SEVERAL MILESTONES IN THE HISTORY OF GAME THEORY

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Introduction

Although the history of a scientific discipline should be studied regardless of which year and anniversary it is, with an attention to the idea development, the interdependence of various topics and problems etc., a jubilee can represent a good opportunity to remind some important events that influenced the forthcoming development, or to find some new and unnoticed relations. Coincidentally, in the domain of game theory, the year 2004 is exceptionally rich in anniversaries; without an exaggeration, most key events in the history of game theory have some jubilee this year.

Before we turn to some of the important historical events, let us remind the definition and properties of one of the fundamental models of a medium generality used for the investigation of real decision situations: a *normal form game.*

Definition 1. *n-player normal form game* is defined as the $(2n+1)$-tuple

$$(Q;\ S_1, S_2, \dots, S_n;\ u_1(s_1, s_2, \dots, s_n), u_2(s_1, s_2, \dots, s_n), \dots, u_n(s_1, s_2, \dots, s_n))\,, \tag{1}$$

where $n \geq 2$ is a natural number; $Q = \{1, 2, \dots, n\}$ is a given finite set whose elements are called *players*; for every $i \in Q$, S_i is an arbitrary set, so-called *set of strategies of player i*, and $u_i : S_1 \times S_2 \times \cdots \times S_n \to \mathbb{R}$ is a real function called *payoff function of player i*. If all sets $S_1, S_2, \dots, S_n$ are finite, the game (1) is called *finite.*

Definition 2. An n-tuple of strategies $\boldsymbol{s}^* = (s_1^*, s_2^*, \dots, s_n^*)$ is called *equilibrium point* of the game (1), iff for every $i \in Q$ end every $s_i \in S_i$ the following condition holds:

$$u_i(s_1^*, \dots, s_{i-1}^*, s_i, s_{i+1}^*, \dots, s_n^*) \leq u_i(s_1^*, \dots, s_{i-1}^*, s_i^*, s_{i+1}^*, \dots, s_n^*). \tag{2}$$

The strategy s_i^* is called *equilibrium strategy of player i.*

Definition 3. Consider a finite n-player normal form game (1). For each $i \in Q$ denote by m_i the number of elements of the strategy set S_i of player i. A *mixed strategy* of player i is defined as the vector of probabilities $\boldsymbol{p}^i = (p_1^i, p_2^i, \dots, p_{m_i}^i)$, where $p_1^i + p_2^i + \cdots + p_{m_i}^i = 1$, $p_j^i \geq 0$ for all $1 \leq j \leq m_i$.

Thus a mixed strategy is a vector whose j-th element represents the probability with which the player chooses the j-th strategy from his strategy set. For distinction, elements of the strategy set S_i are also called *pure strategies.*

Theorem 1 (J. Nash). *Every finite normal form game has at least one equilibrium point in mixed strategies.*

350 ♣ The Beginning of Probability Calculus

As it is clear from the introduction, one of the pillars of game theory is the concept of *mixed strategy* based on the concept of *probability* which is therefore indispensable to reach some interesting results. The beginning of probability calculus is associated with the correspondence of Pierre de Fermat (1607 – 1665) and Blaise Pascal (1623 – 1662) dated in 1654. Hence the true prehistory can start only after this event that happened just 350 years ago and represents the first key milestone in the history of game theory.

320 ♢ James Waldegrave and Mixed Strategies

James Waldegrave (1684 – 1741), born 320 years ago, provided the first known mixed strategy solution of a matrix game. It was related to the game *le Her* and described in Waldegrave's letter to Pierre Rémond de Montmort (1678 – 1719) dated on November 13, 1713. Let us briefly mention that the game *le Her* is played by two people, usually named Peter and Paul. It begins when Peter deals Paul a single card at random from an ordinary deck of cards (ace, two, . . . , king), then deals a single card to himself; neither player sees the card dealt to the other one. The object of the game is to hold the card of the higher value than the opponent. If Paul is not satisfied with the card dealt to him, he may force Peter to exchange with him, with the exception that Peter has a king. If Peter is not satisfied with the card that he holds afterwards, he is permitted to exchange it for a card dealt from the deck at random; if the new card is a king, he must retain his original card. Then the two players compare cards and the one with the higher card wins. If both players hold cards of the same value, then Peter wins.

The game *le Her* was already investigated by de Montmort and Nicholas Bernoulli (1687 – 1759) in their mutual correspondence in 1713. They came to the conclusion that Paul should change every card less than 7 and hold all higher than 7, Peter should change every card less than 8 and hold all higher than 8. In disputable cases, Bernoulli thought both should change, de Monmort believed that no precept could be given.

James Waldegrave was looking for a strategy that maximizes the probability of player's win, whichever strategy was chosen by the opponent, that is, exactly in the sense of today *minimax principle*. He came to the following mixed strategy solution formulated in terms of black and white chips: Paul should choose the strategy "change 7 and lower" with the probability 5/8 and the strategy "hold 7 and higher" with the probability 3/8; Peter should choose the strategy "change 8 and lower" with the probability 3/8 and the strategy "hold 8 and higher" with the probability 5/8.

Although de Montmort published the correspondence related to *le Her*, including Waldegrave's solution, in an appendix to the second edition of the book [21], Waldegrave's solution remained almost unnoticed for a very long time.[1]

80 ♡ Mathematization of the Game Concept

In the period 1921–1928 whose mean value has the eightieth anniversary, after more than two hundred years of a prehistory consisting of various isolated examples, the mathematization of the game concept was accomplished.

In 1921–1927 Émile Borel (1871 – 1956) published a series of notes [4]–[6] on symmetric two-player zero-sum games with a finite number n of pure strategies of each player.[2] Borel was the first who attempted to mathematize the game of strategy (in the mentioned special case); he introduced the concept of *method of play* in the sense of today *pure strategy* and he was looking for a solution in *mixed strategies* in the sense of today *minimax solution*. In his 1921 paper Borel proved the existence of such a solution for $n = 3$. Later, while searching a counterexample, he proved the same for $n = 5$, what he described in [5]; still he believed it can not be done in general. In the paper [6] he formulated the problem positively, nevertheless, he gave no proof.

[1] For more details on the game *le Her* and the relating history see e.g. [10] and [15].

[2] In Definition 1 it is $S_1 = S_2 = S = \{s_1, \dots, s_n\}$; $u_1(s_i, s_j) = -u_2(s_i, s_j) = u_2(s_j, s_i)$ for all $s_i, s_j \in S$. The game can be described by the matrix (a_{ij}) where $a_{ij} = u_1(s_i, s_j)$, $s_i, s_j \in S$.

In 1928 the existence of the solution of any finite two-player zero-sum game in mixed strategies was proved by John von Neumann (1903 – 1957), whose treatise [29] represents a true milestone in the history of game theory: von Neumann provided a mathematization of a general finite n-player zero-sum game in the sense of Definition 1 and for the special case of $n = 2$ he proved the existence of a minimax solution which is the fundamental result of the theory of matrix games.[3] In short:

Theorem (von Neumann). Consider a finite two-person zero-sum game. Denote with $h(\boldsymbol{p}, \boldsymbol{q})$ the expected payoff to player 1 provided the players choose mixed strategies $\boldsymbol{p}$, $\boldsymbol{q}$. Then there always exist mixed strategies $(\boldsymbol{p}^*, \boldsymbol{q}^*)$ for which

$$h(\boldsymbol{p}^*, \boldsymbol{q}^*) = \max_p \min_q h(\boldsymbol{p}, \boldsymbol{q}) = \min_q \max_p h(\boldsymbol{p}, \boldsymbol{q}).$$

In 1953 M. Fréchet published in *Econometrica* Savage's English translations of Borel's notes [4]–[6] and designated Borel the *initiator* and von Neumann the *founder* of game theory. This arouse a dispute over a priority which is not easy to judge. Borel did not come to the fundamental result on solvability and his works had almost none influence – the label *initiator* is therefore questionable. Von Neumann's paper was published few years later but it contained a comprehensive and exact exposition of general concepts of game theory including the proof of minimax theorem and it had a substantial influence on further development – these attributes support his *founder* label.

60 ♠ Game Theory as the Mathematical Discipline

The next important milestone is represented by the publication of the extended monograph *Theory of Games and Economic Behavior* [30] in 1944, which was the result of a fruitful collaboration of von Neumann and the economist Oskar Morgenstern (1902 – 1976). This event is usually considered as the beginning of the existence of game theory as a "fully-fledged" mathematical discipline. Von Neumann and Morgenstern started with a detailed formulation of economical problem and showed the exceptionally broad application possibilities of game theory in economy; then they settled the foundations of an axiomatic utility theory. From all other topics, let us mention the general formal description of a game of strategy, the comprehensive theory of finite two-player zero-sum games and n-player zero-sum cooperative games with transferable payoffs.

The monograph stimulated a massive development of game theory and its applications into various domains. Still, there remained many open problems. Antagonistic games form only a small part of all decision situations; players may have no chance to cooperate or redistribute payoffs. For this reason, special interest shall be devoted to J. F. Nash.

55 ♣ Nash Equilibrium

In 1949 John Forbes Nash wrote his Ph.D. thesis named *Non-Cooperative Games*, where the concept of *equilibrium point* (today also *Nash equilibrium*) in the sense of Definition 2 was introduced and its existence was proven. The most important results of the thesis were published in a short note [24] and in a more detailed paper [26] in 1950 and 1951.

[3] Shortly before the paper [29] was published, a brief report [28] containing an outline of the main ideas appeared in Comptes Rendus. According to his own remarks, von Neumann developed his theory independently of Borel's works.

The second concept associated with the name of John F. Nash is *Nash bargaining solution* concerning two-player cooperative games without transferable payoffs. Nash suggested a system of axioms that a solution should satisfy and proved the existence of unique solution with these properties.

50 ♢ Entrance of Game Theory into Political Sciences

In 1954 Lloyd Shapley and Martin Shubik published the paper [33] which represents one of the earliest explicit applications of game theory to political sciences. The authors used Shapley value, one of the solution concepts for cooperative games introduced by Shapley one year earlier, to determine the power of the members of the United Nations Security Council.

Many other works then appeared; let us mention at least the paper [18] by R. D. Luce and A. A. Rogow from 1956, and the paper [32] by W. H. Riker from 1959. Soon the theory of games found a crucial place in political sciences. Nowadays it is used for modeling various situations related to elections, legislature, politics of interest groups, lobbies, bargaining, etc. Not only facilitates it to solve particular problems, it provides the exact terminology and methods – what is an underbelly of all social sciences. Many of modern monographs on political sciences regard game theory as an inseparable part of this discipline; the presentation of political sciences based on game theory is given for example in publications of J. D. Morrow [22] or P. Ordershook [31].

Although game theory is not a "cure-all" and it can't offer an optimal solution to all problems, it is a strong tool for analysis of a given situation and it induces the decision-maker to think rationally and without emotions; this, in itself, often yields a general acceptable solution.

30 ♡ Entrance of Game Theory into Evolutionary Biology

Nowadays game theory is the main tool for the investigation of conflict and cooperation of animals and plants. As for zoological applications, the theory of games is used for the analysis, modeling and understanding the fight, cooperation and communication of animals, coexistence of alternative traits, mating systems, conflict between the sexes, offspring sex ratio, distribution of individuals in their habitats, etc. Among botanical applications we can find the questions of seed dispersal, seed germination, root competition, nectar production, flower size, sex allocation, etc.

In 1960's several isolated works using a game-theoretical approach in biology appeared.[4] A historical milestone is represented by the short but "epoch-making" paper *The Logic of Animal Conflict* [19] by J. Maynard Smith and G. R. Price. This treatise, published in 1973, stimulated a great deal of successful works and applications of game theory in evolutionary biology; the development of the following decade was summarized in Maynard Smith's book *Evolution and the Theory of Games* [20]. Not only proved game theory to provide the most satisfying explanation of the theory of evolution and the principles of behavior of animals and plants in mutual interactions, it was just *this field* which turned out to provide the most promising applications of the theory of games at all. Is this a paradox? How is it possible that the behavior of animals or plants prescribed on the base of game-theoretical models agree with the action observed in the

[4] Let us mention the works of R. C. Lewontin [17], W. D. Hamilton ([12], [13]) and R. L. Trivers [34]. Some ideas were foreshedowed by R. A. Fisher in 1930 [11], without the terminology of game theory.

nature? Can a fly or a fig tree, for example, be a rational decision-maker who evaluates all possible outcomes and by the tools of game theory selects his optimal strategy? How is it possible that even the less developed the thinking ability of an organism is, the better game theory tends to work?

The explanation is simple: the *players* of the game are not taken to be the organisms under study, but the *genes* in which the instinctive behavior of these organisms is coded. The *strategy* is then the *behavioral phenotype*, i.e. the behavior preprogrammed by the genes – the specification of what an individual will do in any situation in which it may find itself; the *payoff function* is a *reproductive fitness*, i.e. the measure of the ability of a gene to survive and spread in the genotype of the population in question. The main *solution concept* of this model is the *evolutionary stable strategy* which is defined as *a strategy such that, if all the members of a population adopt it, no mutant strategy can invade.*[5] In certain specific situations, this somewhat vague concept is expressed more precisely.[6]

In short, to understand the basic principles of so-called "genocentric" conception of the evolution, it suffices to imagine that about four thousand million years ago a remarkable molecule, so-called *replicator*, was formed by accident, that was able to create copies of itself. The copies started to spread in the environment; During the replication sometimes a mistake or *mutation* occured, some of these mutations led to replicators that were more successful in mutual contests and in reproduction. Some of them could discover how to break up molecules of rival varieties chemically, others started to construct for themselves containers, vehicles for their continued existence. Such replicators survived, that had better and more effective *"survival machines"*. Generation after generation, the *survival machines* of genes, i.e. the organisms controlled by the genes, compete in mutual contests; the genes that choose the best strategy for their *machine* spread themselves and step by step their *learning* proceeds. The result is that these *machines* act in the same way as game theory would calculate – instead of the calculation they have come to the equilibrium strategies by gradual adaptation and natural selection.[7]

An illustrative analogy of the slow evolution process is learning of an individual who finds himself repeatedly in the same conflict situation during his relatively short lifetime. Apparently the most interesting experiment was made in 1979 by B. A. Baldwin and G. B. Meese with the Skinner sty: there is the snout-lever at one end of the sty, the food dispenser at the other. Pressing the lever causes puring the food down a chute. Baldwin and Meese placed two domestic pigs into this sty. Such couple always settles down into a stable *dominant/subordinate* hierarchy. Which pig will press the lever and run across the sty and which will be sitting by the food trough? The situation is schematically illustrated in Figure 1.

The strategy *"If dominant, sit by the food trough; if subordinate, work the lever"* sounds sensible, but would not be stable. The subordinate pig would soon give up pressing the lever, for the habit would never be rewarded. The reverse strategy *"If*

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[5] [20], p. 204.

[6] For example, the pairwise contests in an infinite asexual population can be modelled by a two-player normal form game and the evolutionary stable strategy I is a strategy such that, for all $J \neq I$ it is either $u_1(I,I) > u_1(J,I)$, or $u_1(I,I) = u_1(J,I)$ and $u_1(I,J) > u_1(J,J)$. Due to the symmetry, the pair of strategies (I,I) forms an equilibrium point.

It shall be noted that there exist good field measurements of costs and benefits in nature, which have been plugged into particular models; see e.g. [7]. Hence the payoff can really be measured quantitatively.

[7] The reader is invitated to read the books [8] and [9] by R. Dawkins and [20] by Maynard Smith.

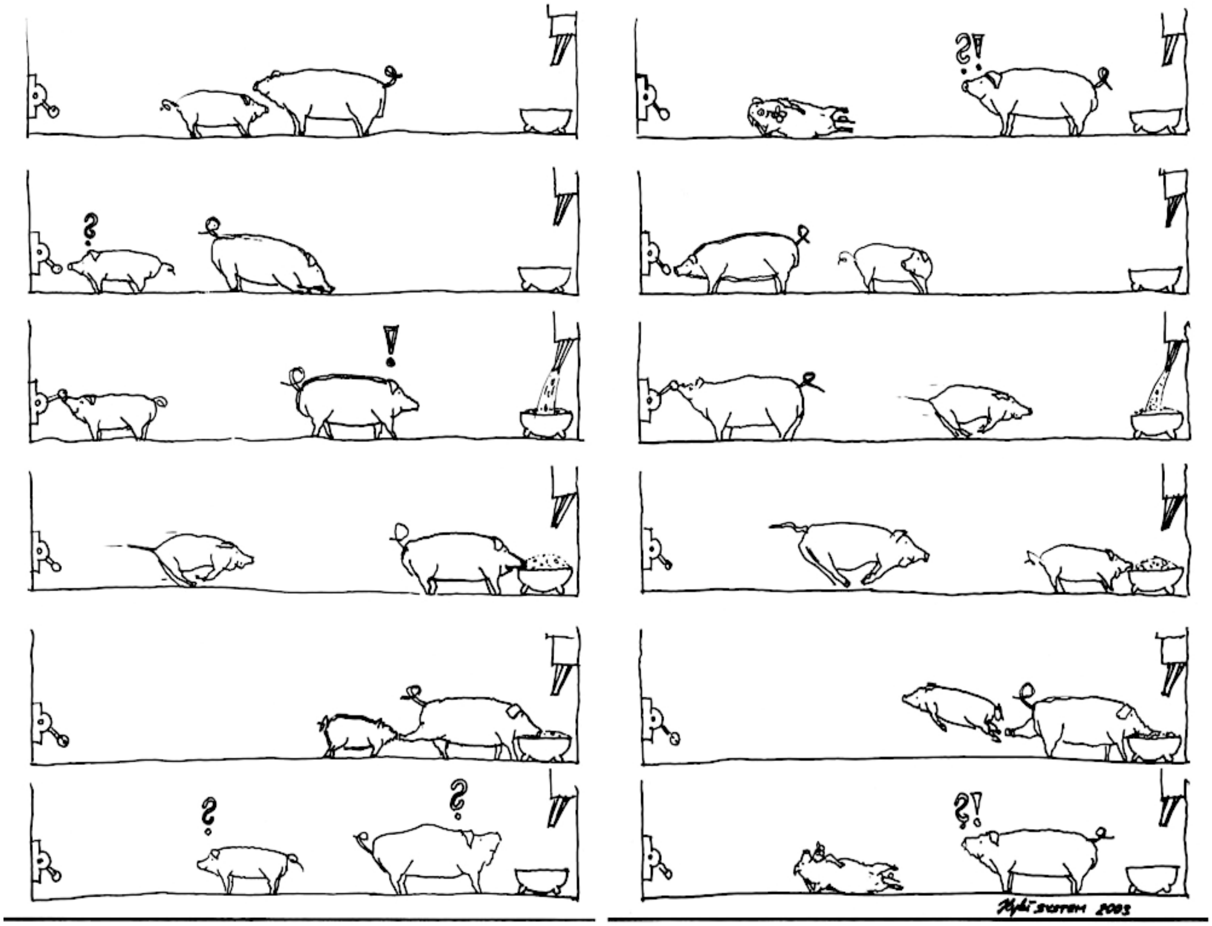
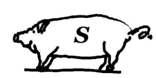
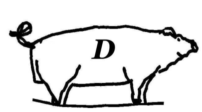


Figure 1 Skinner box

dominant, work the lever; if subordinate, sit by the food trough" would be stable – even though it it has the paradoxical result that the subordinate pig gets most of the dominant pig when he charges up from the other end of the sty. As soon as he arrives, he has no difficulty in tossing the subordinate pig out of the trough. As long as there is a crumb left to reward him, his habit of working the lever will persist.[8]

Using the theory of games, we would model the situation by a *bimatrix game*:[9]





$$
\begin{array}{r|ccc|}
 & \textit{Press the lever} & & \textit{Sit by the trough} \\
\hline
\textit{Press the lever} & (8,-2) & \rightarrow & \boxed{(\mathbf{3},\mathbf{5})} \\
 & & & \uparrow \\
\textit{Sit by the trough} & (10,-2) & \rightarrow & (0,0) \\
\hline
\end{array}
$$

[8]For more details see [8], pp. 286–287, and the original paper [2] by Baldwin and Meese.

[9]We consider the profit from the whole ration to the extent of 10 utility units, the loss caused by the labor connected with pressing the lever and running −2 units and the amount of food which the subordinate pig manages to eat before he is tossed out by the dominant one, 5 units (these units were chosen at random – from the strategic point of view nothings changes when the labor is evaluated with an arbitrary negative number, the waiting subordinate pig receives nonnegative number of units and nonnegative number of units remains for the dominant one.

Rational players would come to the equilibrium strategies in the following way. For the second player – the subordinate pig – the first strategy is dominated by the second one and can be therefore eliminated. The first player – the dominant pig – presumes the rationality of his opponent and hence decides between the profit of 0 or 3 units in the second column, which leads him to the choice of the first strategy. Indeed, the couple of strategies (*press the lever*, *sit by the trough*) is an equilibrium point.

20 ♠ The Evolution of Cooperation

In 1984 Robert Axelrod published the book *The Evolution of Cooperation* [1] where he showed that cooperation based upon reciprocity can evolve and sustain itself even among egoists provided there is sufficient prospect of a long-term interaction. His exposition was based on computer tournaments of various strategies in a repeated game that he organized in 1981, historical cases and mathematical theorems.

Axelrod stimulated a deep interest in repeated games. Among other disciplines, in biology repeated games provide a key tool for the explanation of the existence of an altruism among non-relative individuals who interact in a long run – many instinctively acting species shall be taken as shining examples by humans, often governed by negative emotions. For example, a functioning reciprocal altruism underlies the regular alternation of the sex roles in the hermafrodite sea bass, the reciprocal help between mails of pavian anubi to fight off an attacker during the time one of them is mating, or the blood-sharing by the great mythmakers vampires (the bats eating the cattle blood): the individuals that have returned from an unsuccessful hunt are feeded by successful ones, even non-relative; they recognize each other and preferentially feed those they know.

10 ♣ Nobel Prize for Mathematicians

In 1994 John Forbes Nash, John Harsanyi and Reinhard Selten were awarded Nobel Prize for economics for their contributions to non-cooperative game theory – that is, in fact for mathematics. John F. Nash was appreciated for his equilibrium concept and the foundations for the analysis of non-cooperative games, the other two scientists for extensions of the ideas of Nash. John Harsanyi showed how Nash's concept could be applied in situations with incomplete information, Reinhard Selten refined Nash equilibrium concept for analyzing dynamic strategic interactions.

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