Neurčitý integrál

Pomocí tabulkových integrálů určete dané integrály a uveďte interval, ve kterém je uvedené vyjádření platné:

a) $\int (2x-3)\,\mathrm{d}x$ $\left[x^2 - 3x + c\,;\ x \in \mathbb{R}\right]$

b) $\int \left(\sqrt{x} + \dfrac{2}{x}\right)\,\mathrm{d}x$ $\left[\dfrac{2}{3}\,x\sqrt{x} + 2\ln x + c\,;\ x \in (0, +\infty)\right]$

c) $\int \left(\sqrt{x\sqrt{x}} - \dfrac{1}{x^2}\right)\,\mathrm{d}x$ $\left[\dfrac{4}{7}\,x^{7/4} + \dfrac{1}{x} + c\,;\ x \in (0, +\infty)\right]$

d) $\int (3\mathrm{e}^x + 4\sin x - 5\cos x)\,\mathrm{d}x$ $\left[3\mathrm{e}^x - 4\cos x - 5\sin x + c\,;\ x \in \mathbb{R}\right]$

e) $\int \left(2^x + \dfrac{3}{\sqrt{4 - 4x^2}}\right)\,\mathrm{d}x$ $\left[\dfrac{2^x}{\ln 2} + \dfrac{3}{2}\,\arcsin x + c\,;\ x \in (-1, 1)\right]$

f) $\int \dfrac{4x^2 - 3}{x^2 + 1}\,\mathrm{d}x$ $\left[4x - 7\,\mathrm{arctg}\,x + c\,;\ x \in \mathbb{R}\right]$

Pomocí metody per partes spočtěte:

a) $\int \ln x\,\mathrm{d}x$ $\left[x\ln x - x + c\,;\ x \in (0, +\infty)\right]$

b) $\int x^\alpha \ln x\,\mathrm{d}x$ $\left[\dfrac{x^{\alpha+1}}{\alpha + 1}\,\ln x - \dfrac{x^{\alpha+1}}{(\alpha + 1)^2} + c\,;\ x \in (0, +\infty)\right]$

c) $\int \dfrac{\ln x}{x}\,\mathrm{d}x$ $\left[\dfrac{1}{2}\,\ln^2 x + c\,;\ x \in (0, +\infty)\right]$

d) $\int (3x - 1)\mathrm{e}^x\,\mathrm{d}x$ $\left[(3x - 4)\mathrm{e}^x\,;\ x \in \mathbb{R}\right]$

e) $\int (x - 1)\sin x\,\mathrm{d}x$ $\left[(1 - x)\cos x + \sin x + c\,;\ x \in \mathbb{R}\right]$

f) $\int (x + 2)\cos x\,\mathrm{d}x$ $\left[(x + 2)\sin x + \cos x + c\,;\ x \in \mathbb{R}\right]$

Pomocí metody per partes spočtěte:

a) $\int x^2 \sin x\,\mathrm{d}x$ $\left[-x^2\cos x + 2x\sin x + 2\cos x + c\,;\ x \in \mathbb{R}\right]$

b) $\int \left(2 - x^2\right)\mathrm{e}^x\,\mathrm{d}x$ $\left[\left(2x - x^2\right)\mathrm{e}^x + c\,;\ x \in \mathbb{R}\right]$

c) $\int \mathrm{e}^x \sin x\,\mathrm{d}x$ $\left[\dfrac{1}{2}\,\mathrm{e}^x(\sin x - \cos x)\,;\ x \in \mathbb{R}\right]$

d) $\int x\ln^2 x\,\mathrm{d}x$ $\left[\dfrac{x^2}{4}\left(2\ln^2 x - 2\ln x + 1\right) + c\,;\ x \in (0, +\infty)\right]$

Pomocí substituční metody vypočtěte:

a) $\int \sin(3x + 1)\,\mathrm{d}x$ $\left[-\dfrac{1}{3}\,\cos(3x + 1)\,;\ x \in \mathbb{R}\right]$

b) $\int \dfrac{\mathrm{d}x}{\cos^2 5x}$ $\left[\dfrac{1}{5}\,\mathrm{tg}\,5x\right]$

c) $\int \dfrac{\mathrm{d}x}{2x - 5}$ $\left[\dfrac{1}{2}\,\ln|2x - 5|\right]$

d) $\int \dfrac{\cos x\,\mathrm{d}x}{1 + \sin x}$ $\left[\ln|1 + \sin x|\right]$

e) $\int e^{-2x+3}\,dx$ $\left[-\dfrac{1}{2}\,e^{-2x+3}\right]$

f) $\int \dfrac{x\,dx}{\sqrt{x^2-4}}$ $\left[\sqrt{x^2-4}\right]$

g) $\int \dfrac{dx}{x^2+9}$ $\left[\dfrac{1}{3}\,\operatorname{arctg}\dfrac{x}{3}\right]$

h) $\int \dfrac{dx}{x\ln x}$ $\left[\ln|\ln x|\right]$

Vypočtěte:

a) $\int \sin^3 x\cos x\,dx$ $\left[\dfrac{1}{4}\,\sin^4 x\right]$

b) $\int \sin^2 x\cos^2 x\,dx$ $\left[\dfrac{x}{8}-\dfrac{\sin 4x}{32}\right]$

Vypočtěte:

a) $\int \dfrac{dx}{5x-3}$ $\left[\dfrac{1}{5}\,\ln|5x-3|\right]$

b) $\int \dfrac{dx}{(2x-1)^3}$ $\left[-\dfrac{1}{4(2x-1)^2}\right]$

c) $\int \dfrac{dx}{(x-1)(x+2)}$ $\left[\dfrac{1}{3}\,\ln\left|\dfrac{x-1}{x+2}\right|\right]$

d) $\int \dfrac{dx}{(x-1)^2(x+1)}$ $\left[-\dfrac{1}{2(x-1)}+\dfrac{1}{4}\,\ln\left|\dfrac{x+1}{x-1}\right|\right]$

Vypočtěte:

a) $\int \dfrac{3x-2}{x^2+3}\,dx$ $\left[\dfrac{3}{2}\,\ln|x^2+3|-\dfrac{2}{\sqrt{3}}\,\operatorname{arctg}\dfrac{x}{\sqrt{3}}\right]$

b) $\int \dfrac{dx}{x^4+x^2}$ $\left[-\dfrac{1}{x}-\operatorname{arctg}x\right]$

c) $\int \dfrac{dx}{x^3+1}$ $\left[\dfrac{1}{3}\,\ln\dfrac{|x+1|}{\sqrt{x^2-x+1}}+\dfrac{1}{\sqrt{3}}\,\operatorname{arctg}\dfrac{2x-1}{\sqrt{3}}\right]$

d) $\int \dfrac{x^3\,dx}{x^2+2x+3}$ $\left[\dfrac{x^2}{2}-2x+\dfrac{1}{2}\,\ln\left(x^2+2x+3\right)+\dfrac{5}{\sqrt{2}}\,\operatorname{arctg}\dfrac{x+1}{\sqrt{2}}\right]$

Vypočtěte:

a) $\int \dfrac{dx}{2+\sqrt{x+1}}$ $\left[2\sqrt{x+1}-4\ln\left(2+\sqrt{x+1}\right)\right]$

b) $\int \dfrac{\sqrt{x-2}\,dx}{(x^2-3x+2)\sqrt{x-1}}$ $\left[2\sqrt{\dfrac{x-2}{x-1}}\right]$

Vypočtěte:

a) $\int \dfrac{dx}{\sin x}$ $\left[\ln\left|\operatorname{tg}\dfrac{x}{2}\right|\right]$

b) $\int \dfrac{\sin x-\cos x}{\sin x+\cos x}\,dx$ $\left[-\ln|\sin x+\cos x|\right]$

c) $\int \dfrac{dx}{2+3\cos^2 x}$ $\left[\dfrac{1}{\sqrt{10}}\,\operatorname{arctg}\left(\sqrt{\dfrac{2}{5}}\,\operatorname{tg}x\right)\right]$

d) $\displaystyle\int \frac{\mathrm{d}x}{5 - 3\cos x}$ $\qquad\qquad \left[\dfrac{1}{2}\,\operatorname{arctg}\left(2\,\operatorname{tg}\dfrac{x}{2}\right)\right]$

Vypočtěte:

a) $\displaystyle\int \frac{\mathrm{d}x}{2 - \mathrm{e}^x - \mathrm{e}^{2x}}$ $\qquad\qquad \left[\dfrac{x}{2} - \dfrac{1}{6}\ln\left(\mathrm{e}^x + 2\right) - \dfrac{1}{3}\ln\left|\mathrm{e}^x - 1\right|\right]$

b) $\displaystyle\int \frac{\mathrm{d}x}{4 + 3\sinh^2 x}$ $\qquad\qquad \left[\dfrac{1}{4}\ln\dfrac{3\mathrm{e}^{2x} + 1}{\mathrm{e}^{2x} + 3}\right]$

Vypočtěte $\displaystyle\int \frac{\mathrm{d}x}{x\sqrt{1 - \ln^2 x}}$. $\qquad\qquad [\arcsin(\ln x)]$

Vypočtěte:

a) $\displaystyle\int \sqrt{1 - x^2}\,\mathrm{d}x$ $\qquad\qquad \left[\dfrac{1}{2}\,\arcsin x + \dfrac{1}{4}\,\sin(2\arcsin x)\right]$

b) $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{x^2 - 1}}$ $\qquad\qquad [\operatorname{argcosh} x]$

c) $\displaystyle\int \sqrt{x^2 + 1}\,\mathrm{d}x$ $\qquad\qquad \left[\dfrac{1}{2}\,\operatorname{argsinh} x + \dfrac{1}{4}\,\sinh(2\operatorname{argsinh} x)\right]$

d) $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{x^2 + x + 1}}$ $\qquad\qquad \left[\operatorname{argsinh}\left(\dfrac{2x + 1}{\sqrt{3}}\right)\right]$

Odvoďte rekurentní formuli pro integrál $K_n = \displaystyle\int \frac{\mathrm{d}x}{\left(1 + x^2\right)^n}$.

$$\left[K_{n+1} = \frac{2n - 1}{2n}\,K_n + \frac{x}{2n\left(1 + x^2\right)^n}\right]$$

Pomocí tabulkových integrálů vypočtěte:

a) $\displaystyle\int (2x + 1)\,\mathrm{d}x$ $\qquad\qquad \left[x^2 + x\ ;\ x \in \mathbb{R}\right]$

b) $\displaystyle\int (3x^2 + x - 1)\,\mathrm{d}x$ $\qquad\qquad \left[x^3 + \dfrac{x^2}{2} - x\ ;\ x \in \mathbb{R}\right]$

c) $\displaystyle\int (x^4 - 3x^2 + x - 5)\,\mathrm{d}x$ $\qquad\qquad \left[\dfrac{x^5}{5} - x^3 + \dfrac{x^2}{2} - 5x\right]$

d) $\displaystyle\int \left(\sqrt{x} + \dfrac{1}{x}\right)\mathrm{d}x$ $\qquad\qquad \left[\dfrac{2}{3}\,x\sqrt{x} + \ln x\ ;\ x \in (0, +\infty)\right]$

e) $\displaystyle\int \left(\dfrac{1}{x\sqrt{x}} - \dfrac{1}{x^2}\right)\mathrm{d}x$ $\qquad\qquad \left[-\dfrac{2}{\sqrt{x}} + \dfrac{1}{x}\ ;\ x \in (0, +\infty)\right]$

f) $\displaystyle\int \left(\dfrac{1}{\sqrt[3]{x}} + x\sqrt{x} + 2\right)\mathrm{d}x$ $\qquad\qquad \left[\dfrac{3}{2}\,\sqrt[3]{x^2} + \dfrac{2}{5}\,x^2\sqrt{x} + 2x\ ;\ x \in (0, +\infty)\right]$

g) $\displaystyle\int \left(\sqrt{x\sqrt{x}} - \dfrac{2}{\sqrt{x}} + \dfrac{1}{x^3}\right)\mathrm{d}x$ $\qquad\qquad \left[\dfrac{4}{7}\,x\sqrt[4]{x^3} - 4\sqrt{x} - \dfrac{1}{2x^2}\ ;\ x \in (0, +\infty)\right]$

h) $\displaystyle\int x^2(x^2 + 1)\,\mathrm{d}x$ $\qquad\qquad \left[\dfrac{x^5}{5} + \dfrac{x^3}{3}\ ;\ x \in \mathbb{R}\right]$

Pomocí tabulkových integrálů vypočtěte:

a) $\displaystyle\int \left(2\mathrm{e}^x - \sqrt[3]{x^2}\right)\mathrm{d}x$ $\qquad\qquad \left[2\mathrm{e}^x - \dfrac{2}{5}\,x\sqrt[3]{x^2}\right]$

b) $\displaystyle\int (a^x - 2\sin x)\, \mathrm{d}x$ $\qquad\qquad$ $\left[\dfrac{a^x}{\ln a} + 2\cos x\right]$

c) $\displaystyle\int \left(\cos x + 2\sqrt[5]{x^3}\right)\, \mathrm{d}x$ $\qquad\qquad$ $\left[\sin x + \dfrac{5}{4}\, x\sqrt[5]{x^3}\right]$

d) $\displaystyle\int \left(2^x + \sqrt{\dfrac{1}{x}}\right)\, \mathrm{d}x$ $\qquad\qquad$ $\left[\dfrac{2^x}{\ln 2} + 2\sqrt{x}\right]$

e) $\displaystyle\int \left(-\sin x + \dfrac{3}{\sqrt{4 - 4x^2}}\right)\, \mathrm{d}x$ $\qquad\qquad$ $\left[\cos x + \dfrac{3}{2}\, \arcsin x\right]$

f) $\displaystyle\int \left(\dfrac{1}{\sqrt{2 - 2x^2}} - 3^{-x}\right)\, \mathrm{d}x$ $\qquad\qquad$ $\left[\dfrac{\arcsin x}{\sqrt{2}} + \dfrac{1}{3^x \ln 3}\right]$

g) $\displaystyle\int \dfrac{x^2\, \mathrm{d}x}{3(1 + x^2)}$ $\qquad\qquad$ $\left[\dfrac{x}{3} - \dfrac{\operatorname{arctg} x}{3}\right]$

h) $\displaystyle\int \dfrac{x^2 - 3}{2 + 2x^2}\, \mathrm{d}x$ $\qquad\qquad$ $\left[\dfrac{x}{2} - 2\operatorname{arctg} x\right]$

Metodou per partes vypočtěte:

a) $\displaystyle\int x^2 \ln x\, \mathrm{d}x$ $\qquad\qquad$ $\left[\dfrac{x^3}{3}\, \ln x - \dfrac{x^3}{9}\right]$

b) $\displaystyle\int \dfrac{\ln x}{x^2}\, \mathrm{d}x$ $\qquad\qquad$ $\left[-\dfrac{\ln x + 1}{x}\right]$

c) $\displaystyle\int x^2 \cos x\, \mathrm{d}x$ $\qquad\qquad$ $\left[(x^2 - 2)\sin x + 2x\cos x\right]$

d) $\displaystyle\int x \operatorname{arctg} x\, \mathrm{d}x$ $\qquad\qquad$ $\left[\dfrac{x^2 + 1}{2}\, \operatorname{arctg} x - \dfrac{x}{2}\right]$

e) $\displaystyle\int x^2 \mathrm{e}^x\, \mathrm{d}x$ $\qquad\qquad$ $\left[\mathrm{e}^x \left(x^2 - 2x + 2\right)\right]$

f) $\displaystyle\int \dfrac{x\, \mathrm{d}x}{\sin^2 x}$ $\qquad\qquad$ $\left[-x\operatorname{cotg} x + \ln |\sin x|\right]$

Pomocí vhodné substituce vypočtěte:

a) $\displaystyle\int \cos 5x\, \mathrm{d}x$ $\qquad\qquad$ $\left[\dfrac{\sin 5x}{5}\right]$

b) $\displaystyle\int \dfrac{\mathrm{d}x}{2 - x}$ $\qquad\qquad$ $\left[-\ln |x - 2|\right]$

c) $\displaystyle\int \dfrac{\mathrm{d}x}{\sin^2 3x}$ $\qquad\qquad$ $\left[-\dfrac{\operatorname{cotg} 3x}{3}\right]$

d) $\displaystyle\int \dfrac{\mathrm{d}x}{5 + 3x}$ $\qquad\qquad$ $\left[\dfrac{1}{3}\, \ln |3x + 5|\right]$

e) $\displaystyle\int \mathrm{e}^{4x}\, \mathrm{d}x$ $\qquad\qquad$ $\left[\dfrac{1}{4}\, \mathrm{e}^{4x}\right]$

f) $\displaystyle\int x\mathrm{e}^{x^2}\, \mathrm{d}x$ $\qquad\qquad$ $\left[\dfrac{1}{2}\, \mathrm{e}^{x^2}\right]$

g) $\displaystyle\int x^2\sqrt{1 + x^3}\, \mathrm{d}x$ $\qquad\qquad$ $\left[\dfrac{2}{9}\, (x^3 + 1)\sqrt{x^3 + 1}\right]$

h) $\displaystyle\int x\sqrt{1 - x^2}\, \mathrm{d}x$ $\qquad\qquad$ $\left[-\dfrac{1}{3}\, (1 - x^2)\sqrt{1 - x^2}\right]$

i) $\displaystyle\int \dfrac{x\, \mathrm{d}x}{1 + x^4}$ $\qquad\qquad$ $\left[\dfrac{1}{2}\, \operatorname{arctg} x^2\right]$

j) $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{9-x^2}}$ $\qquad\qquad\left[\arcsin\dfrac{x}{3}\right]$

Pomocí vhodné substituce a integrací metodou per partes vypočtěte:

a) $\displaystyle\int x^2 \mathrm{e}^{-x}\,\mathrm{d}x$ $\qquad\qquad\left[-\mathrm{e}^{-x}(x^2+2x+2)\right]$

b) $\displaystyle\int \mathrm{e}^{-x}\cos x\,\mathrm{d}x$ $\qquad\qquad\left[\dfrac{1}{2}\,\mathrm{e}^{-x}(\sin x-\cos x)\right]$

c) $\displaystyle\int \operatorname{arctg} x\,\mathrm{d}x$ $\qquad\qquad\left[x\operatorname{arctg} x-\dfrac{1}{2}\,\ln(1+x^2)\right]$

d) $\displaystyle\int \arcsin x\,\mathrm{d}x$ $\qquad\qquad\left[x\arcsin x+\sqrt{1-x^2}\right]$

e) $\displaystyle\int \sqrt{1+x^2}\,\mathrm{d}x$ $\qquad\qquad\left[\dfrac{x}{2}\sqrt{1+x^2}+\dfrac{1}{2}\,\ln\left(x+\sqrt{1+x^2}\right)\right]$

f) $\displaystyle\int \operatorname{arctg}\sqrt{x}\,\mathrm{d}x$ $\qquad\qquad\left[(x+1)\operatorname{arctg}\sqrt{x}-\sqrt{x}\right]$

Vypočtěte:

a) $\displaystyle\int \cos^5 x\sin x\,\mathrm{d}x$ $\qquad\qquad\left[-\dfrac{1}{6}\,\cos^6 x\right]$

b) $\displaystyle\int \sin^3 2x\,\mathrm{d}x$ $\qquad\qquad\left[\dfrac{1}{6}\,\cos^3 2x-\dfrac{1}{2}\,\cos 2x\right]$

c) $\displaystyle\int \sin^6 x\,\mathrm{d}x$ $\qquad\left[\dfrac{5x}{16}-\dfrac{1}{4}\,\sin 2x+\dfrac{1}{48}\,\sin^3 2x+\dfrac{3}{64}\,\sin 4x\right]$

d) $\displaystyle\int \cos^6 x\,\mathrm{d}x$ $\qquad\left[\dfrac{5x}{16}+\dfrac{1}{4}\,\sin 2x-\dfrac{1}{48}\,\sin^3 2x+\dfrac{3}{64}\,\sin 4x\right]$

Vypočtěte:

a) $\displaystyle\int \frac{2x-1}{x^2+4}\,\mathrm{d}x$ $\qquad\qquad\left[\ln(x^2+4)-\dfrac{1}{2}\,\operatorname{arctg}\dfrac{x}{2}\right]$

b) $\displaystyle\int \frac{\mathrm{d}x}{(x^2+2)(x-1)^2}$ $\quad\left[\dfrac{1}{9}\,\ln(x^2+2)-\dfrac{1}{9\sqrt{2}}\,\operatorname{arctg}\dfrac{x}{\sqrt{2}}-\dfrac{2}{9}\,\ln|x-1|-\dfrac{1}{3(x-1)}\right]$

c) $\displaystyle\int \frac{\mathrm{d}x}{x^3-1}$ $\quad\left[\dfrac{1}{3}\,\ln|x-1|-\dfrac{1}{6}\,\ln(x^2+x+1)-\dfrac{1}{\sqrt{3}}\,\operatorname{arctg}\dfrac{2x-1}{\sqrt{3}}\right]$

d) $\displaystyle\int \frac{x-1}{x(x^2+x+1)^2}\,\mathrm{d}x$ $\quad\left[-\ln x+\dfrac{1}{2}\,\ln(x^2+x+1)+\sqrt{3}\,\operatorname{arctg}\dfrac{2x+1}{\sqrt{3}}+\dfrac{x}{x^2+x+1}\right]$

Vypočtěte:

a) $\displaystyle\int \frac{\mathrm{d}x}{1+\sqrt{x}}\,\mathrm{d}x$ $\qquad\qquad\left[2\sqrt{x}-2\ln(1+\sqrt{x})\right]$

b) $\displaystyle\int \frac{\sqrt{x}}{x+4}\,\mathrm{d}x$ $\qquad\qquad\left[2\sqrt{x}-4\operatorname{arctg}\dfrac{\sqrt{x}}{2}\right]$

c) $\displaystyle\int \frac{\mathrm{d}x}{x\sqrt{x+1}}$ $\qquad\qquad\left[\ln\left|\dfrac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right|\right]$

d) $\displaystyle\int x\sqrt[3]{x-1}\,\mathrm{d}x$ $\qquad\qquad\left[\dfrac{3}{7}\,(x-1)^2\sqrt[3]{x-1}+\dfrac{3}{4}\,(x-1)\sqrt[3]{x-1}\right]$

e) $\displaystyle\int x\sqrt{x+1}\,\mathrm{d}x$ $\qquad\qquad\left[\dfrac{2}{5}\,(x+1)^2\sqrt{x+1}-\dfrac{2}{3}\,(x+1)\sqrt{x+1}\right]$

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Vypočtěte:

a) $\displaystyle\int \frac{\mathrm{d}x}{\sin x \cos^2 x}$
$\qquad\left[\dfrac{1}{\cos x} + \ln\left|\operatorname{tg}\dfrac{x}{2}\right|\right]$

b) $\displaystyle\int \frac{\mathrm{d}x}{\cos^3 x}$
$\qquad\left[\dfrac{\sin x}{2\cos^2 x} + \dfrac{1}{4}\,\ln\left|\dfrac{1+\sin x}{1-\sin x}\right|\right]$

c) $\displaystyle\int \frac{\mathrm{d}x}{\sin^3 x}$
$\qquad\left[-\dfrac{\cos x}{2\sin^2 x} + \dfrac{1}{2}\,\ln\left|\operatorname{tg}\dfrac{x}{2}\right|\right]$

d) $\displaystyle\int \frac{\mathrm{d}x}{\sin^4 x \cos^4 x}$
$\qquad\left[\dfrac{1}{3}\,\operatorname{tg}^3 x + 3\operatorname{tg} x - 3\operatorname{cotg} x - \dfrac{1}{3}\,\operatorname{cotg}^3 x\right]$

e) $\displaystyle\int \frac{\mathrm{d}x}{1 + \sin^2 x}$
$\qquad\left[\dfrac{1}{\sqrt{2}}\,\operatorname{arctg}\left(\sqrt{2}\,\operatorname{tg} x\right)\right]$

f) $\displaystyle\int \frac{\mathrm{d}x}{\sin x + \cos x}$
$\qquad\left[\dfrac{1}{\sqrt{2}}\left(\ln\left|\operatorname{tg}\dfrac{x}{2} - 1 + \sqrt{2}\right| - \ln\left|\operatorname{tg}\dfrac{x}{2} - 1 - \sqrt{2}\right|\right)\right]$

g) $\displaystyle\int \frac{\mathrm{d}x}{5 + 4\sin x}$
$\qquad\left[\dfrac{2}{3}\,\operatorname{arctg}\left(\dfrac{5}{3}\,\operatorname{tg}\dfrac{x}{2} + \dfrac{4}{3}\right)\right]$

Vypočtěte:

a) $\displaystyle\int \cos x \cdot \cos 3x \,\mathrm{d}x$
$\qquad\left[\dfrac{1}{8}\,\sin 4x + \dfrac{1}{4}\,\sin 2x\right]$

b) $\displaystyle\int \sin 2x \cdot \cos 5x \,\mathrm{d}x$
$\qquad\left[-\dfrac{1}{14}\,\cos 7x + \dfrac{1}{6}\,\cos 3x\right]$

Vypočtěte:

a) $\displaystyle\int \frac{\mathrm{d}x}{\mathrm{e}^x - 1}$
$\qquad\left[\ln\left|\mathrm{e}^x - 1\right| - x\right]$

b) $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{\mathrm{e}^x + 1}}$
$\qquad\left[\ln\left(\sqrt{\mathrm{e}^x + 1} - 1\right) - \ln\left(\sqrt{\mathrm{e}^x + 1} + 1\right)\right]$

c) $\displaystyle\int \frac{\mathrm{d}x}{\mathrm{e}^x + \mathrm{e}^{-x}}$
$\qquad\left[\operatorname{arctg}\mathrm{e}^x\right]$

Vypočtěte:

a) $\displaystyle\int \sqrt{2x - 3}\,\mathrm{d}x$
$\qquad\left[\dfrac{1}{3}\,(2x-3)^{3/2}\right]$

b) $\displaystyle\int \sqrt[3]{3x + 2}\,\mathrm{d}x$
$\qquad\left[\dfrac{1}{4}\,(3x+2)^{4/3}\right]$

c) $\displaystyle\int \mathrm{e}^{\sqrt{x}}\,\mathrm{d}x$
$\qquad\left[2\mathrm{e}^{\sqrt{x}}\left(\sqrt{x} - 1\right)\right]$

d) $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,\mathrm{d}x$
$\qquad\left[-2\cos\sqrt{x}\right]$

e) $\displaystyle\int \frac{\mathrm{d}x}{\mathrm{e}^x + \sqrt{\mathrm{e}^x}}$
$\qquad\left[-2\sqrt{\mathrm{e}^{-x}} + 2\ln\left(\sqrt{\mathrm{e}^x} + 1\right) - x\right]$

f) $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{\mathrm{e}^x - 1}}$
$\qquad\left[2\operatorname{arctg}\sqrt{\mathrm{e}^x - 1}\right]$

Vypočtěte:

a) $\displaystyle\int \frac{\sqrt{x}}{1 + \sqrt{x}}\,\mathrm{d}x$
$\qquad\left[x - 2\sqrt{x} + 2\ln\left(1 + \sqrt{x}\right)\right]$

b) $\displaystyle\int \frac{\sqrt{x}}{1-\sqrt[3]{x}}\,\mathrm{d}x$ $\qquad\left[-\dfrac{6}{7}\,x^{7/6}-\dfrac{6}{5}\,x^{5/6}-2\sqrt{x}-6x^{1/6}+3\ln\left(1+\sqrt{x}\right)-3\ln\left(1-\sqrt[6]{x}\right)\right]$

c) $\displaystyle\int \frac{\mathrm{d}x}{x+\sqrt[3]{x^2}}$ $\qquad\left[3\ln\left(1+\sqrt[3]{x}\right)\right]$

d) $\displaystyle\int \frac{\sqrt{x+2}}{x}\,\mathrm{d}x$ $\qquad\left[2\sqrt{x+2}+\sqrt{2}\ln\left|\sqrt{x+2}-\sqrt{2}\right|-\sqrt{2}\ln\left|\sqrt{x+2}+\sqrt{2}\right|\right]$

e) $\displaystyle\int \frac{1+\sqrt{x+1}}{\sqrt{x+1}-1}\,\mathrm{d}x$ $\qquad\left[x+4\sqrt{x+1}+4\ln\left|\sqrt{x+1}-1\right|\right]$

f) $\displaystyle\int x\sqrt{1+x}\,\mathrm{d}x$ $\qquad\left[\dfrac{2}{5}\,(x+1)^{5/2}-\dfrac{2}{3}\,(x+1)^{3/2}\right]$

Vypočtěte:

a) $\displaystyle\int \frac{x^3}{x^8+1}\,\mathrm{d}x$ $\qquad\left[\dfrac{1}{4}\,\operatorname{arctg} x^4\right]$

b) $\displaystyle\int \frac{x^2+1}{x(x^2-1)}\,\mathrm{d}x$ $\qquad\left[\ln\left|\dfrac{x^2-1}{x}\right|\,\mathrm{d}x\right]$

c) $\displaystyle\int \frac{x^3}{\sqrt{1+2x^2}}\,\mathrm{d}x$ $\qquad\left[\left(\dfrac{1}{12}\,(1+2x^2)-\dfrac{1}{4}\right)\sqrt{1+2x^2}\right]$

d) $\displaystyle\int \frac{\mathrm{d}x}{x\sqrt{1+\ln x}}$ $\qquad\left[2\sqrt{1+\ln x}\right]$

e) $\displaystyle\int \frac{\sqrt{1-x}}{x}\,\mathrm{d}x$ $\qquad\left[2\sqrt{1-x}+\ln\left|\sqrt{1-x}-1\right|-\ln\left|\sqrt{1-x}+1\right|\right]$

f) $\displaystyle\int \mathrm{e}^{\sqrt{x}}\sqrt{x}\,\mathrm{d}x$ $\qquad\left[2\mathrm{e}^{\sqrt{x}}\left(x-2\sqrt{x}+2\right)\right]$

Vypočtěte:

a) $\displaystyle\int \ln\left(1+x^2\right)\,\mathrm{d}x$ $\qquad\left[x\ln\left(1+x^2\right)-2x+2\operatorname{arctg} x\right]$

b) $\displaystyle\int \frac{\arcsin x}{\sqrt{1+x}}\,\mathrm{d}x$ $\qquad\left[2\sqrt{1+x}\,\arcsin x+4\sqrt{1-x}\right]$

c) $\displaystyle\int \frac{x\arcsin x}{\sqrt{1-x^2}}\,\mathrm{d}x$ $\qquad\left[x-\sqrt{1-x^2}\,\arcsin x\right]$

Vypočtěte:

a) $\displaystyle\int \frac{x^2}{\sqrt{1+x^2}}\,\mathrm{d}x$ $\qquad\left[\dfrac{1}{4}\,\sinh(2\operatorname{argsinh} x)-\dfrac{1}{2}\,\operatorname{argsinh} x\right]$

b) $\displaystyle\int \frac{\mathrm{d}x}{x\sqrt{1-x^2}}$ $\qquad\left[\dfrac{1}{2}\,\ln\left|\cos(\arcsin x)-1\right|-\dfrac{1}{2}\,\ln\left|\cos(\arcsin x)+1\right|\right]$

c) $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{x^2-2x-1}}$ $\qquad\left[\operatorname{argcosh}\dfrac{x-1}{\sqrt{2}}\right]$