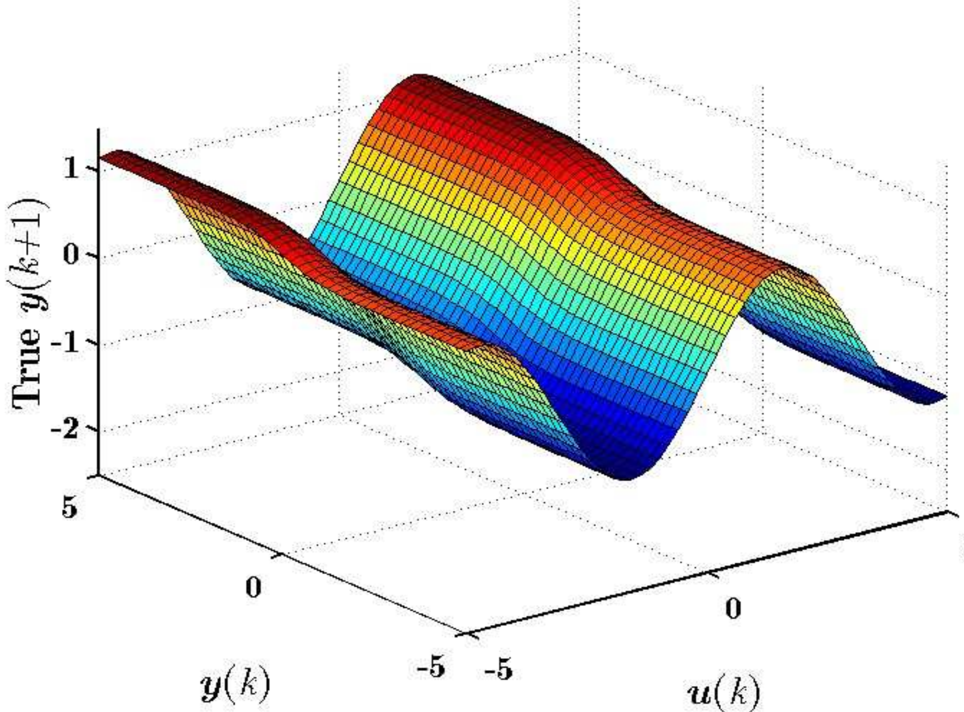


$$\hat{y}(k) = f(\hat{y}(k-1), \hat{y}(k-2), \ldots, u(k-1), \ldots)$$

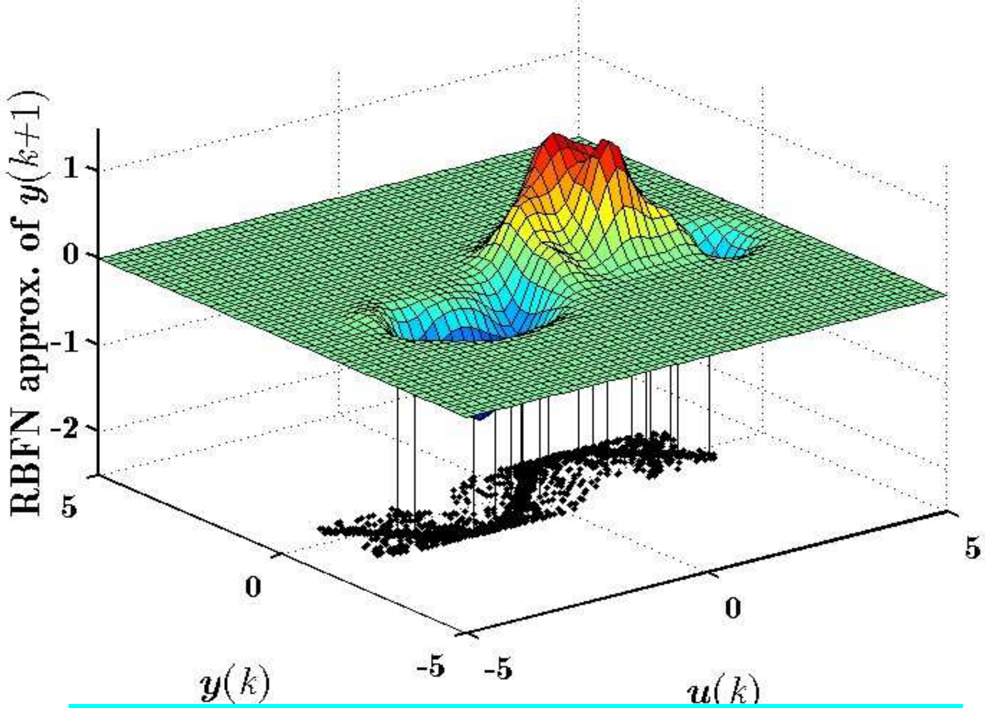
RBF network – Example

$$y(k+1) = 0.2\tanh(y(k)) + \sin(u(k))$$

2207 I/O data, 20 basis functions distributed randomly in operating area



Original function



Approksimation with 20 basis functions

Experimental modelling of nonlinear systems

- 1990s: ANN = nonlinear systems identification
- Rule:

  Do not estimate what you already know!
- white box model, grey box model (physical modelling, semi-physical modelling), black box model
- Nonlinear black-box models: artificial neural networks, fuzzy models, wavelet models, etc.

Experimental modelling of nonlinear systems

- Used terms (system theory vs. ANN/Machine learning:
  - estimate, identify = train, learn
  - validate = generalize
  - model structure = network
  - estimation data, identification data = training set
  - validation data = generalization set
  - overfit = overtraining

Some practical concerns about nonlinear systems identification

- Identification procedure cannot/must not be fully automatised!
- Neccessary: S/W, I/O data.
- We need experience on similar identification cases.

  Computer simulations of similar cases.

Experimental modelling of nonlinear systems

Nonlinear systems identification problem:

$$y(k) = g(u(k-1), u(k-2), \ldots, y(k-1), y(k-2), \ldots) + v(k)$$

$$\hat{y}(k \mid \boldsymbol{\theta}) = g(\boldsymbol{\psi}(k), \boldsymbol{\theta})$$

$\boldsymbol{\psi}(t)$ = vector of regressors

θ = vector of parameters

- 2 subproblems
  - Selection of regressors ψ(k)
  - Selection of nonlinear mapping **g(φ)**

Regressors

- linear systems

$$A(z^{-1})y(z^{-1}) = \frac{B(z^{-1})}{F(z^{-1})}u(z^{-1}) + \frac{C(z^{-1})}{D(z^{-1})}e(z^{-1})$$

- FIR (A=F=D=1,C=0)
- ARX (F=C=D=1)
- OE (A=C=D=1)
- ARMAX (F=D=1)
- BJ (A=1)
- State-space $\mathbf{x}(k) = \mathbf{A}\mathbf{x}(k-1) + \mathbf{B}\mathbf{u}(k-1)$

$$\hat{y}(k) = \boldsymbol{\theta}^T \boldsymbol{\psi}(k, \boldsymbol{\theta})$$

Regressors

- nonlinear systems $$\hat{y}(k) = g(\psi(k), \theta)$$

regressors ψ determine different models:

- NFIR: $u(k-i)$
- NARX: $u(k-i), y(k-i)$
- NOE: $u(k-i), \hat{y}(k-i)$
- NARMAX: $u(k-i), y(k-i), \varepsilon(k-i) = y(k-i) - \hat{y}(k-i)$
- NBJ: $u(k-i), y(k-i), \varepsilon(k-i), \varepsilon_n(k-i) = y(k-n) - \hat{y}_n(k-i)$
- State-space

- other possible regressors

Nonlinear mappings

$$g(\psi,\theta)=\sum \alpha_k g_k(\psi)$$

- Fourier series (scalar case)
- $g_k(\psi)$ is a basis function
- “Known” structures:
  - wavelet functions
  - B splines
  - ARTIFICIAL NEURAL NETWORKS
    - Multilayer perceptron
    - radial basis function network
    - etc
  - FUZZY MODELS

Forms of known structures:

- Neural networks with sigmoid activation function

$$g_k(\psi) = \sigma(\beta_k \psi + \gamma_k)$$

γ - position

β - direction scale

- Radial basis function networks

$$g_k(\psi) = r(\beta_k(\psi - \gamma_k))$$

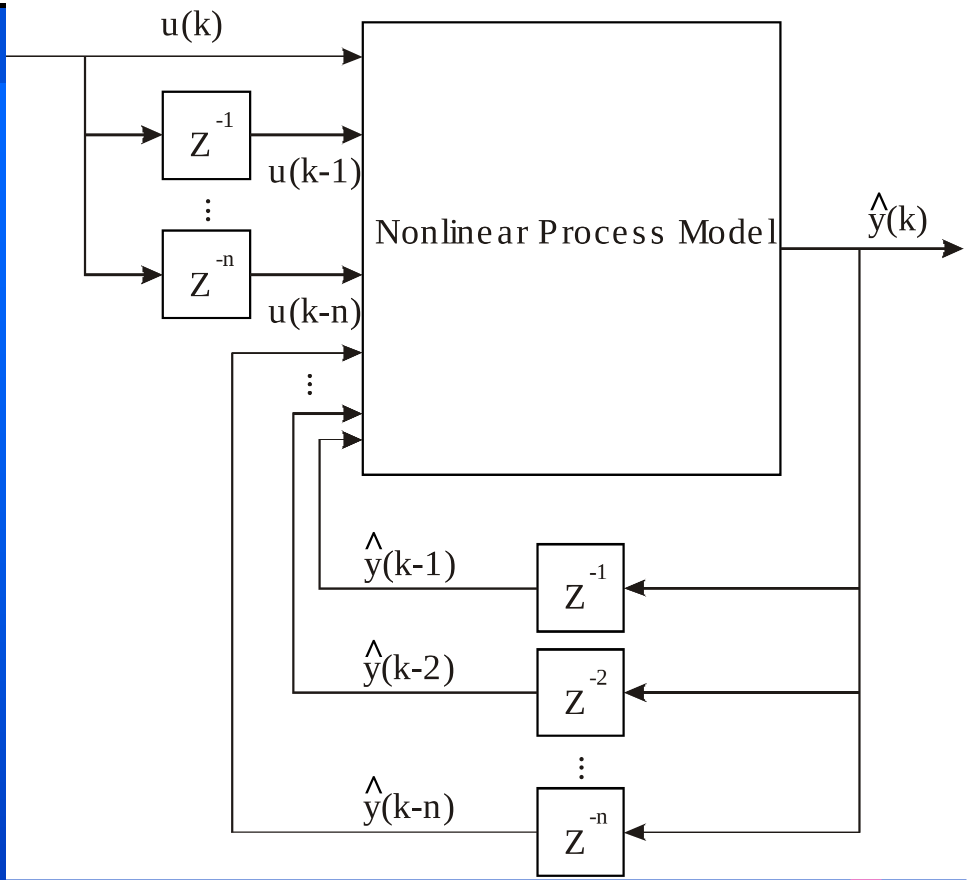
- Fuzzy models

$$g(\psi) = \sum_j y_j (\prod \mu_A(\psi))$$

- Multilayer networks

- Recurrent networks

$$\varphi(k) = g(\psi(k-i), \theta)$$



- Algorithms for estimation of parameters:
  - Gauss-Newton algorithms are very efficient,
  - Off-line identification, as well as on-line (recursive),
  - Gradient optimisation methods – time consuming.

Subproblems:

- Systematic selection of regressors
  - *u(k)* - static nonlinearity
  - *u(k-i)*
  - *u(k-i),y(k-i)*
  - ...
- Selection of basis function
  - Most of them are universal “approximators”
  - There exist no exact criteria for basis function selection except subjective ones
  - “Radial” functions for low number of regressors
    (e.g. wavelet function for max. 3 regressors)
  - “ridge” functions for larger number of regressors
    (e.g. neural networks with sigmoid functions)
  - Fuzzy models, where heuristic knowledge exsists.
- Models order (2n+1, Takan’s theorem);

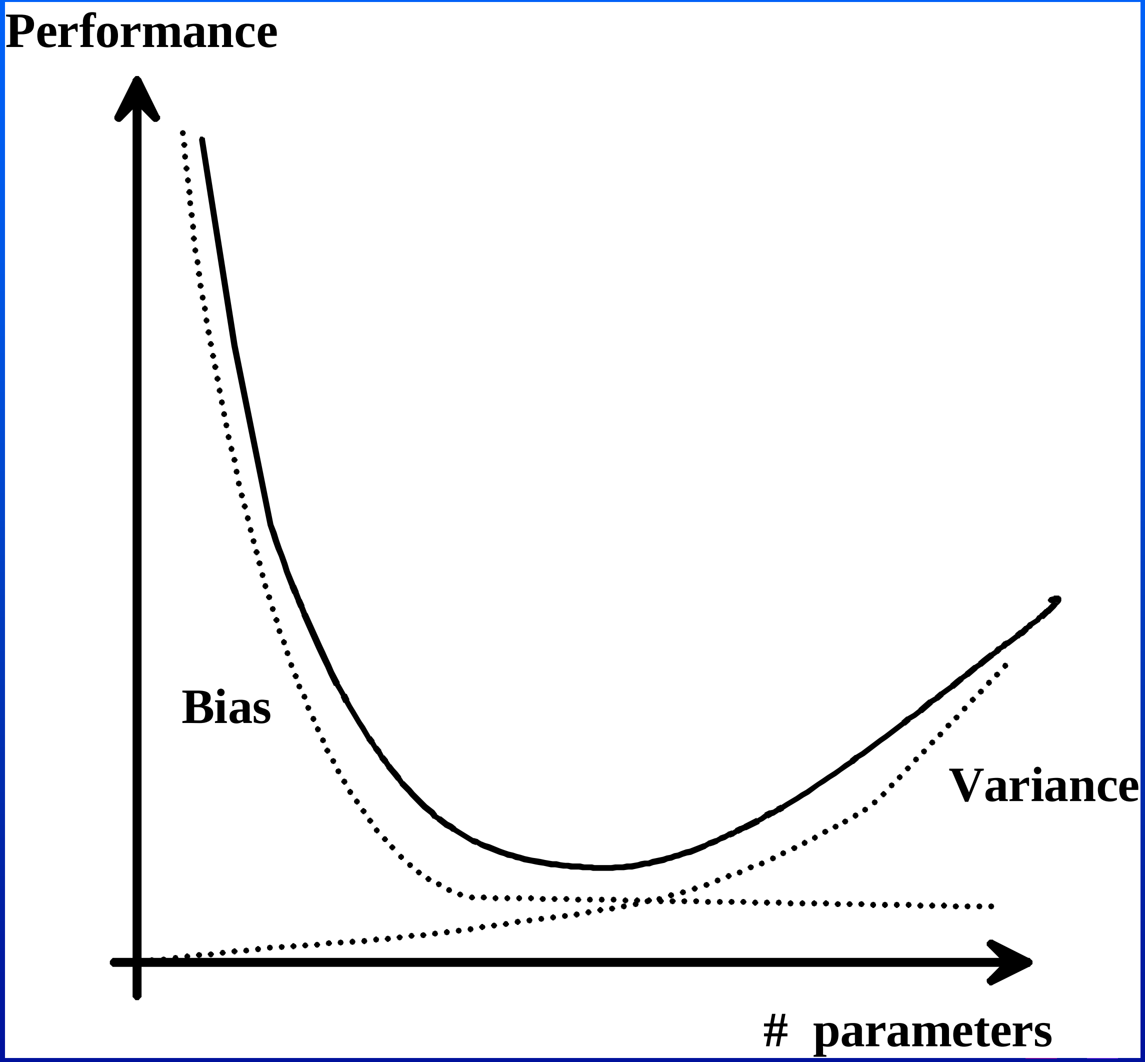
Measures of model quality

- Measure of model quality (example)

$$\bar{V}(\boldsymbol{\theta}) = E\left\| y(t) - g(\boldsymbol{\psi}(t), \boldsymbol{\theta}) \right\|^2 = \lambda + E\left\| g_0(\boldsymbol{\psi}(t)) - g(\boldsymbol{\psi}(t), \boldsymbol{\theta}) \right\|^2$$

- Three sources of differences with true system:
  - noise $e(t)$; variance $\lambda = E(e^2(t))$
  - bias $V = E\left\| g_0(\hat{\boldsymbol{\theta}}_N, \boldsymbol{\psi}(t)) - g^*(\boldsymbol{\psi}(t)) \right\|^2$
  - Variance of estimation $V = \lambda \frac{\dim \boldsymbol{\theta}}{N}$

There is always a limitation to model fit.



Recommendatins for practice:

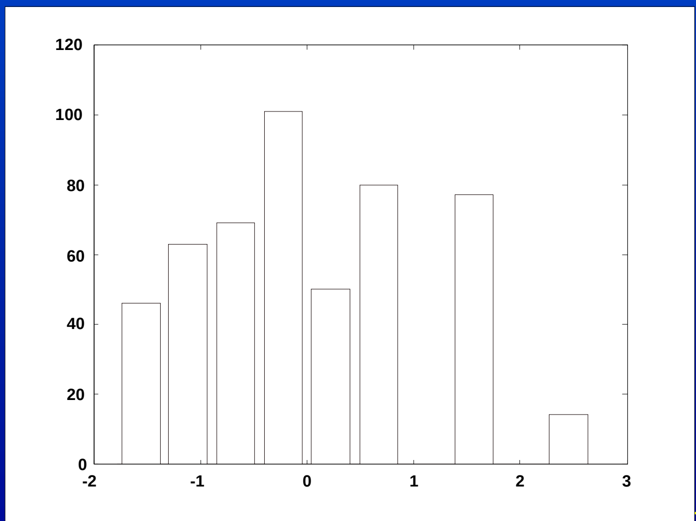
- Look at the data
  (detection of nonlinearity, time constants).
- Try simple things first
  (try simple structures and model orders first,
  at first linear models and small number of estimation parameters).
- Look into the physics (ideas for regressors).
- Validation and estimation data.
- Center and scale the data.
- The bias-variance trade-off.
- The notion of efficient number of parameters (shrinking).

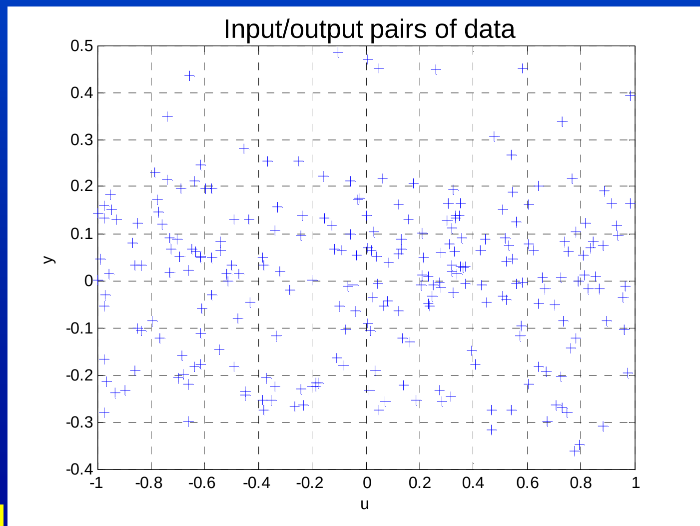
- Sampling time selection (same rule as for linear systems).

Sampling time should be selected to grasp all interesting process dynamics.

- Input signal selection

(look at magnitude distribution and input/output distribution of the data)





Model validation

- One-step-ahead prediction

$$\hat{y}(k) = g(y(k-1), y(k-2), \ldots u(k-1), u(k-2), \ldots)$$

nnvalid

- Simulation

$$\hat{y}(k) = g(\hat{y}(k-1), \hat{y}(k-2), \ldots u(k-1), u(k-2), \ldots)$$

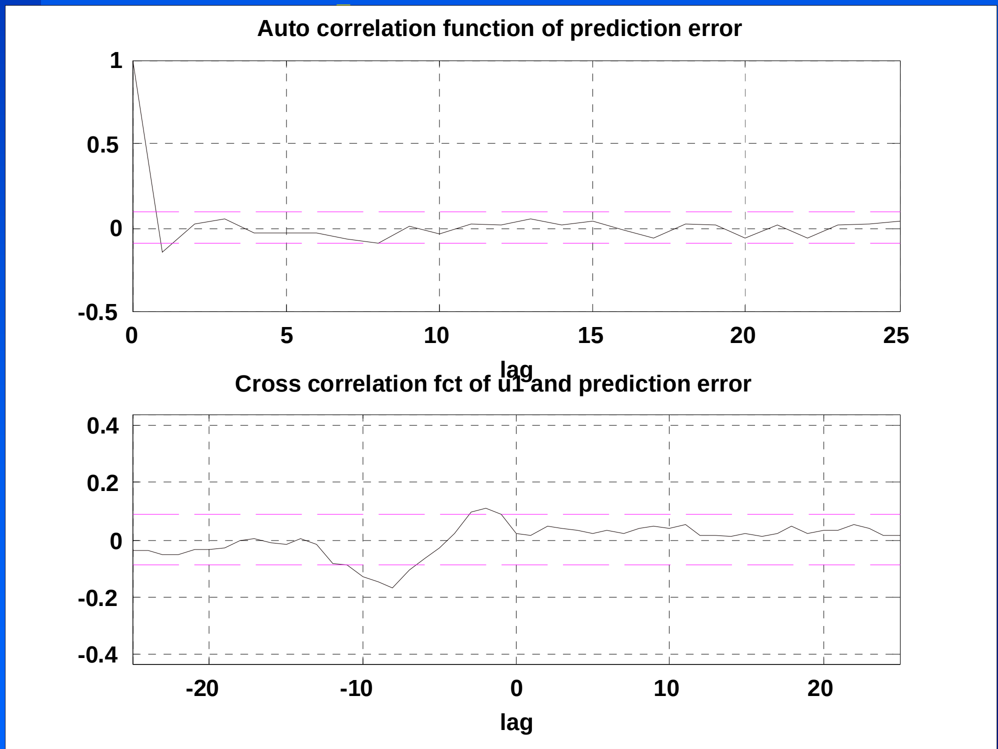
nnsimul

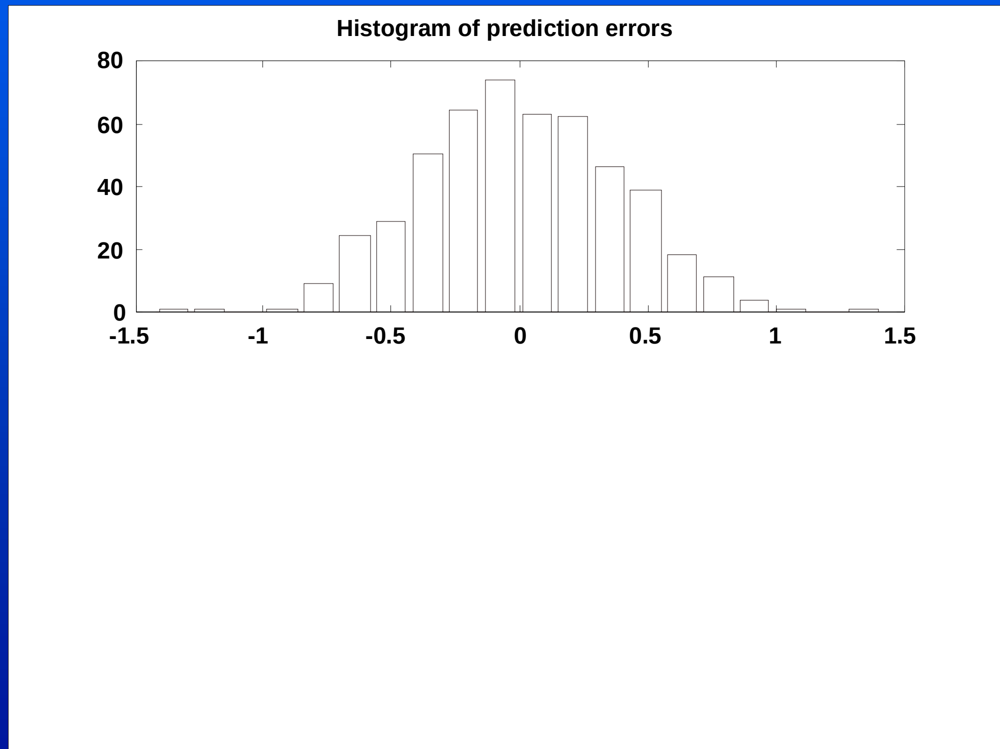
crossco

hist

Model validation

- Validation of residuals for one-step-ahead





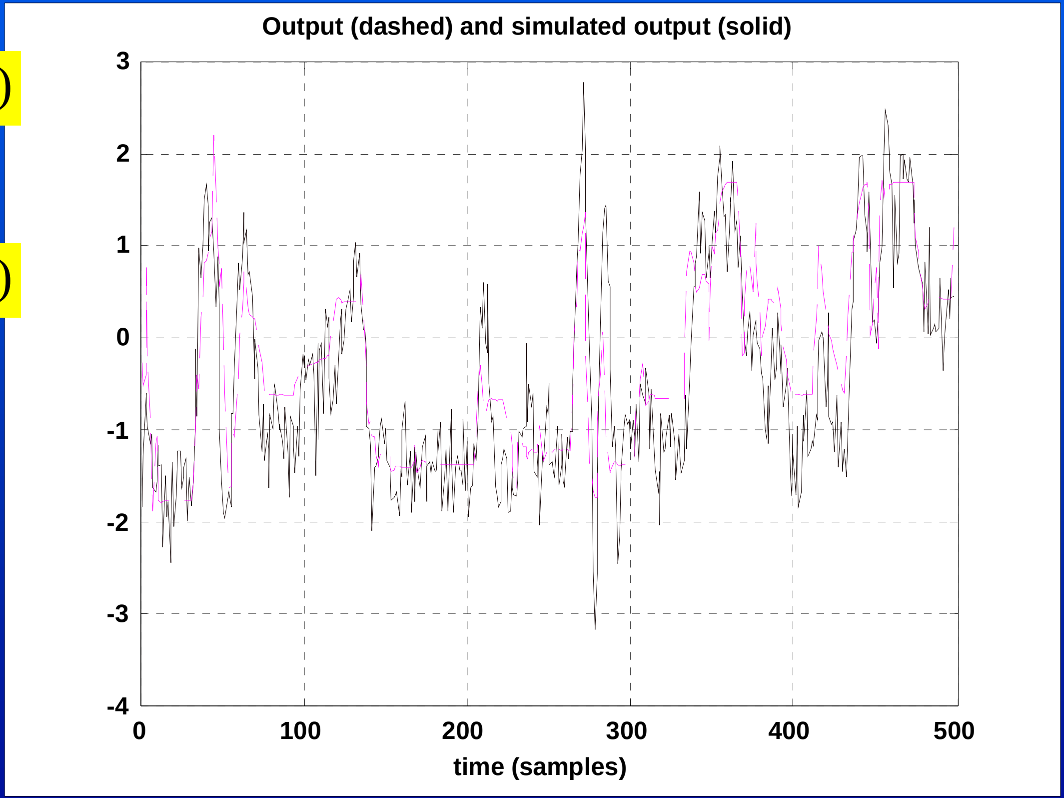
Model validation

- The consistency of input/output response
  - One-step-ahead prediction

$$\hat{y}(k) = g(y(k-1), y(k-2), \ldots u(k-1), u(k-2), \ldots)$$

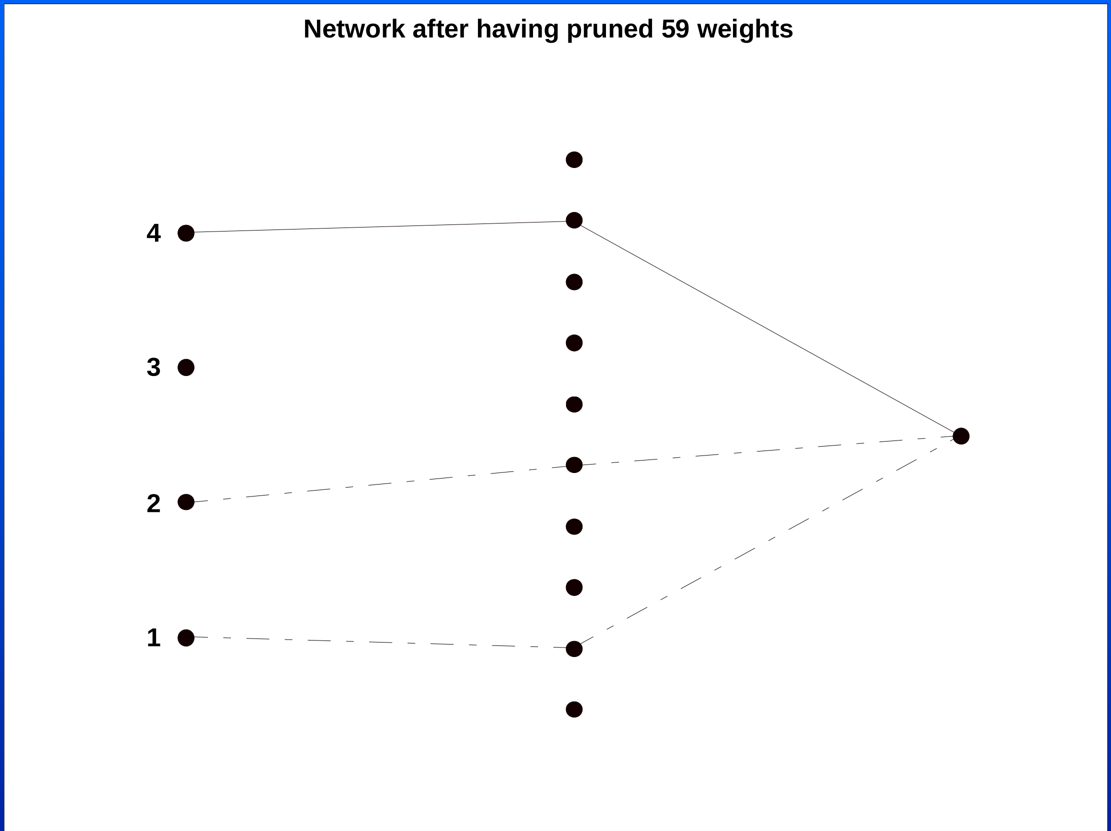
  - Simulation ✓

$$\hat{y}(k) = g(\hat{y}(k-1), \hat{y}(k-2), \ldots u(k-1), u(k-2), \ldots)$$



Model validation

- Modela reduction (pruning)



- Model purposiveness or usefulness

Example of identification of 1st order system

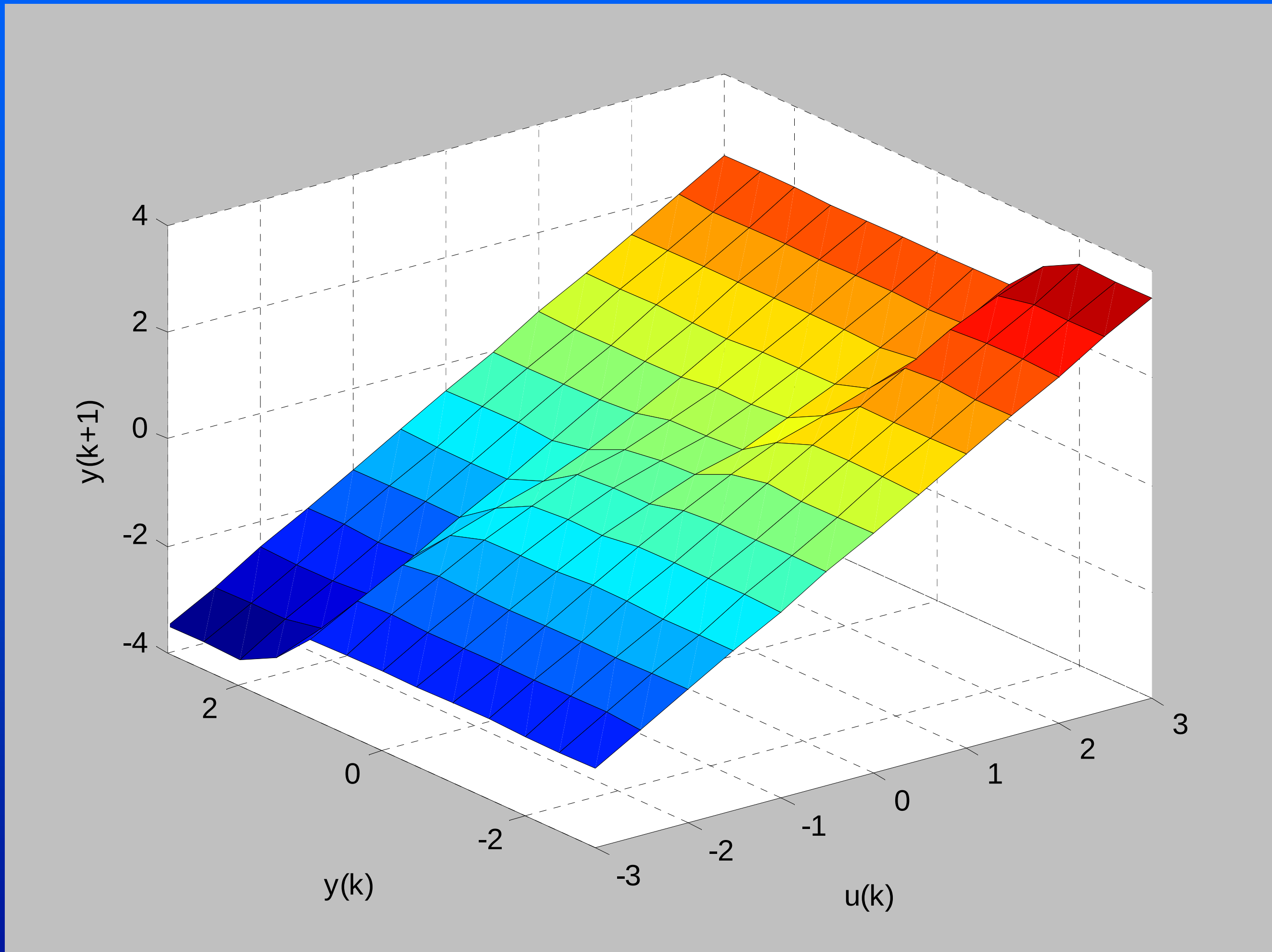
Mathematical model of the process with parameters

$$y(k) = y(k-1) - 0.5\tanh\left(y(k-1) + u(k-1)^3\right)$$

u – input signal

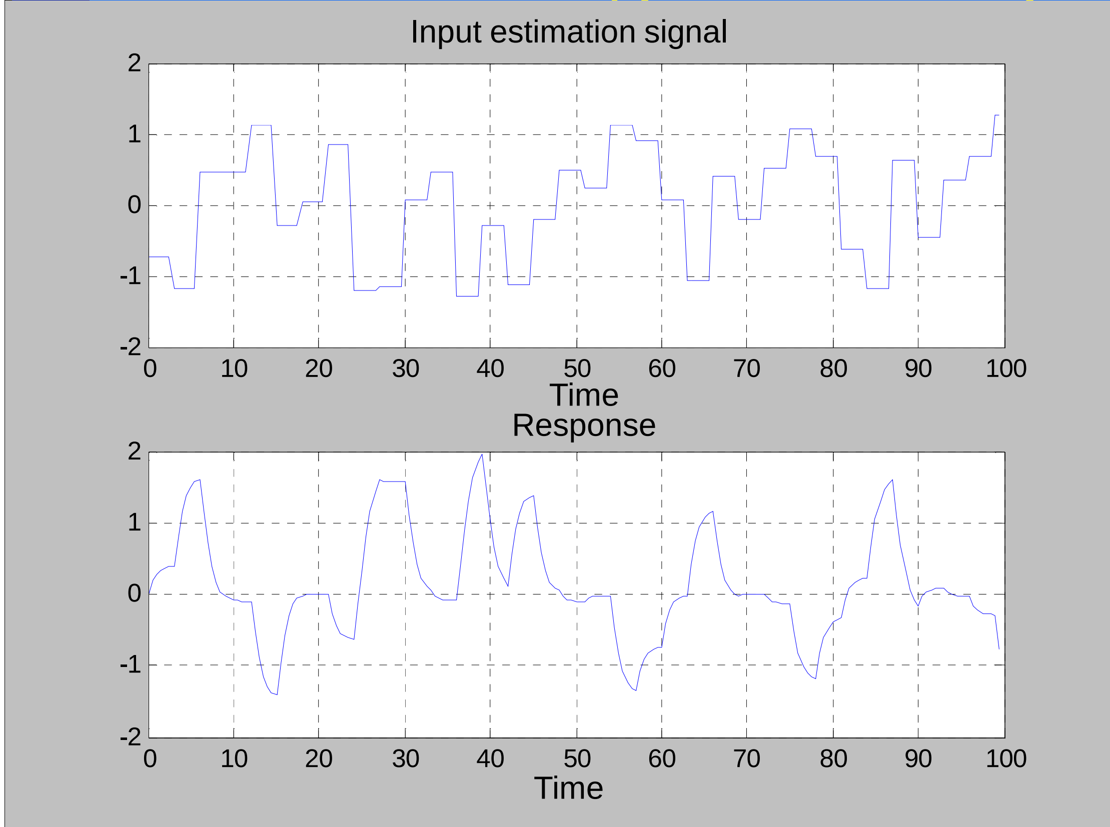
y – output signal

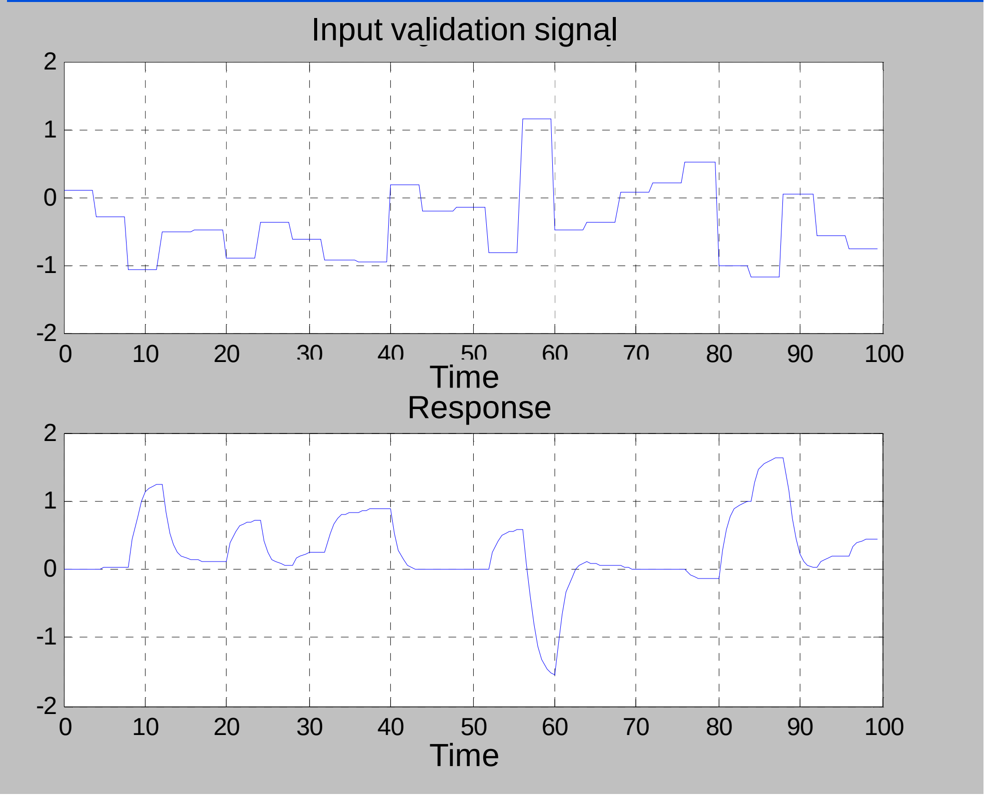
Nonlinearity of the system



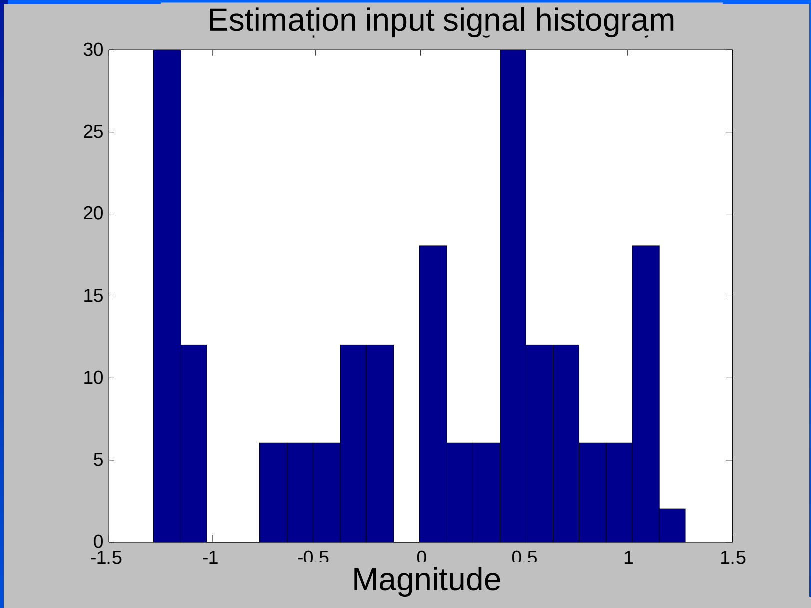
Estimation signal and response

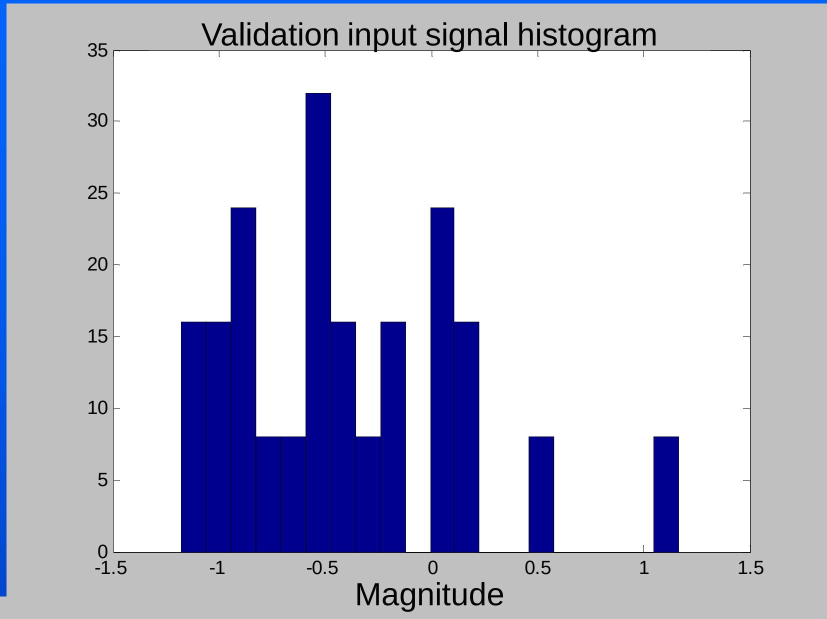
Validation signal and response

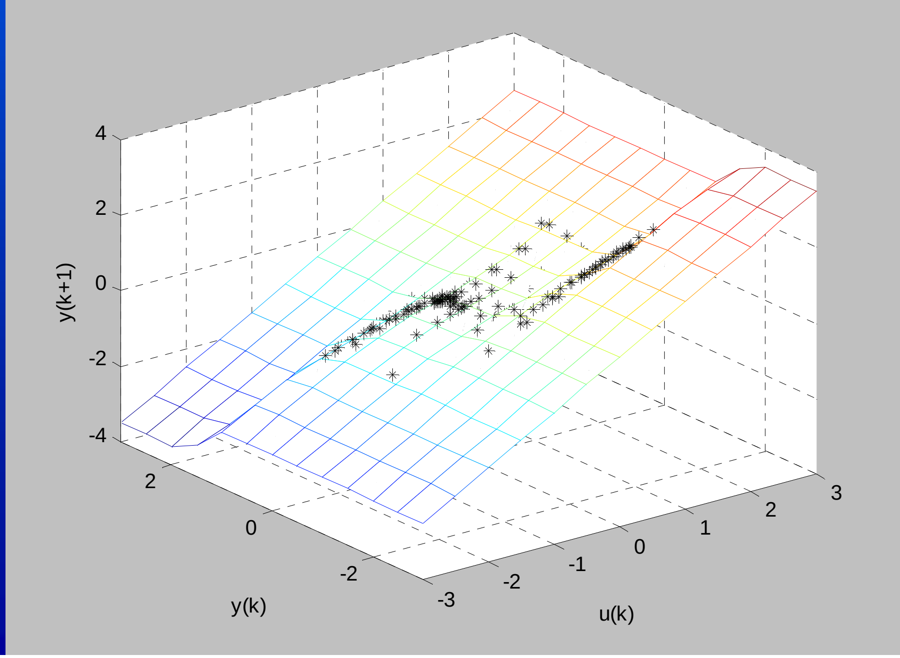




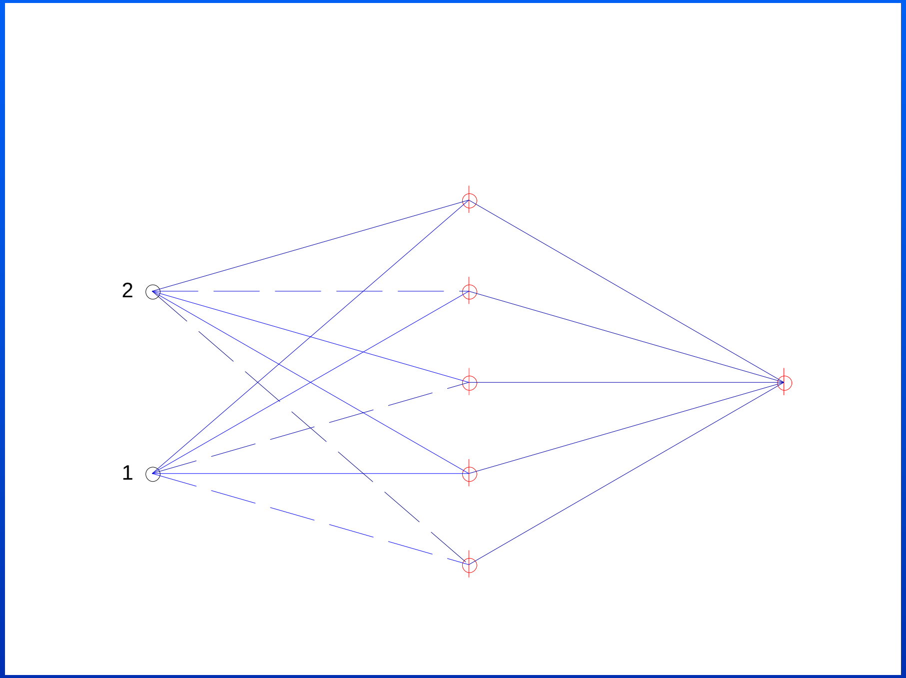
Histograms of magnitudes and input/output data







Neural network, regressors, structure and parameters



Used software:
NNSYSID Toolbox for Matlab

Regressors:
y(k-1), u(k-1)

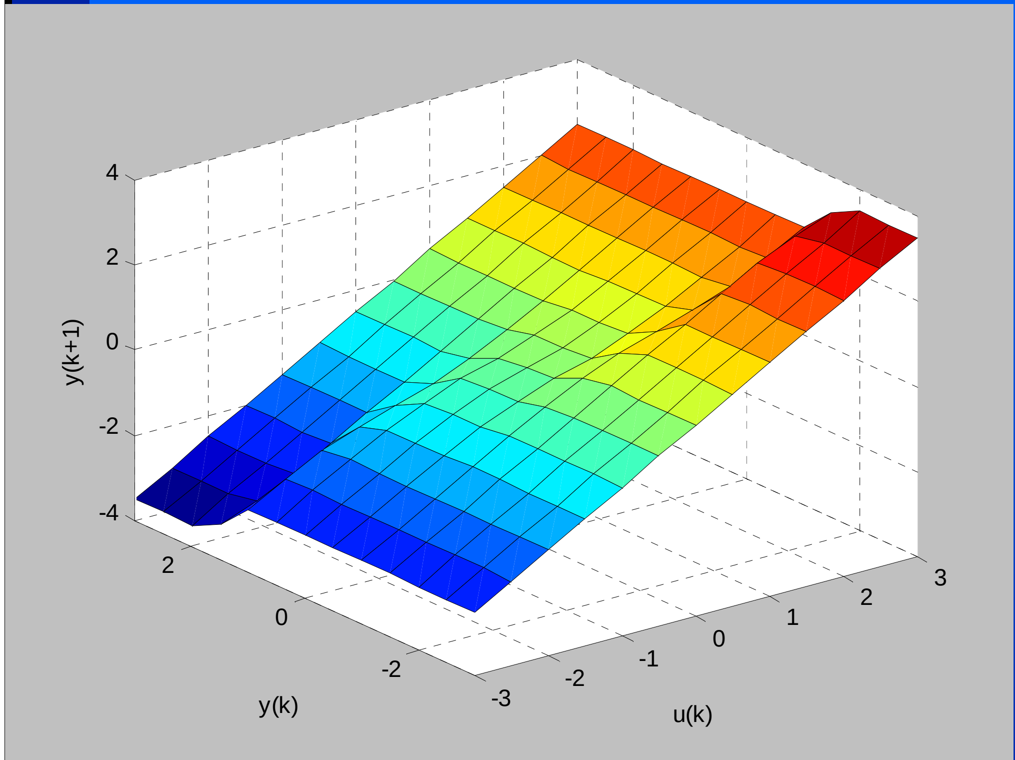
Structure:
ARX (model error method)

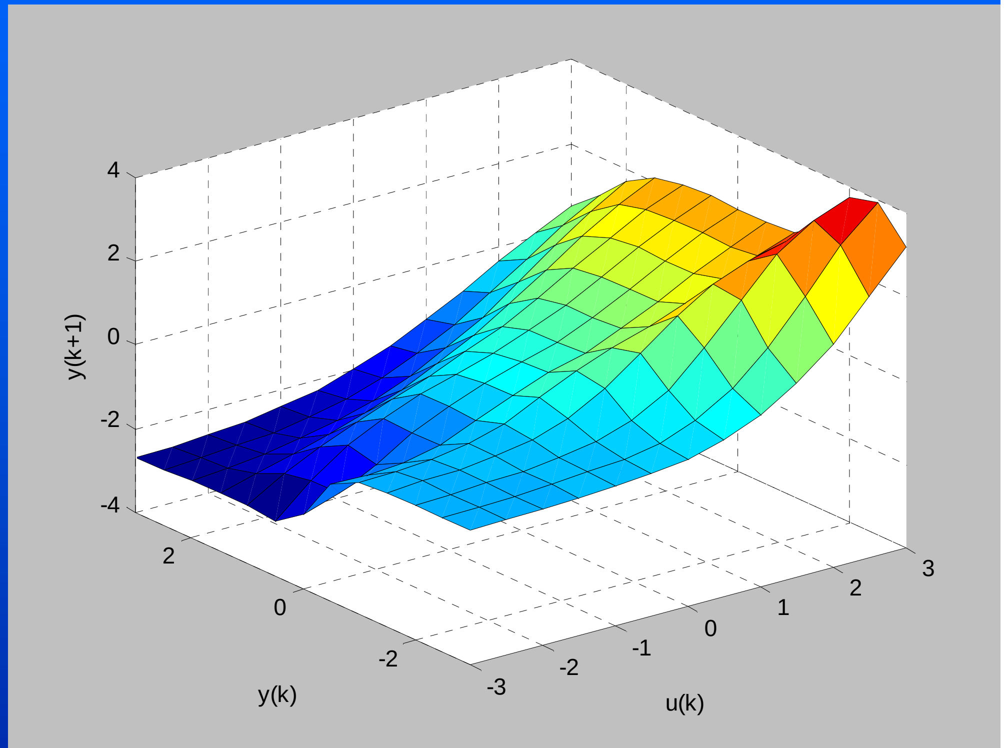
Optimisation method:
Levenberg-Marquardt

$$\mathbf{W}_1 = \begin{bmatrix} -0.5588 & -2.0621 & -1.9530 \\ 0.5155 & 0.0499 & -0.8670 \\ -1.5149 & 0.3190 & 0.4768 \\ 0.3366 & -1.2029 & 1.8379 \\ 0.8411 & 1.3841 & 1.7123 \end{bmatrix}$$

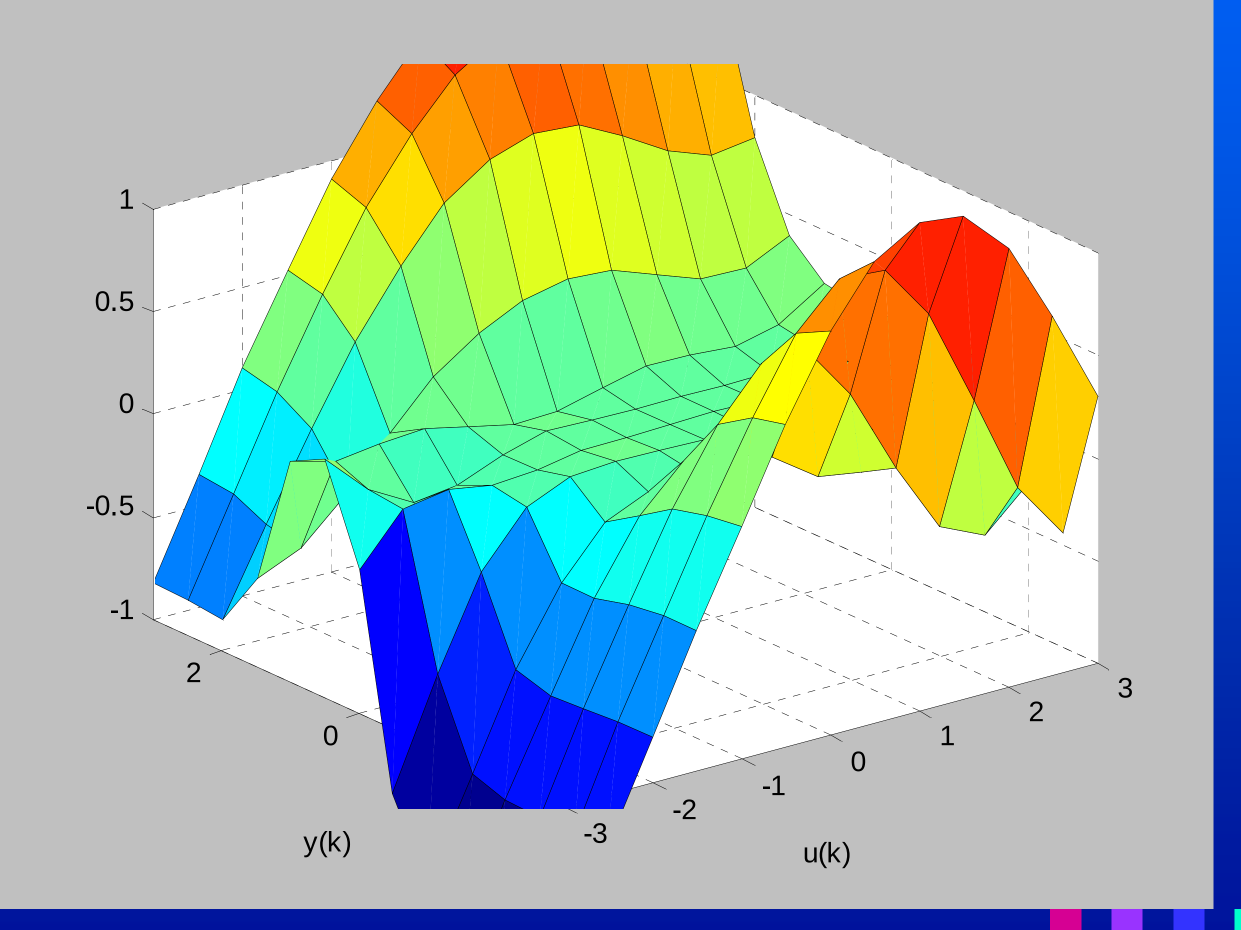
$$\mathbf{W}_2 = \begin{bmatrix} 1.2054 & 1.7784 & 0.0810 & 1.1704 & 1.4048 & -0.0580 \end{bmatrix}$$

Model validation (one-step-ahead prediction)

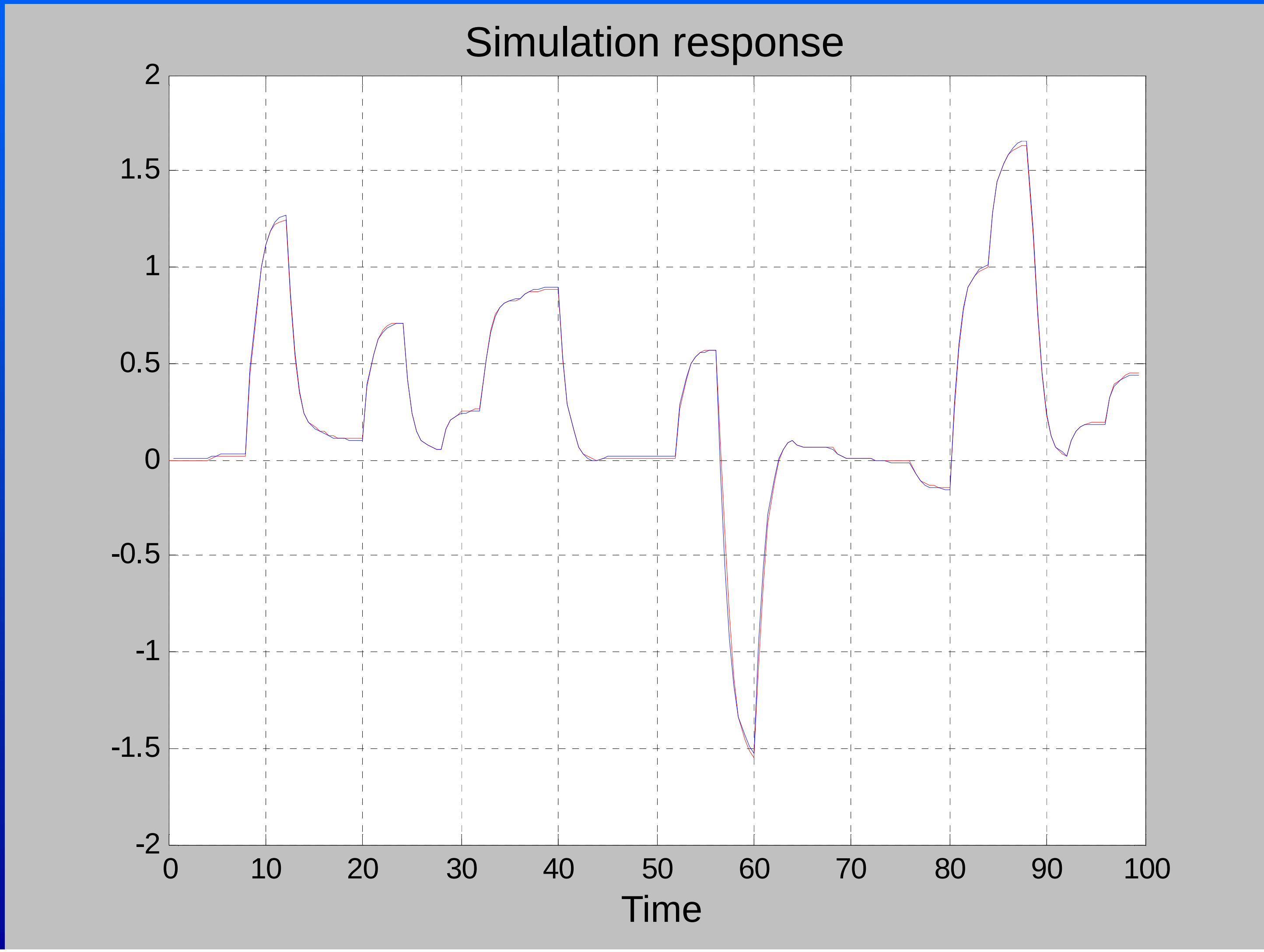




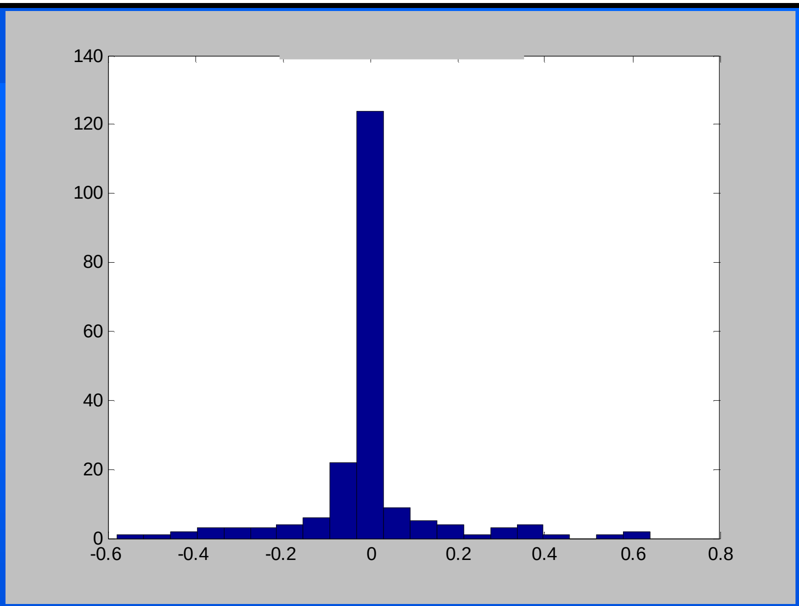
Validation of residuals (one-step-ahead prediction)

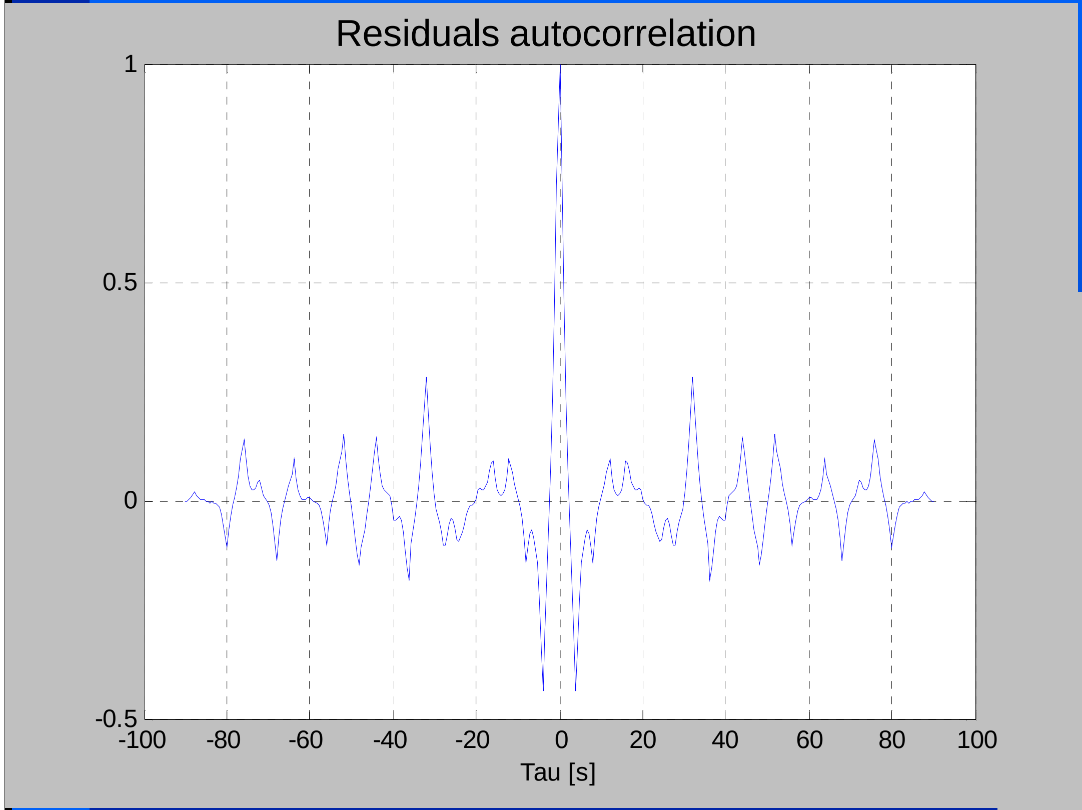


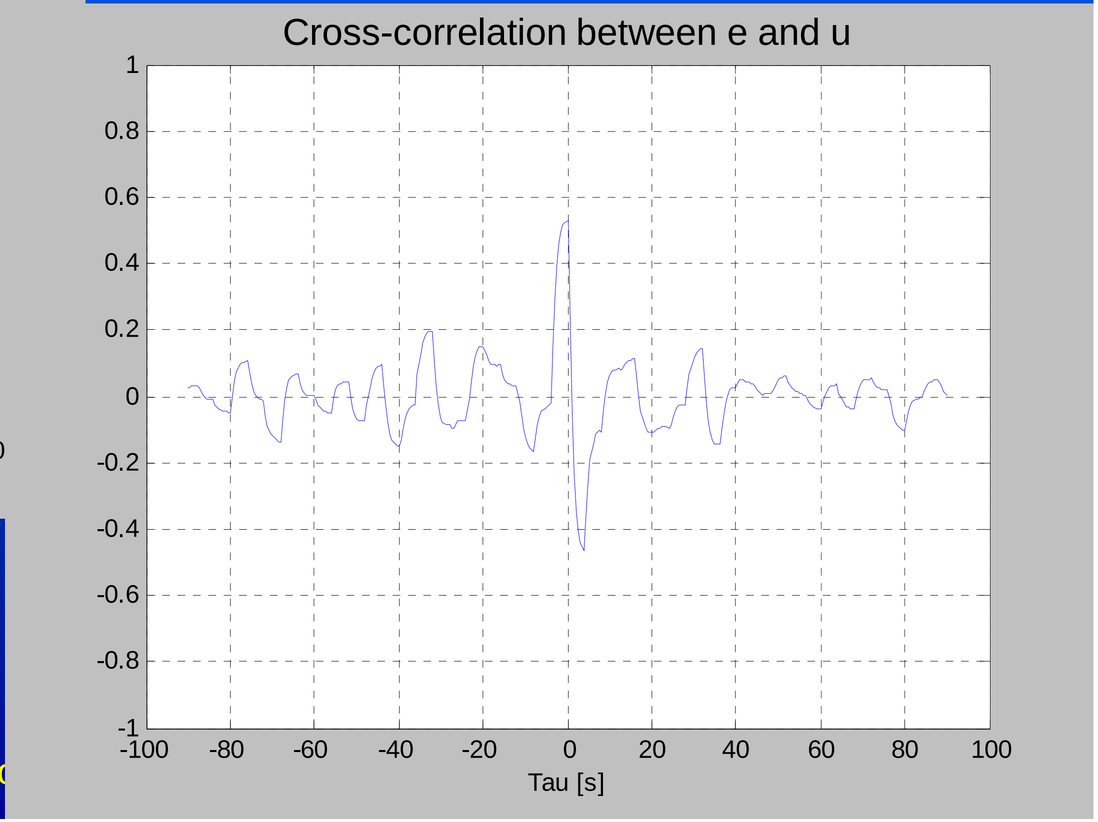
Model response and original system's response on validation data (simulation, not one-step-ahead prediction)



Residuals of simulated valdiation data







Examination assignment

Select a discrete nonlinear system of the 1st order, identify it and make a written report. The report should contain the following items:

- mathematical model of the original system with all parameters;
- Simulation scheme or Matlab code that enables rerun of data acquisition (no masks);
- Show the nonlinearity of the system in 3D plot;
- Estimation signal and response, sampling time;
- Validation signal and response, sampling time;

- Histograms of magnitudes and input/output data pairs;
- Type of neural network and optimisation method;
- Figure of the final network structure;
- Values of parameters (weights);
- Figure of comparison between original system simulation response and model simulation response on validation signal.
- Simulation residuals validation on validation data

(the figure of residuals, residuals histogram, ϕee, ϕue).

Eksperiments should be repeatable based on your report only. Reports should not be longer than 6 pages.