Local model networks, velocity-based linearisation and blended multimodel systems

References:

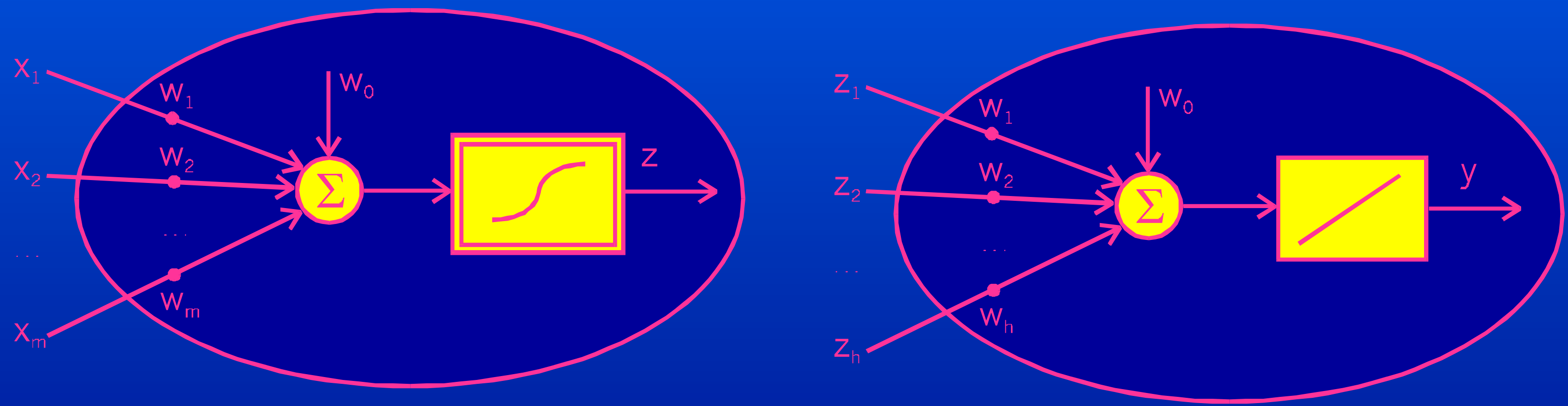
R. Murray-Smith and T.-A. Johansen [Eds.] (1997): Multiple Model Approaches to Modelling and Control, Taylor & Francis, London.

D.J. Leith and W.E. Leithead (1999): Analytic framework for blended multiple model systems using linear local models, International Journal of Control, Vol. 72, str. 605-619.

Multilayer networks

Ridge basis function ⇨ multilayer perceptron

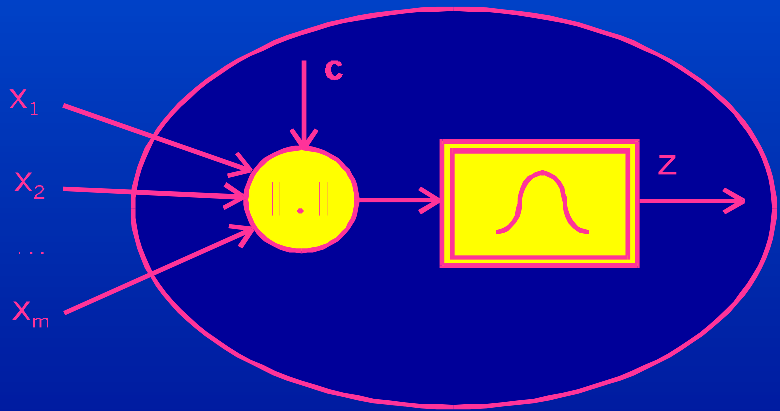
$$z_i = \sigma\left(\sum_{j=1}^{m} w_{ij} x_j(k) \; + \; w_{i0}\right)$$

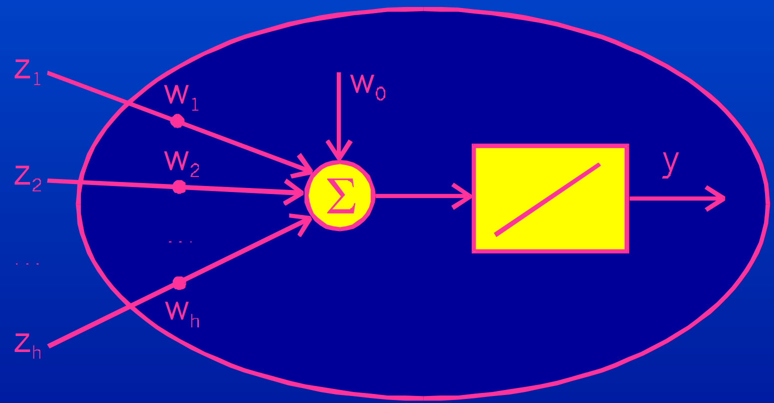


Multilayer networks

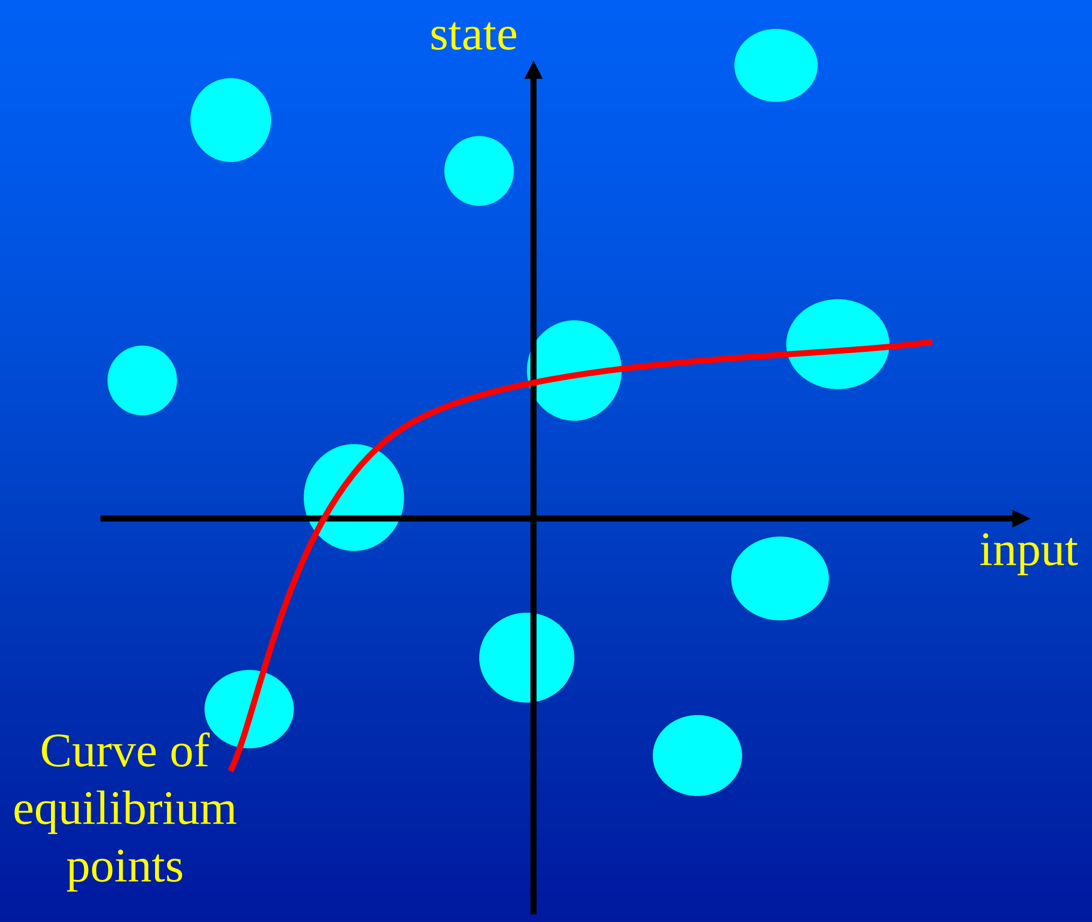
Radial basis function ⇨ RBF network

$$z_j = \exp\left(-\frac{1}{2} \sum_{l=1}^{m} \frac{(x_l - c_{jl})^2}{\sigma_{jl}^2} \right)$$

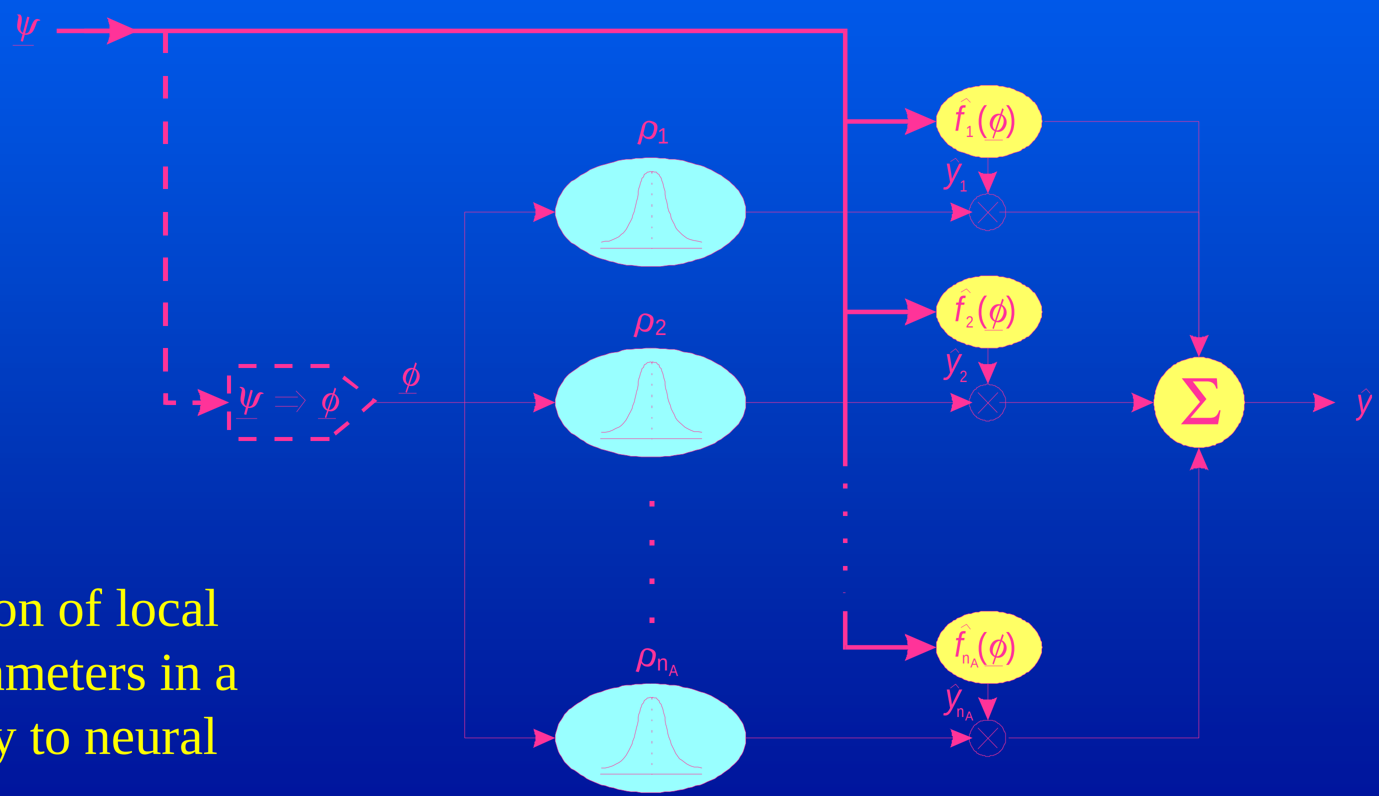




RBF networks:

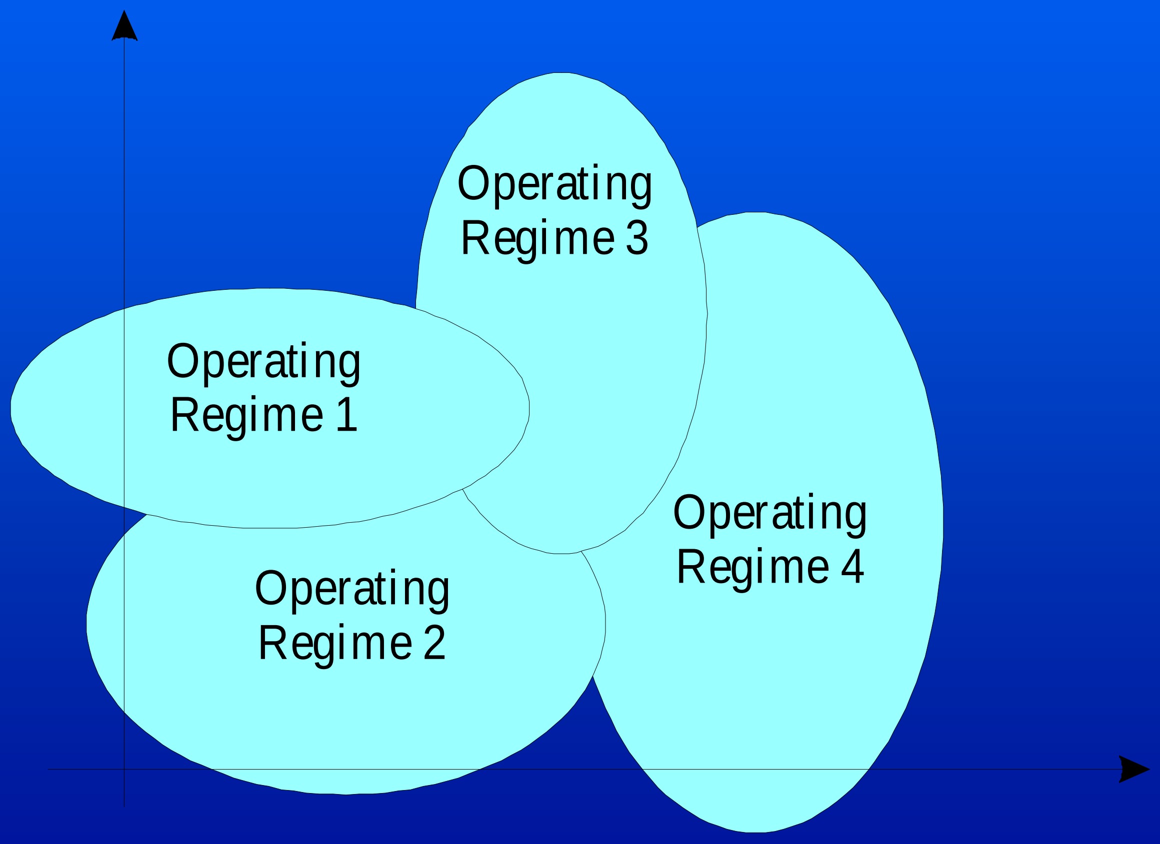


Local linearisations of nonlinear process connected in network ⇨ Local model network

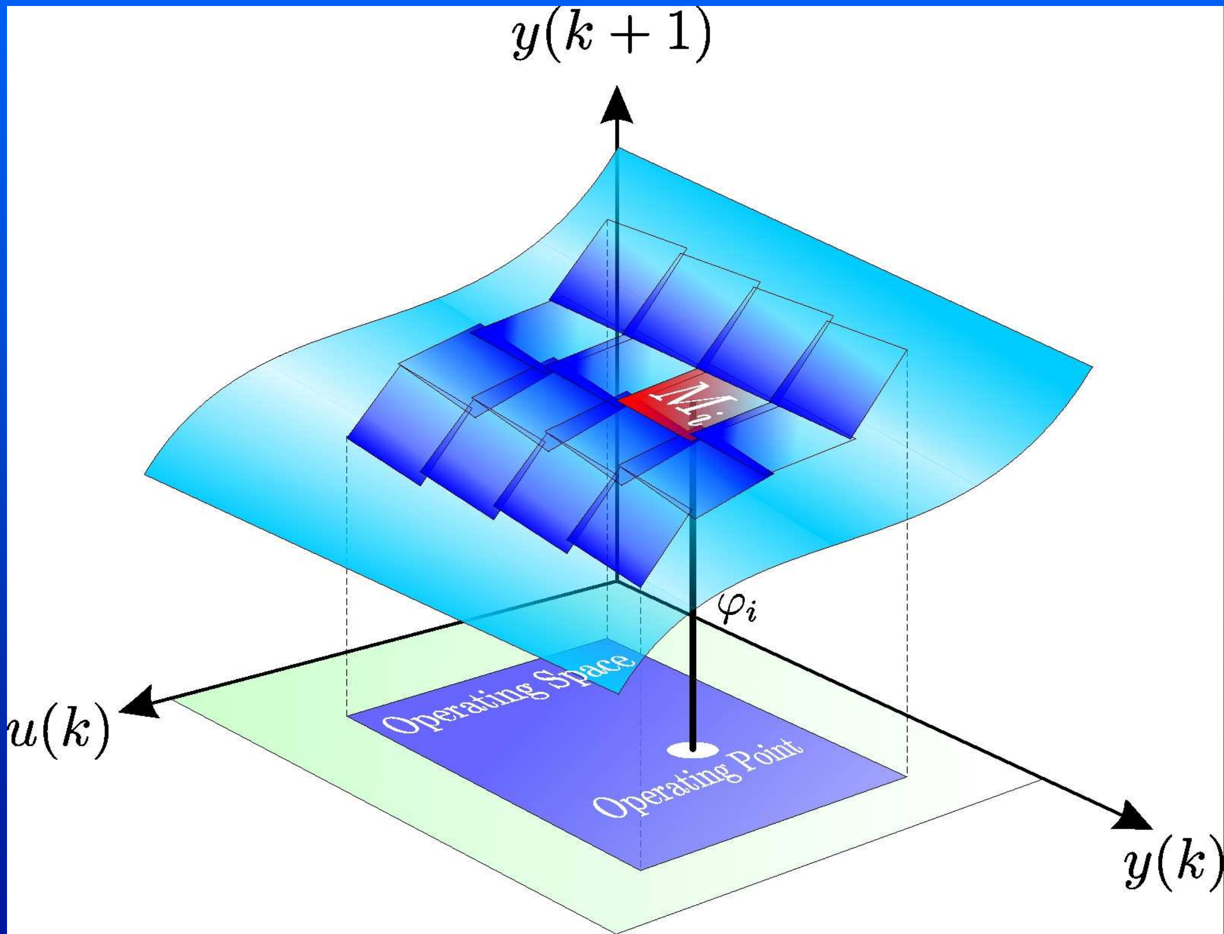


Optimisation of local model parameters in a similar way to neural networks.

Divison of operating area based on different operating regimes.



Uniform division of operating area to local linear areas



Dynamic systems

Linear is simple ...

Nonlinear system approximated local with linear model.

All real systems are nonlinear ⇒ linearisation is the basic step of all linear based controller designes.

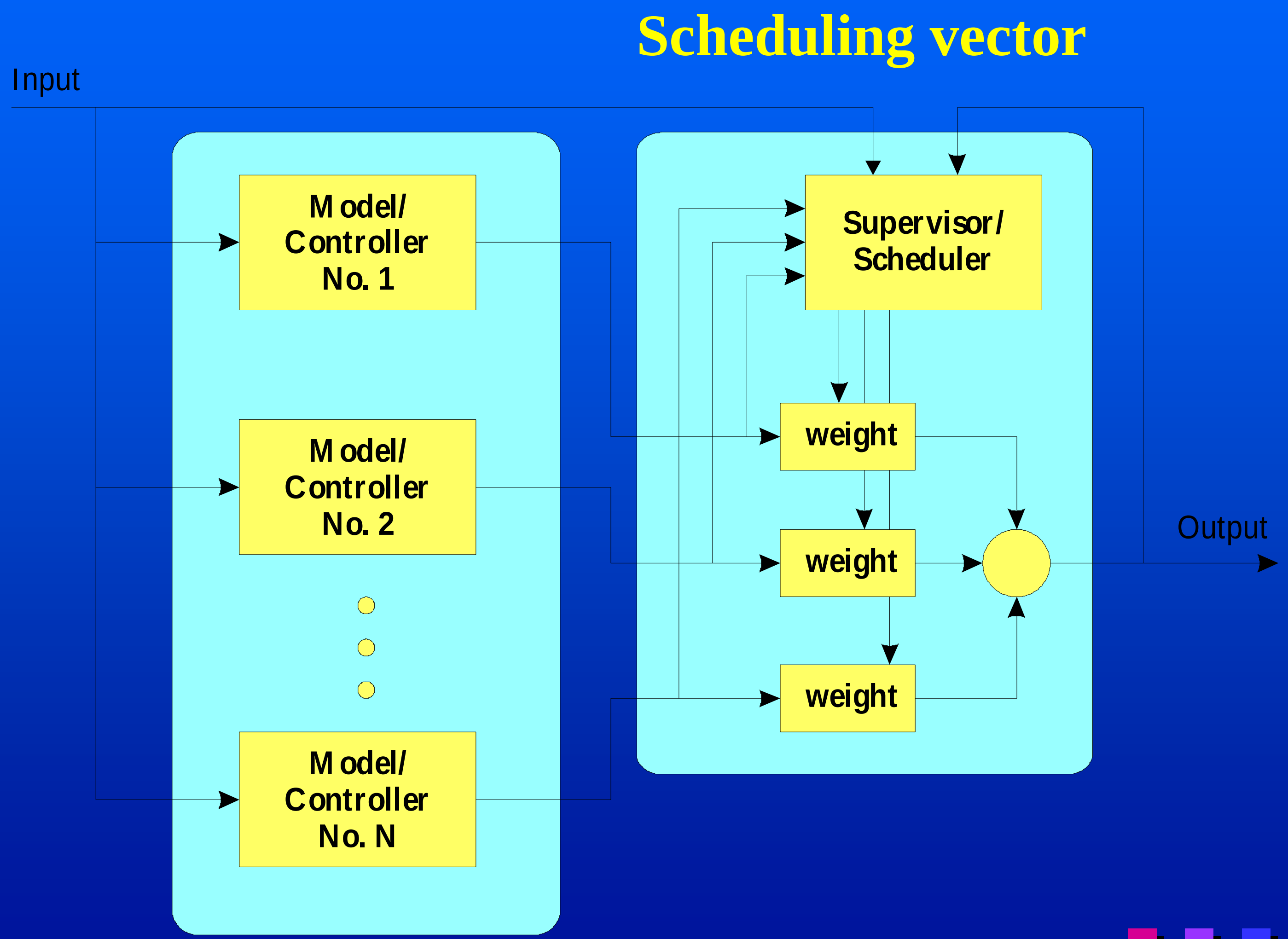
Standard method: the first order Taylor linearisation in operating point or linear system identification. Valid only in the vicinity of selected equilibrium point.

Divide and conquer ...

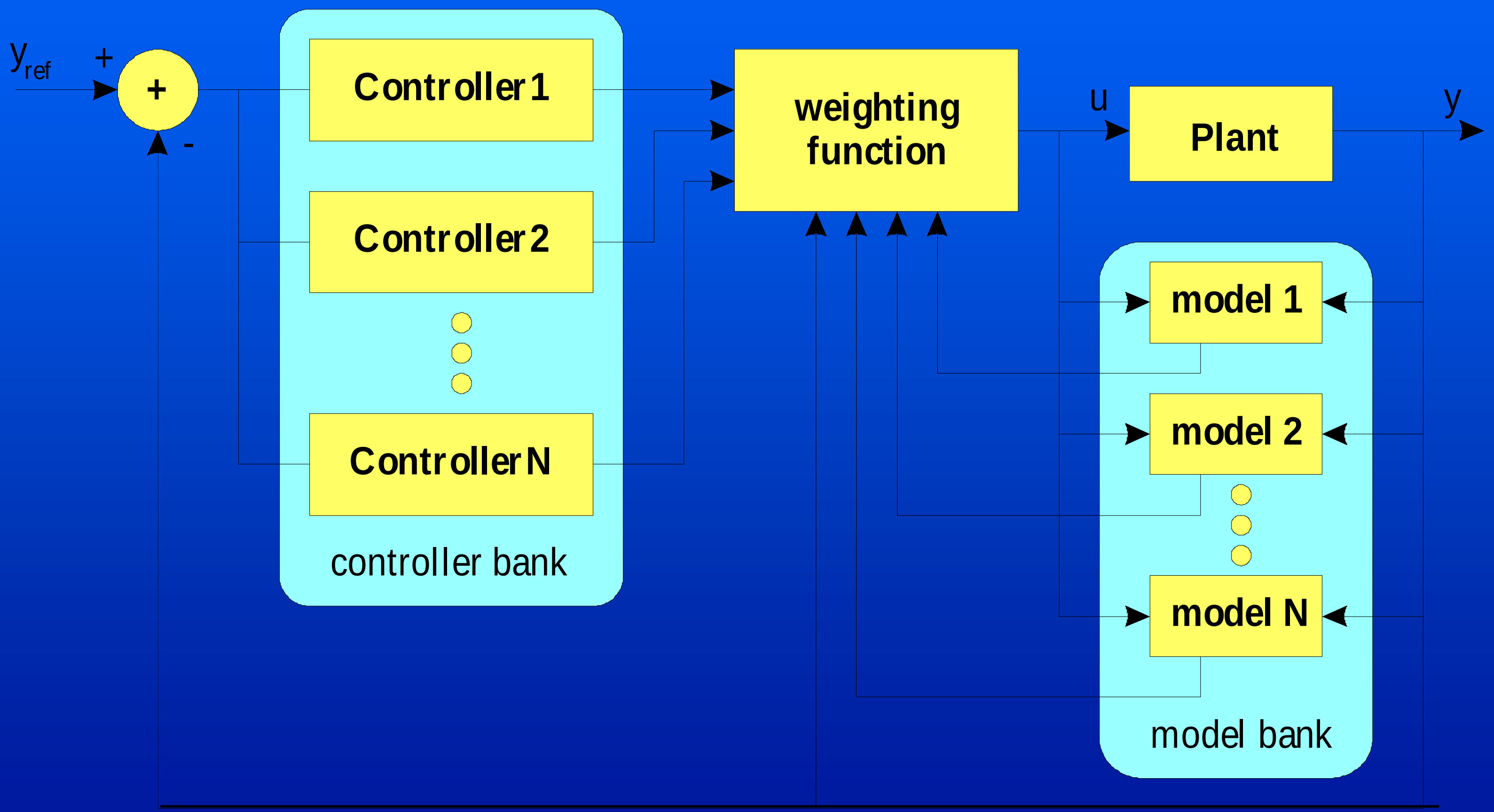
Standard analysis method of nonlinear systems: analysis of linearised models arround representative number of equilibrium points.

Standard method for design of nonlinear systems control: design of local controllers for linearised process models and integration into a single nonlinear controller.

Block sheme of local model network as it is frequently used for controllers:



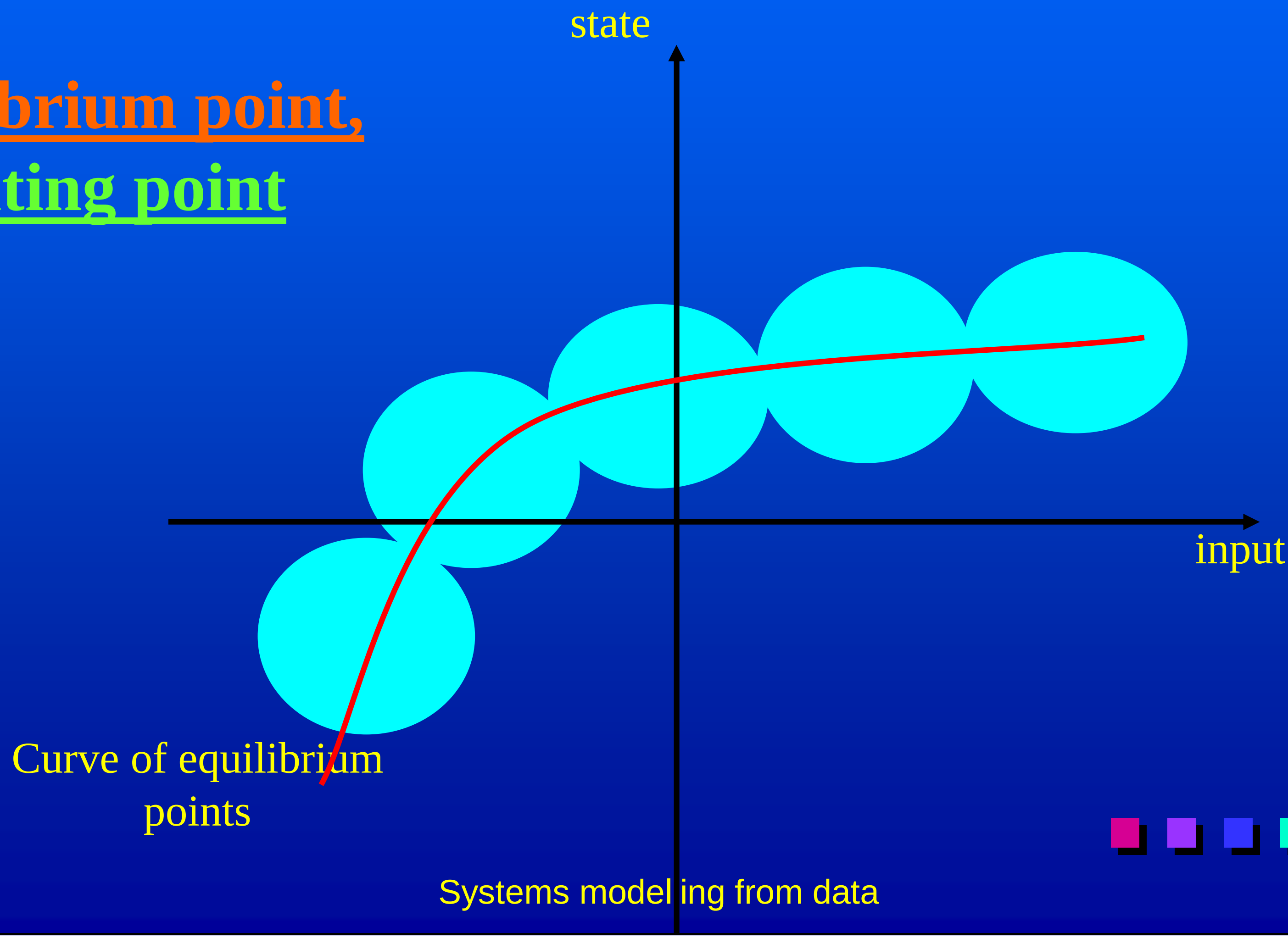
Closed-loop scheme of frequently used implementation of divide-and-conquer control design.



Nonlinear system’s model as a family of linear systems obtained with linearisations in equilibrium points

Equilibrium point,

Operating point



Linear in parameters (affine) versus linear

- Result of linearisation with Taylor expansion is an affine system – linear in parameters.

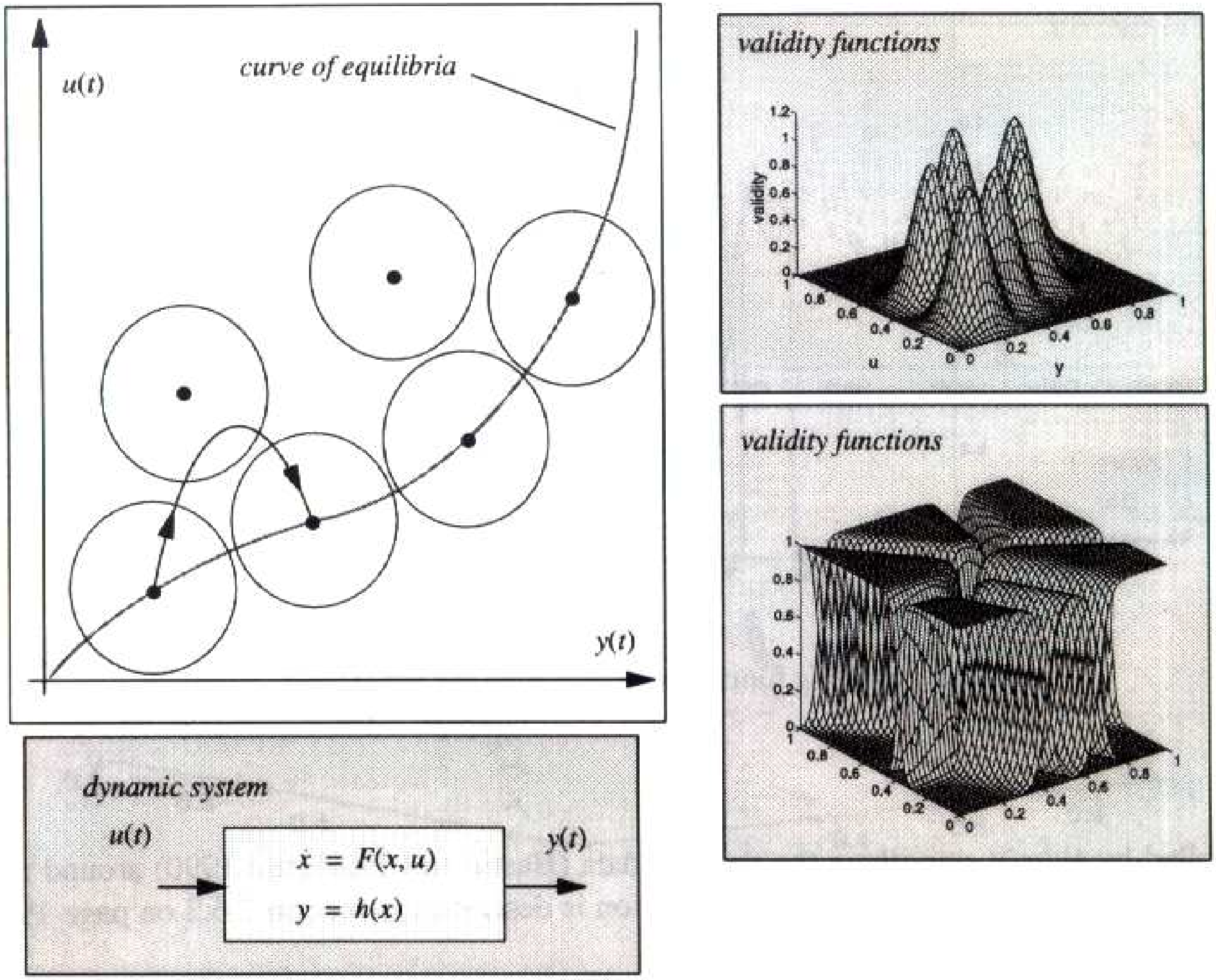
$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x},\mathbf{r}) \Rightarrow \dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}_0,\mathbf{r}_0) + \nabla_x \mathbf{F}(\mathbf{x}_0,\mathbf{r}_0)(\mathbf{x}-\mathbf{x}_0)$$

$$+\nabla_r \mathbf{F}(\mathbf{x}_0,\mathbf{r}_0)(\mathbf{r}-\mathbf{r}_0) + \text{higher order derivations}$$

The constant element – operating point ⇒ superposition condition is not valid, the system is not linear.

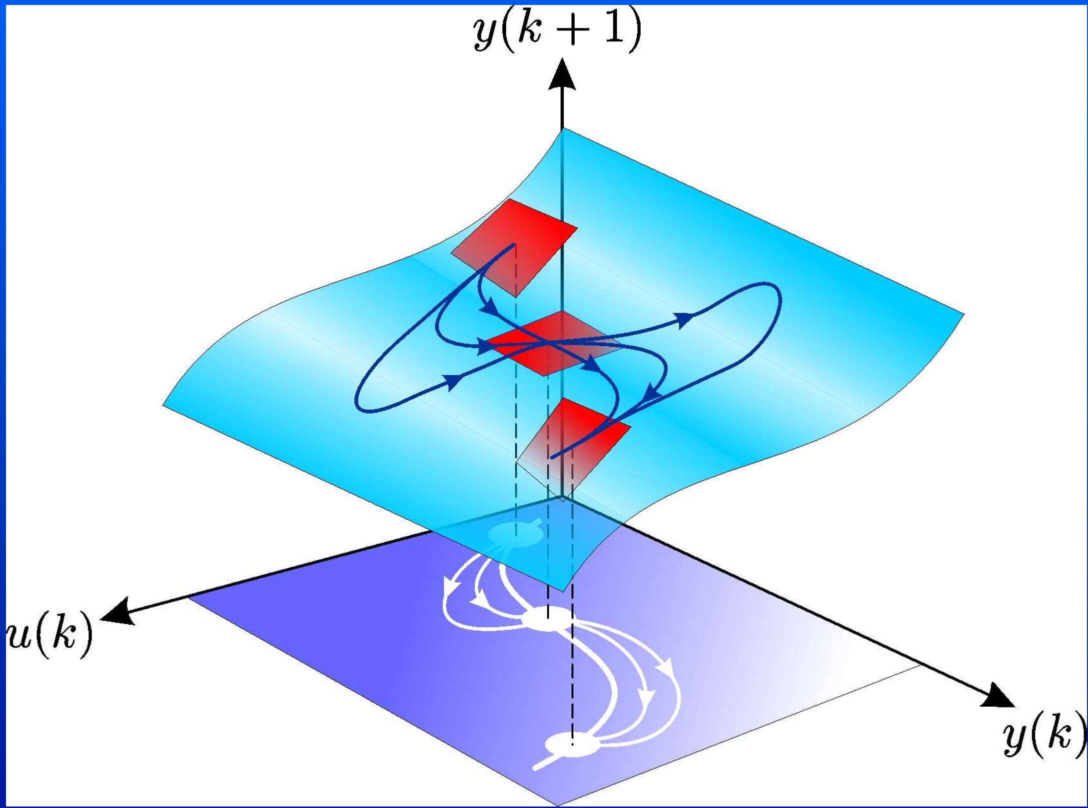
The constant element can be very large ⇒ it is not a constant "disturbance".

The constant element is changing when we change operating points ⇒ it can not be neglected, its contribution to dynamics is considerable.

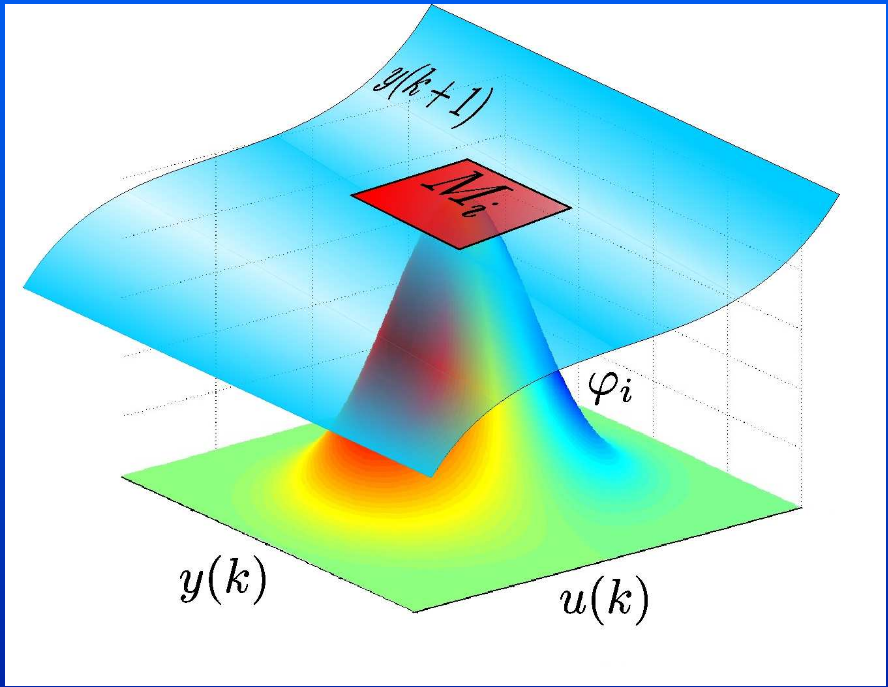


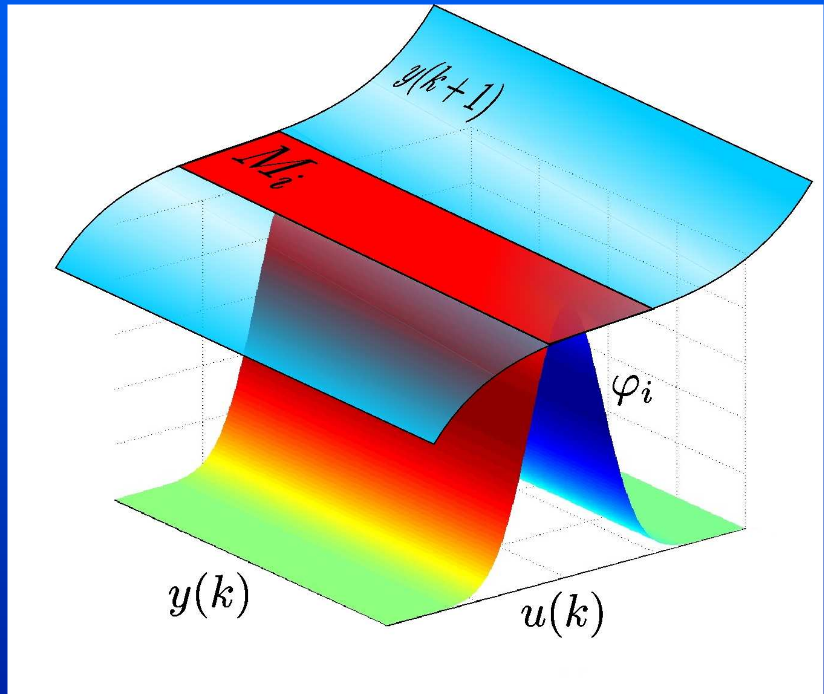
Interpolation – blending of local models arround equilibrium points

Moving between local models



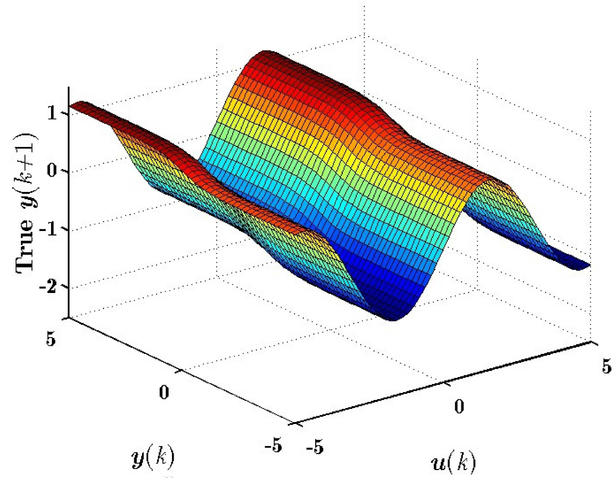
Selection of scheduling vector

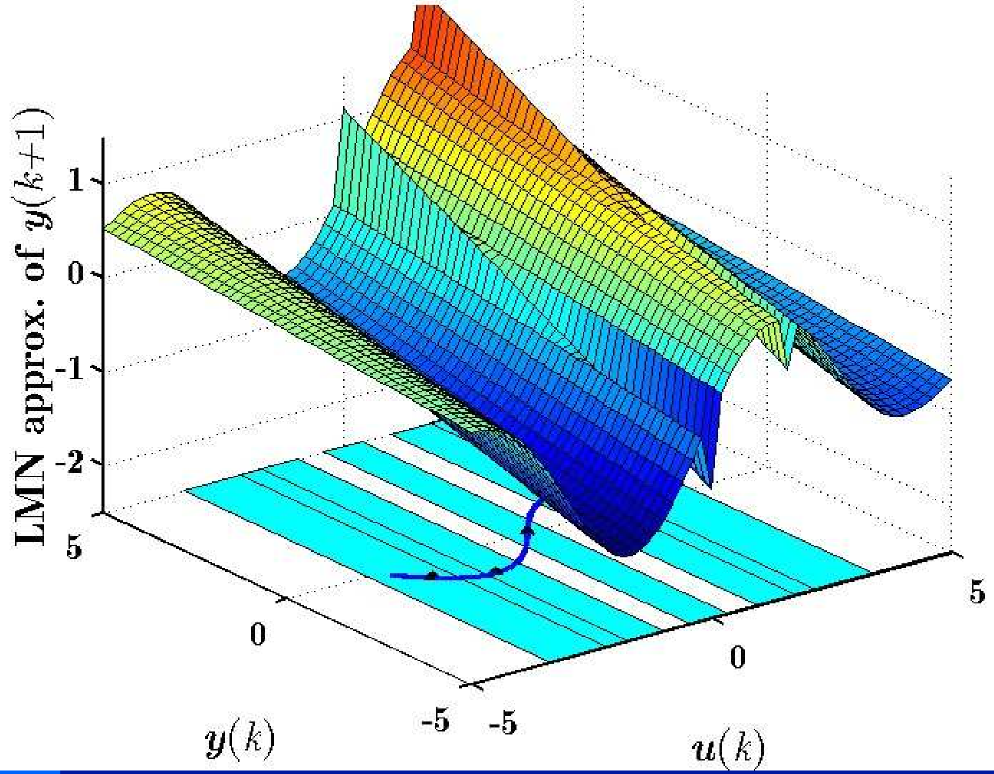


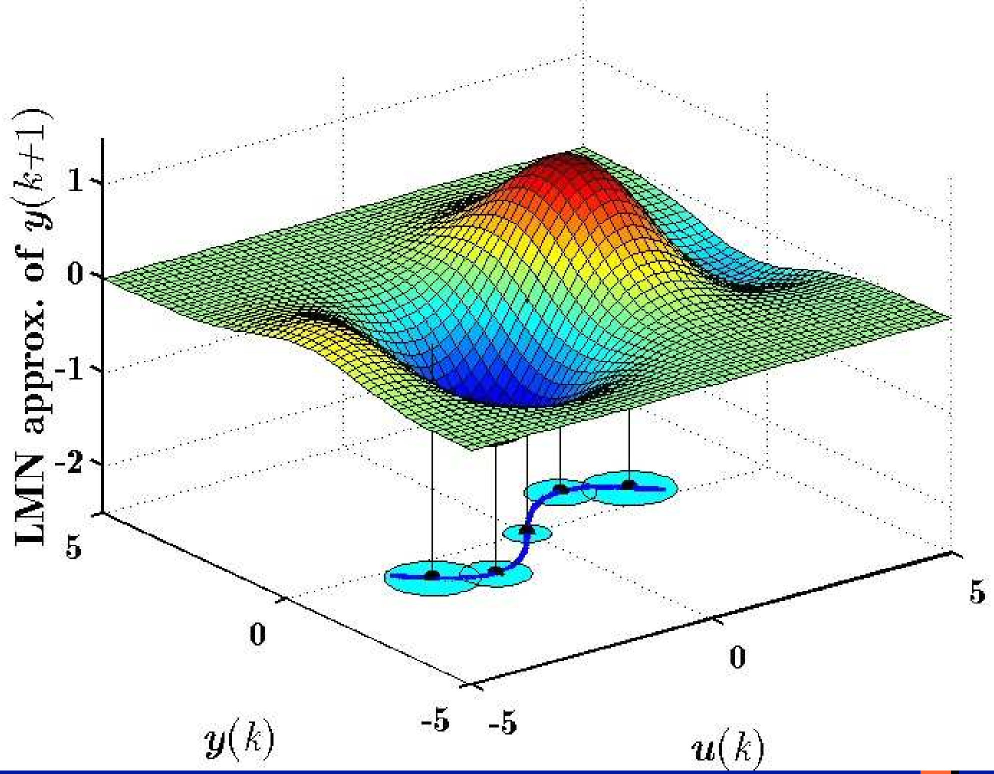


Important!

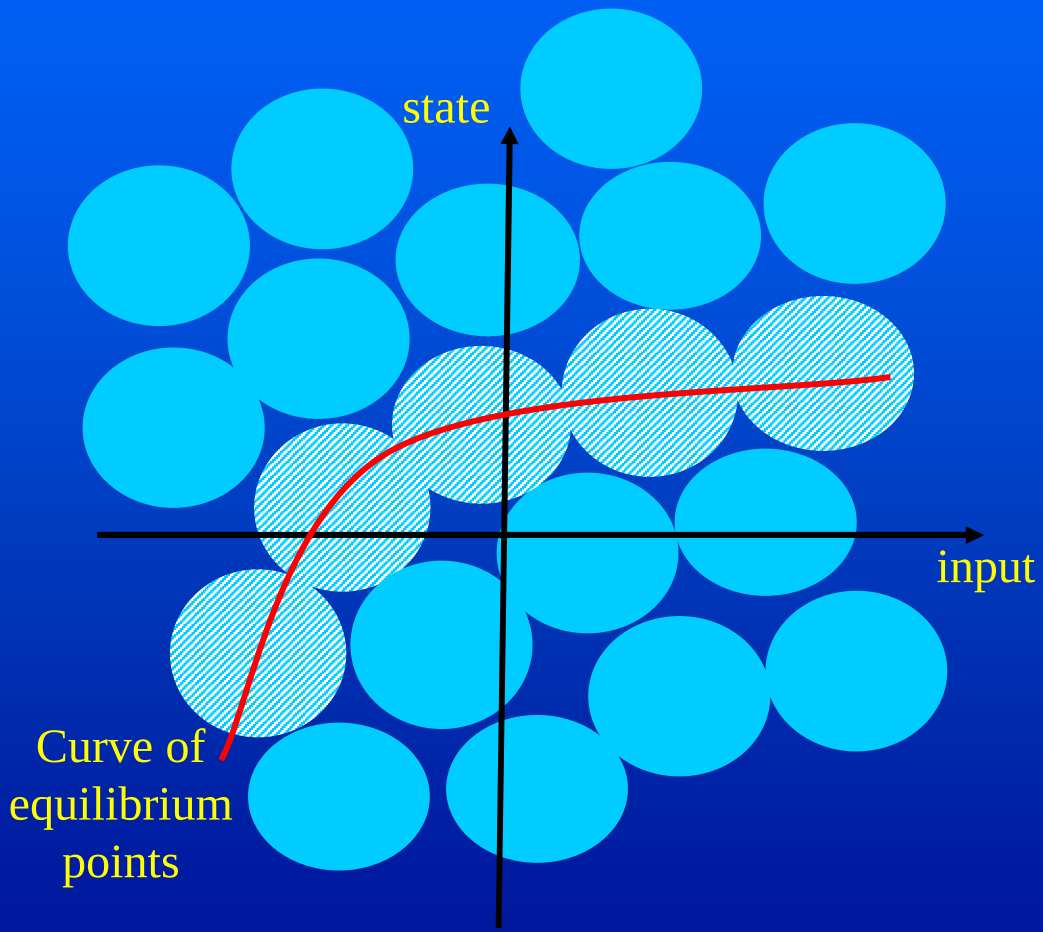
An example of reduced scheduling vector







Velocity-based linearisation



Velocity-based linearisation

Nonlinear system

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x},\mathbf{r}), \quad \mathbf{y} = \mathbf{G}(\mathbf{x},\mathbf{r})$$

or equivalently

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{r} + \mathbf{f}(\boldsymbol{\rho}),$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{r} + \mathbf{g}(\boldsymbol{\rho})$$

$$\boldsymbol{\rho} = \boldsymbol{\rho}(\mathbf{x},\mathbf{r}) \text{ with } \nabla_{\mathbf{x}}\boldsymbol{\rho}, \nabla_{\mathbf{r}}\boldsymbol{\rho} = \text{const.}$$

We derive ...

$$\dot{\mathbf{x}} = \mathbf{w}$$

$$\dot{\mathbf{w}} = (\mathbf{A} + \nabla\mathbf{f}(\boldsymbol{\rho})\nabla_{\mathbf{x}}\boldsymbol{\rho})\mathbf{w} + (\mathbf{B} + \nabla\mathbf{f}(\boldsymbol{\rho})\nabla_{\mathbf{r}}\boldsymbol{\rho})\dot{\mathbf{r}}$$

$$\dot{\mathbf{y}} = (\mathbf{C} + \nabla\mathbf{g}(\boldsymbol{\rho})\nabla_{\mathbf{x}}\boldsymbol{\rho})\mathbf{w} + (\mathbf{D} + \nabla\mathbf{g}(\boldsymbol{\rho})\nabla_{\mathbf{r}}\boldsymbol{\rho})\dot{\mathbf{r}}$$

After “freezing” in a operating point, we get velocity-based linearised model

$$\dot{\mathbf{x}} = \mathbf{w}$$

$$\dot{\mathbf{w}} = (\mathbf{A} + \nabla\mathbf{f}(\boldsymbol{\rho}_1)\nabla_{\mathbf{x}}\boldsymbol{\rho})\mathbf{w} + (\mathbf{B} + \nabla\mathbf{f}(\boldsymbol{\rho}_1)\nabla_{\mathbf{r}}\boldsymbol{\rho})\dot{\mathbf{r}}$$

$$\dot{\mathbf{y}} = (\mathbf{C} + \nabla\mathbf{g}(\boldsymbol{\rho}_1)\nabla_{\mathbf{x}}\boldsymbol{\rho})\mathbf{w} + (\mathbf{D} + \nabla\mathbf{g}(\boldsymbol{\rho}_1)\nabla_{\mathbf{r}}\boldsymbol{\rho})\dot{\mathbf{r}}$$

- A linear system (the 'velocity-based linearisation') is associated with every operating point of a nonlinear system (not just the equilibrium points).
- A family of velocity-based linearisations is therefore associated with the nonlinear system. This family embodies the entire dynamics of the nonlinear system and so is an alternative representation. It is emphasised that this representation is valid globally and does not involve any restriction to the vicinity of the equilibrium points.
- Large transients and sustained non-equilibrium operation can both be accommodated.

- We retained the direct connection with linear subsystems.
- We obtained a “transparent” system.

Remark: The method introduces some new problems, e.g. derivation of input signal, but this can be circumvented in modelling as well as later in design phase.

Blended multimodel systems

- Finite number of local models
- More practical.
- Advantages of velocity-based linearisation over common LMN:
  - Linear localne models (not linear in parameters - affine).
  - Direct relation between local and global dynamics.
  - Global dynamics is approximated with weighted combination of local models properties and dynamics.

Blended model based on velocity-based linearisation:

$$\dot{\mathbf{x}} = \mathbf{w}$$

$$\dot{\mathbf{w}} = \sum_{i=1}^{n} \{(\mathbf{A} + \nabla f(\rho_i)\nabla_x \rho)\mathbf{w} + (B + \nabla f(\rho_i)\nabla_r \rho)\dot{r}\}\mu_i(\rho)$$

1. $$\dot{\mathbf{x}}_1 = \mathbf{w}_1$$
$$\dot{\mathbf{w}}_1 = (\mathbf{A} + \nabla f(\rho_1)\nabla_x \rho)\mathbf{w}_1 + (B + \nabla f(\rho_1)\nabla_r \rho)\dot{r}$$

2. $$\dot{\mathbf{x}}_2 = \mathbf{w}_2$$
$$\dot{\mathbf{w}}_2 = (\mathbf{A} + \nabla f(\rho_2)\nabla_x \rho)\mathbf{w}_2 + (B + \nabla f(\rho_2)\nabla_r \rho)\dot{r}$$

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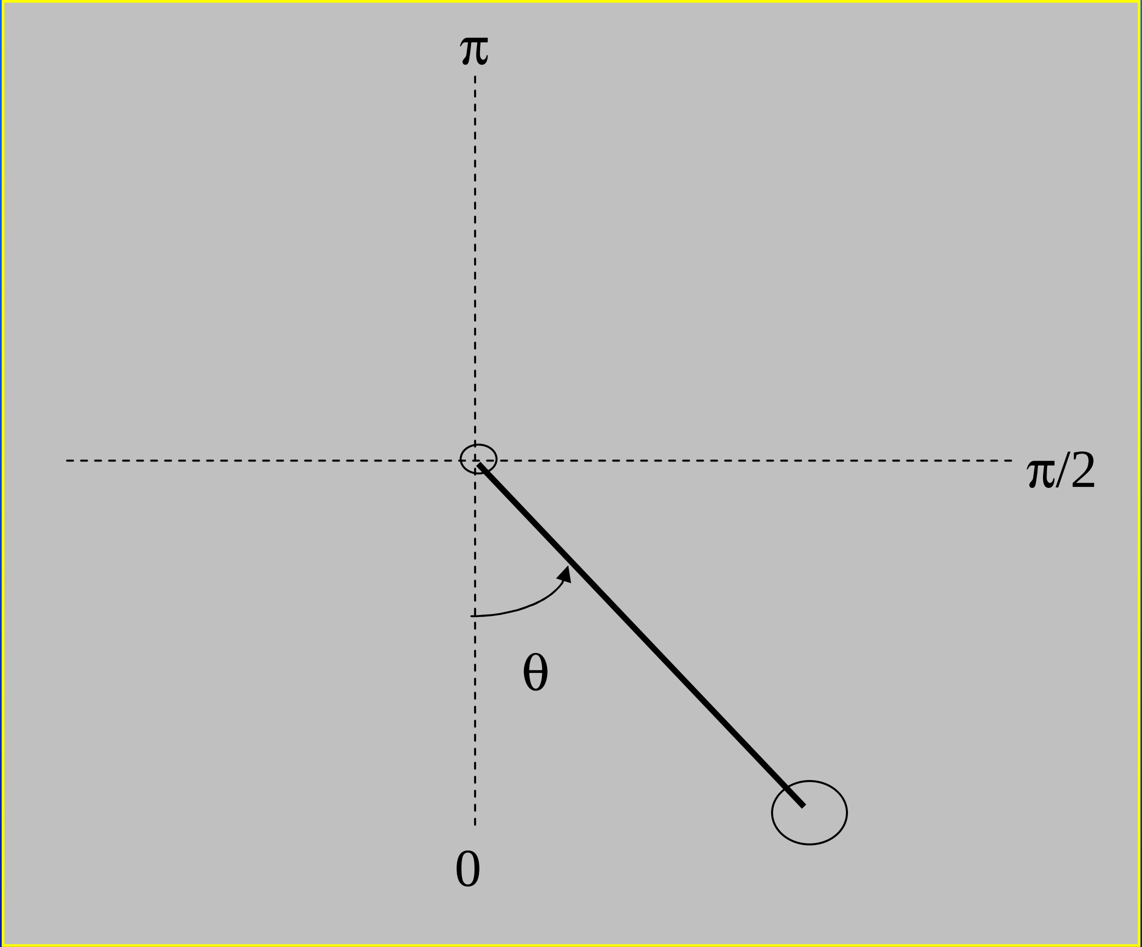
n. $$\dot{\mathbf{x}}_n = \mathbf{w}_n$$
$$\dot{\mathbf{w}}_n = (\mathbf{A} + \nabla f(\rho_n)\nabla_x \rho)\mathbf{w}_n + (B + \nabla f(\rho_n)\nabla_r \rho)\dot{r}$$

Weighted combination of solutions

$$\tilde{\mathbf{w}} = \sum_{i=1}^{n} \mathbf{w}_i \mu_i(\rho)$$

Example: pendulum

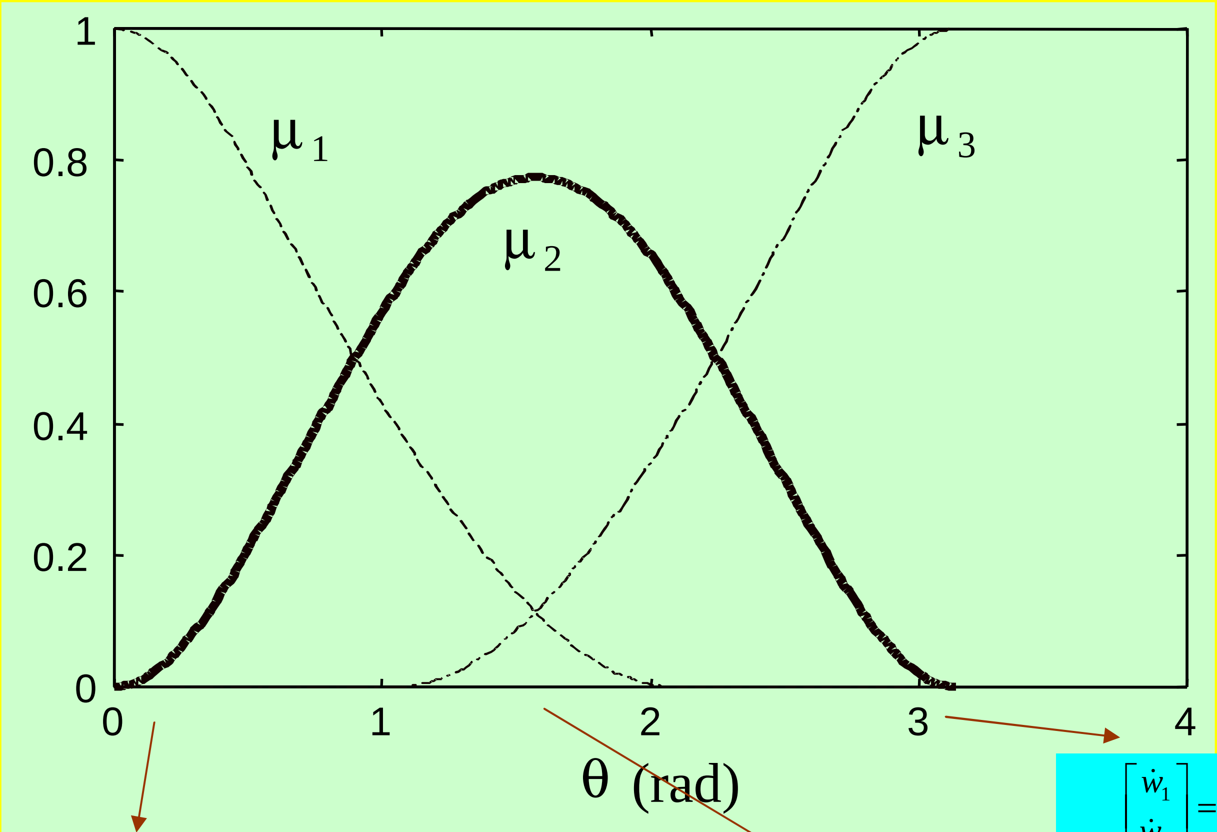
$$\ddot{\theta} = -Q\dot{\theta} - Q\sin\theta + bF$$



Approximate nonlinear system with three blended velocity-based linearised local models at angles 0, π/2 in π.

$$\dot{\mathbf{x}} = \mathbf{w}$$

$$\dot{\mathbf{w}} = \sum_{i=1}^{3} \{(A + \nabla f(\rho_i)\nabla_x \rho)\mathbf{w} + (B + \nabla f(\rho_i)\nabla_r \rho)\dot{r}\}\mu_i(\rho)$$



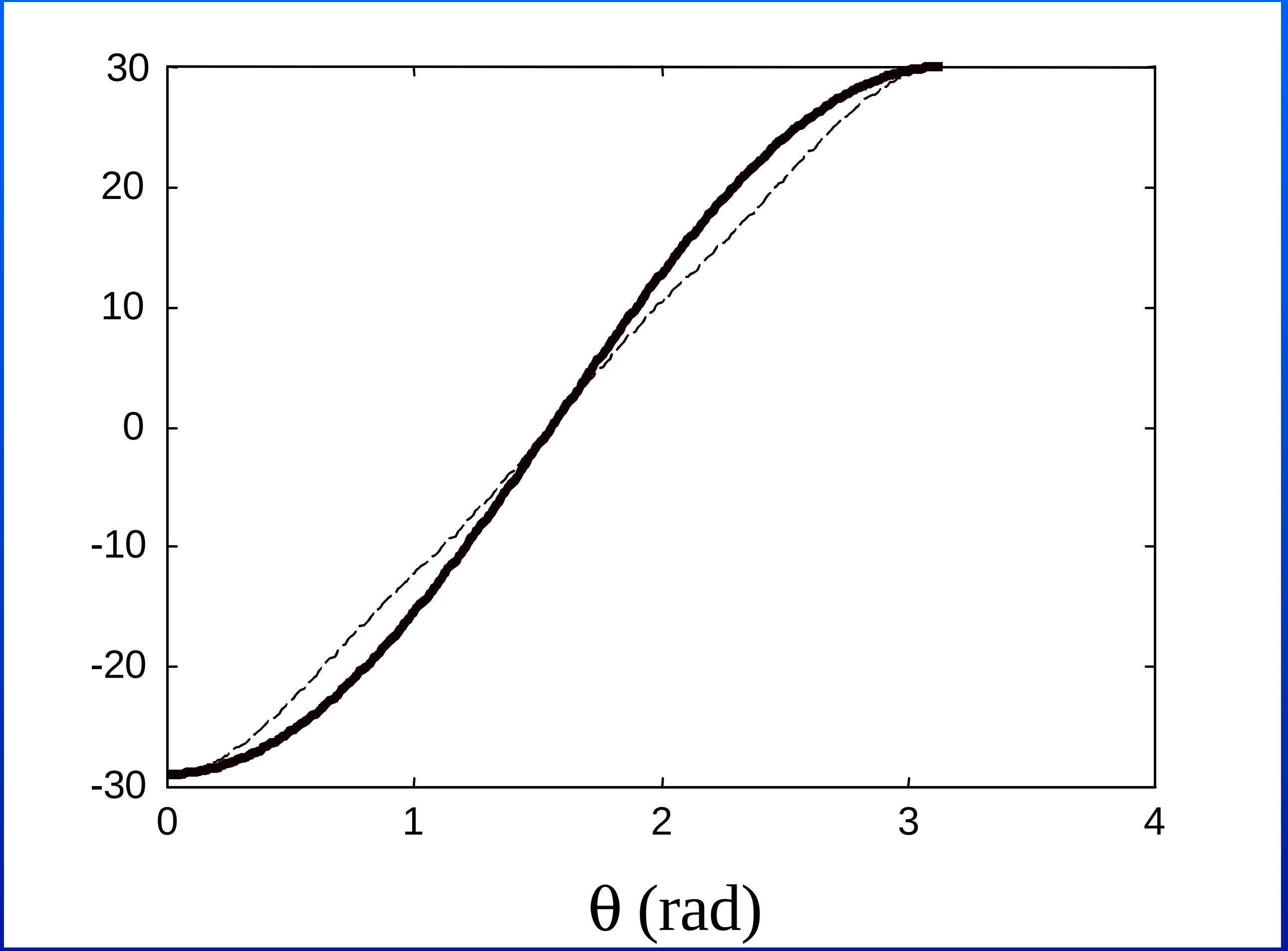
Very small number of local models: only three local models to cover the entire operating region $[0, \pi]$

$$\begin{bmatrix}\dot{w}_1\\ \dot{w}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -Q\cos(0) & -Q\end{bmatrix}\begin{bmatrix}w_1\\ w_2\end{bmatrix} + \begin{bmatrix}0\\ b\end{bmatrix}\dot{r}$$

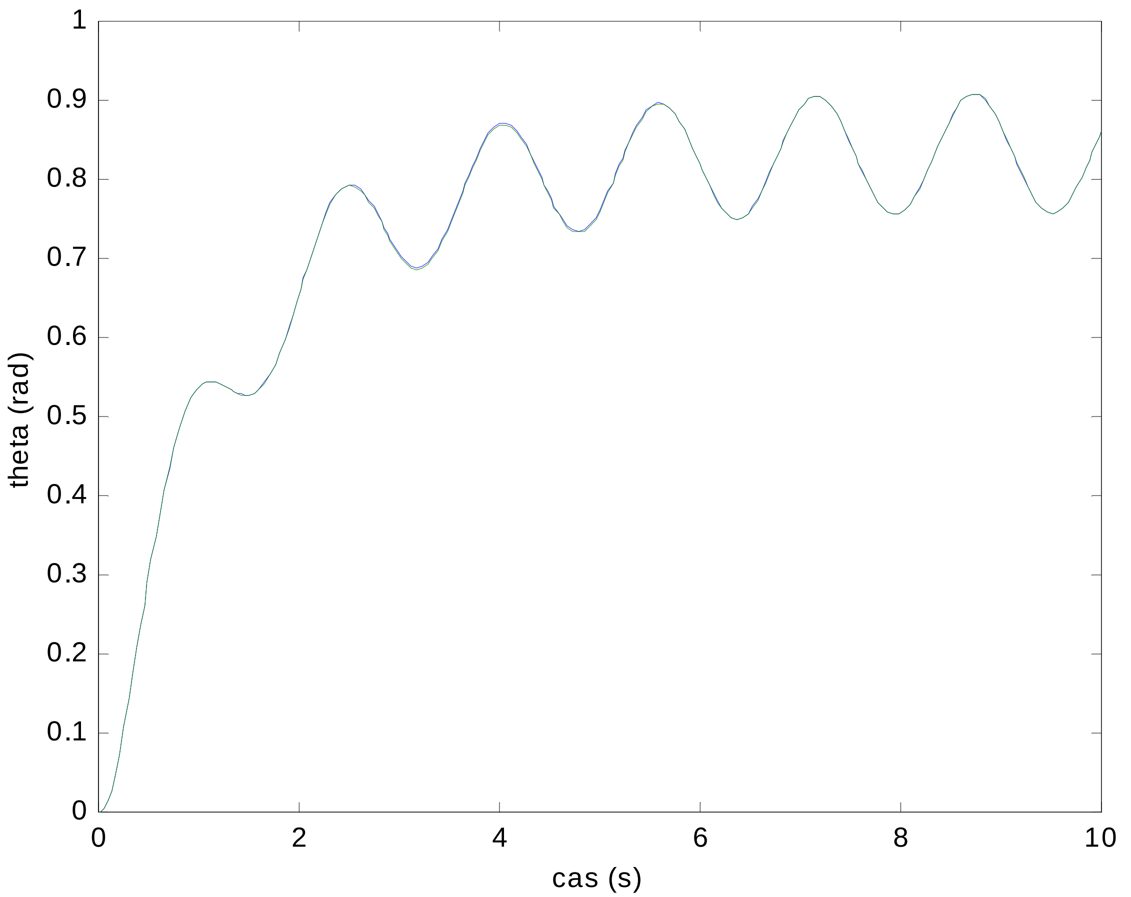
$$\begin{bmatrix}\dot{w}_1\\ \dot{w}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -Q\cos(\pi/2) & -Q\end{bmatrix}\begin{bmatrix}w_1\\ w_2\end{bmatrix} + \begin{bmatrix}0\\ b\end{bmatrix}\dot{r}$$

$$\begin{bmatrix}\dot{w}_1\\ \dot{w}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -Q\cos(\pi) & -Q\end{bmatrix}\begin{bmatrix}w_1\\ w_2\end{bmatrix} + \begin{bmatrix}0\\ b\end{bmatrix}\dot{r}$$

Comparison of w_2 signal of original (full curve) and blended system (dashed curve)



Comparison of output response θ to specific input signal – operating area arround π /4 rad, which is the most tricky region



Residuals