# EXERCISES 9 – Partial derivatives

1. Find partial derivatives $f'_x = \dfrac{\partial f}{\partial x}$ and $f'_y = \dfrac{\partial f}{\partial y}$ of the given function $f(x, y)$.

   a) $f(x, y) = (5x - y)^4$. $\quad [f'_x = 20(5x - y)^3\,, \quad f'_y = -4(5x - y)^3.]$

   b) $f(x, y) = xy^2 - \dfrac{y}{x} + 2\sqrt{x}$. $\quad \left[f'_x = y^2 + \dfrac{y}{x^2} + \dfrac{1}{\sqrt{x}}\,, \quad f'_y = 2xy - \dfrac{1}{x}\,, \quad x > 0.\right]$

   c) $f(x, y) = \dfrac{xy}{y - x}$. $\quad \left[f'_x = \dfrac{y^2}{(y - x)^2}\,, \quad f'_y = \dfrac{-x^2}{(y - x)^2}\,, \quad x \neq y.\right]$

   d) $f(x, y) = \ln(y + x^2)$. $\quad \left[f'_x = \dfrac{2x}{y + x^2}\,, \quad f'_y = \dfrac{1}{y + x^2}\,, \quad y + x^2 > 0.\right]$

   e) $f(x, y) = e^{-xy}$. $\quad [f'_x = -ye^{-xy}\,, \quad f'_y = -xe^{-xy}.]$

   f) $f(x, y) = x\sqrt{x^2 + y^2}$. $\quad \left[f'_x = \dfrac{2x^2 + y^2}{\sqrt{x^2 + y^2}}\,, \quad f'_y = \dfrac{xy}{\sqrt{x^2 + y^2}}\,, \quad y^2 + x^2 \neq 0.\right]$

   g) $f(x, y) = \ln(x - ye^x)$. $\quad \left[f'_x = \dfrac{1 - ye^x}{x - ye^x}, \quad f'_y = \dfrac{-e^x}{x - ye^x}\,, \quad x - ye^x > 0.\right]$

   h) $f(x, y) = (x - y)\ln(2x + y)$.
   $$\left[f'_x = \ln(2x + y) + \frac{2(x - y)}{2x + y}, \quad f'_y = -\ln(2x + y) + \frac{x - y}{2x + y}\,, \quad 2x + y > 0.\right]$$

   i) $f(x, y) = \dfrac{3x + y}{1 - xy}$. $\quad \left[f'_x = \dfrac{3 + y^2}{(1 - xy)^2}\,, \quad f'_y = \dfrac{1 + 3x^2}{(1 - xy)^2}\,, \quad 1 - xy \neq 0.\right]$

2. Find partial derivatives $f'_x = \dfrac{\partial f}{\partial x}$, $f'_y = \dfrac{\partial f}{\partial y}$ and $f'_z = \dfrac{\partial f}{\partial z}$ of the given function $f(x, y, z)$.

   a) $f(x, y, z) = x^2 e^y \sin z$.
   $$[f'_x = 2xe^y \sin z\,, \quad f'_y = x^2 e^y \sin z\,, \quad f'_z = x^2 e^y \cos z\,, \quad (x, y, z) \in \mathbb{R}^3.]$$

   b) $f(x, y, z) = \ln(x^2 - y + 3z)$.
   $$\left[f'_x = \frac{2x}{x^2 - y + 3z}\,,\ f'_y = \frac{-1}{x^2 - y + 3z}\,,\ f'_z = \frac{3}{x^2 - y + 3z}\,,\ x^2 - y + 3z > 0.\right]$$

   c) $f(x, y, z) = (\cos xy)^{xyz}$.
   $$\left[\begin{array}{lcl} f'_x & = & (\cos(xy))^{xyz} yz(\ln\cos(xy) - xy\tan(xy))\,, \\ f'_y & = & (\cos(xy))^{xyz} xz(\ln\cos(xy) - xy\tan(xy))\,, \\ f'_z & = & (\cos(xy))^{xyz} xy\ln\cos(xy)\,, \qquad \cos(xy) > 0. \end{array}\right]$$

   d) $f(x, y, z) = z\sin(xyz) - y\cos(yz) + \sin(xz)$.
   $$\left[\begin{array}{lcl} f'_x & = & yz^2\cos(xyz) + z\cos(xz)\,, \\ f'_y & = & xz^2\cos(xyz) - \cos(yz) + yz\sin(yz)\,, \\ f'_z & = & \sin(xyz) + xyz\cos(xyz) + y^2\sin(yz) + x\cos(xz)\,, \quad (x, y, x) \in \mathbb{R}^3. \end{array}\right]$$