Stationary and non-stationary signals

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Linear state variable system $\dot{\mathbf{x}}(t) = \mathbf{A}(t)\,\mathbf{x}(t) + \mathbf{B}(t)\,\mathbf{u}(t)$ $\mathbf{y}(t) = \mathbf{C}(t)\,\mathbf{x}(t) + \mathbf{D}(t)\mathbf{u}(t)$ $\mathbf{A}(t)$ system matrix $(n \times n)$ $\mathbf{B}(t)$ matrix of inputs $(n \times r)$ $\mathbf{C}(t)$ matrix of outputs $(m \times n)$ $\mathbf{D}(t)$ matrix of outputs $(m \times r)$	Linear state variable system $\mathbf{x}(n+1) = \mathbf{M}(n)\,\mathbf{x}(n) + \mathbf{N}(n)\,\mathbf{u}(n)$ $\mathbf{y}(n) = \mathbf{C}(n)\,\mathbf{x}(n) + \mathbf{D}(n)\mathbf{u}(n)$ $\mathbf{M}(n)$ system matrix $\mathbf{N}(n)$ matrix of inputs $\mathbf{C}(n)$ matrix of outputs $\mathbf{D}(n)$ matrix of outputs

Stationary and non-stationary

Continuous system	**Discrete system**
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Stationary and non-stationary

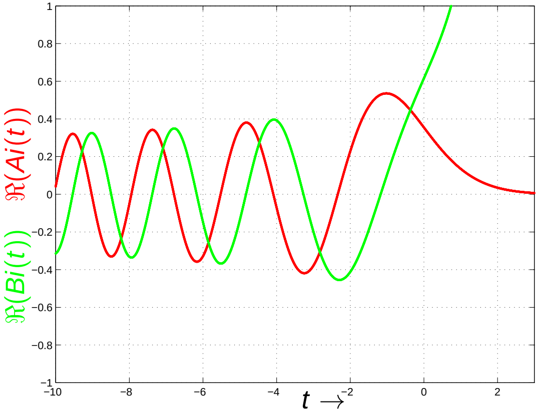
- Non-stationary signals $\Leftrightarrow$ differential/difference equations with time-varying coefficients

$$\ddot{y}(t) - t\,y(t) = 0$$

- Airy's functions

$$\begin{aligned} Ai(t) &= \frac{1}{3}\sqrt{t}\,\left[I_{-1/3}\left(\frac{2}{3}\,t^{3/2}\right) - I_{1/3}\left(\frac{2}{3}\,t^{3/2}\right)\right] \\ Bi(t) &= \frac{1}{3}\sqrt{t}\,\left[I_{-1/3}\left(\frac{2}{3}\,t^{3/2}\right) + I_{1/3}\left(\frac{2}{3}\,t^{3/2}\right)\right] \end{aligned}$$

Stationary and non-stationary



Stationary and non-stationary

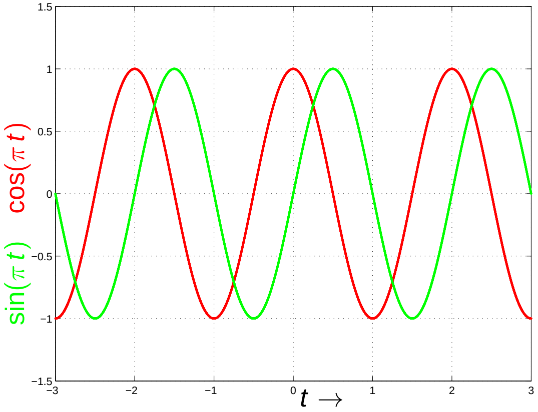
- Stationary signals $\Leftrightarrow$ differential/difference equations with constant coefficients

$$\ddot{y}(t) + \omega_0^2 y(t) = 0$$

- Harmonic wave (periodic functions)

$$\cos(\omega_0 t) \quad \sin(\omega_0 t)$$

Stationary and non-stationary



About stationarity

A deterministic signal is said to be stationary if it can be written as a discrete sum of cosine waves or exponentials :

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi k f_0 t + \Phi_k) \quad (1)$$

$$x(t) = \sum_{k=-N}^{N} C_k \exp(2\pi \jmath k f_0 t + \Phi_k) \quad (2)$$

i.e. as a sum of elements which have constant instantaneous amplitude and instantaneous frequency.

About stationarity

In the random case, a signal $\{x(n)\}$ is said to be wide-sense stationary (or stationary up to the second order) if its variance is independent of time

$$\sigma^2 = E[(x-\mu)^2] = \frac{1}{N}\sum_{n=0}^{N-1}(x-\mu)^2(n)$$

About stationarity

The autocorrelation function for a discrete process of length N $\{x(n)\}$ with known mean μ and variance σ,

$$\varrho_{xx}(n, n+m) = \frac{1}{N\sigma^2} \sum_{n=1}^{N} (x(n) - \mu)(x(n+m) - \mu)$$

depends only on the time difference m.

...and non-stationarity

A signal is said to be non-stationary if one of these fundamental assumptions is no longer valid. For example, a finite duration signal, and in particular a transient signal (for which the length is short compared to the observation duration), is non-stationary.

Shor time Fourier transform of a non-stationarity signal

1. removing the mean of a signal
2. moving average filtering
3. segmentation of a signal using window functions
4. Fourier transform

Moving average filtering

- Assume we have a EEG signal corrupted with noise
- Set the mean to zero
- Apply moving average filter to the noisy signal (use filter order=3 and 5)
- The higher filter order will remove more noise, but it will also distort the signal more (i.e. remove the signal parts also)
- So, a compromise has to be found (normally by trial and error)

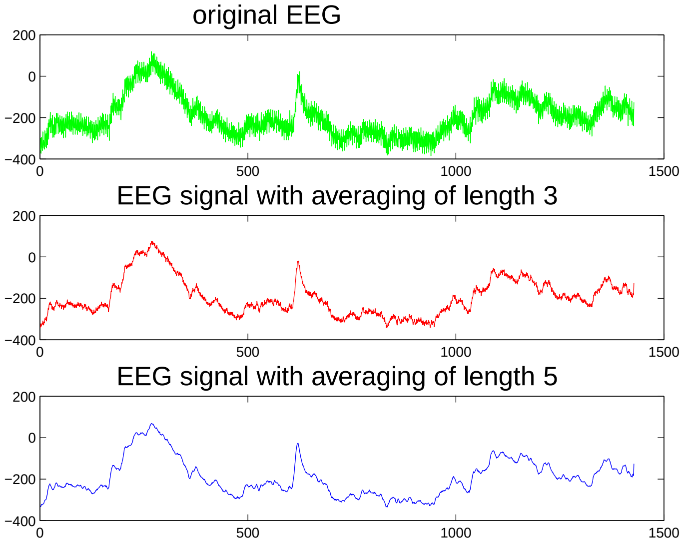
Moving average filtering - MATLAB file

```
% moving average filtering
% December 3, 2009
load('zdroj.mat');
y=EEG(3).Data(:,1);
N=length(y);
% length of average window is 3
for i=1:N-2,
signal3(i)=(y(i)+y(i+1)+y(i+2))/3;
end
signal3(N-1)=(y(N-1)+y(N))/2;
signal3(N)=y(256);
%length of average window is 5
for i=1:N-4,
signal5(i)=(y(i)+y(i+1)+y(i+2)+y(i+3)+y(i+4))/5;
end
```

Moving average filtering - MATLAB file

```
signal5(N-3)= (y(N-3)+y(N-2)+y(N-1)+y(N))/4;
signal5(N-2)=(y(N-2)+y(N-1)+y(N))/3;
signal5(N-1)=(y(N-1)+y(N))/2;
signal5(N)=y(N);
subplot(3,1,1), plot(y, 'g '); title('original
EEG')
subplot(3,1,2), plot(signal3,'r'); title('EEG
signal with averaging of length 3')
subplot(3,1,3), plot(signal5,'b'); title('EEG
signal with averaging of length 5')
print -depsc figureEEG
```

Moving average filtering



STFT, Wavelets, Huang Transform

	STFT	Wavelets	Huang
inversion	yes	yes, but...	no inversion
resolution in time	limited	good	good
resolution in frequency	good	bad	floating frequency

Thank you for your attention