Dynamic Systems Identification
Part 1 - Linear systems

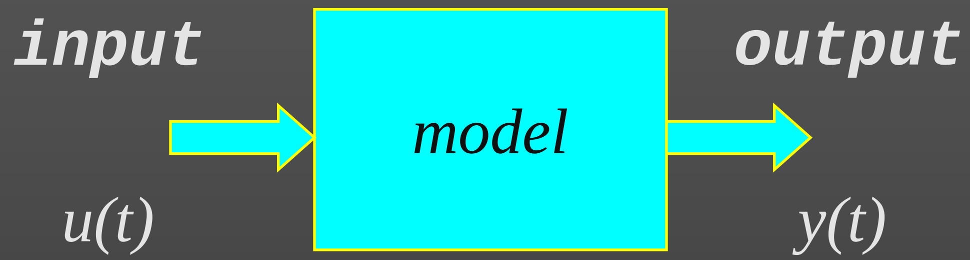
Reference:

J. Sjöberg et al. (1995):Non-linear Black-Box Modeling in System Identification: a Unified Overview, Automatica, Vol. 31, 12, Sections 1 and 3.1.

Identification of dynamic systems

- experimental modelling of dynamic systems
- Basic rule:
  Do not estimate what you already know!
- results of research and engineering practice
- white box model, grey box model, black box model
- available literature and software
- black box linear models: linear systems identification (Ljung, Isermann, etc.)

Static / dynamic model



- *Static model*

$$F[u(t), y(t)] = 0$$

- *Dynamic model*

$$F[t, u(t), u'(t), u''(t), \ldots, u^{(m)}(t), y(t), y'(t), y''(t), \ldots, y^{(n)}(t)] = 0$$

$$F[k, u(k), u(k-1), \ldots, u(k-m), y(k), y(k-1), \ldots, y(k-n)] = 0$$

Linear regression for dynamic systems

$$y(k)=-a_1y(k-1)-a_2y(k-2)\ldots-a_ny(k-n)+b_1u(k-1)+b_2u(k-2)\ldots+b_mu(k-m)$$

$$y(k)=[-y(k-1)\ -y(k-2)\ \ldots\ -y(k-n)\ u(k-1)\ u(k-2)\ldots u(k-m)]\,[a1\ a2\ \ldots\ an\ b1\ b2\ldots\ bm]^T$$

$$\mathbf{y}=\boldsymbol{\psi\theta}$$

$$\hat{\boldsymbol{\theta}}=\left[\boldsymbol{\psi}^T\boldsymbol{\psi}\right]^{-1}\boldsymbol{\psi}^T\mathbf{y}$$

Autoregressive model with exogenous inputs *(ARX)*

$$y(k)+\sum_{j=1}^{n} a_j y(k-j) = \sum_{i=1}^{m} b_i u(k-i-d) + e(k)$$

$$A(z^{-1})y(z^{-1}) = B(z^{-1})u(z^{-1}) + e(z^{-1})$$

$$y(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})}u(z^{-1}) + \frac{1}{A(z^{-1})}e(z^{-1})$$

- filtering

Division of identification methods

- *class of mathematical models*
  - nonparametric models
  - parametric models
- *class of used signals*
  - continuous, discrete
  - deterministic, random, pseudorandom
- *error between the systemand its model*
  - input error
  - output error
  - generalised error
- *concurrency*
  - offline
  - online
- *data processing*
  - nonrecursive
    - direct
    - iterative
  - recursive
- *model structure*
  - linear models
  - nonlinear models

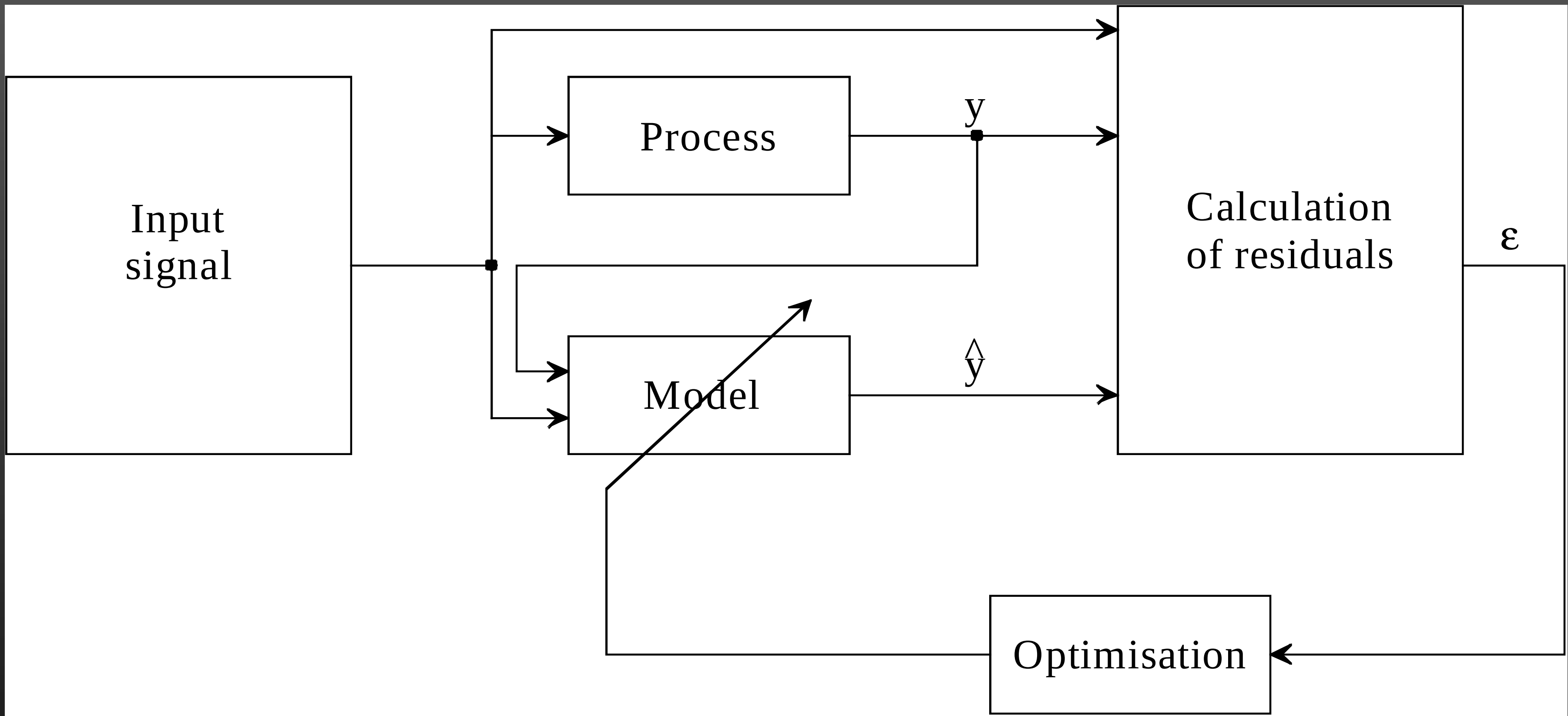
Nonparametric models

- *mostly linear models*
- *I/O characteristics as numeric tables or curves*
  - frequency responses (Bode diagrams)
  - impulse response, step response
  - Fourier analysis, analysis of frequency response, correlation analysis, spectral analysis

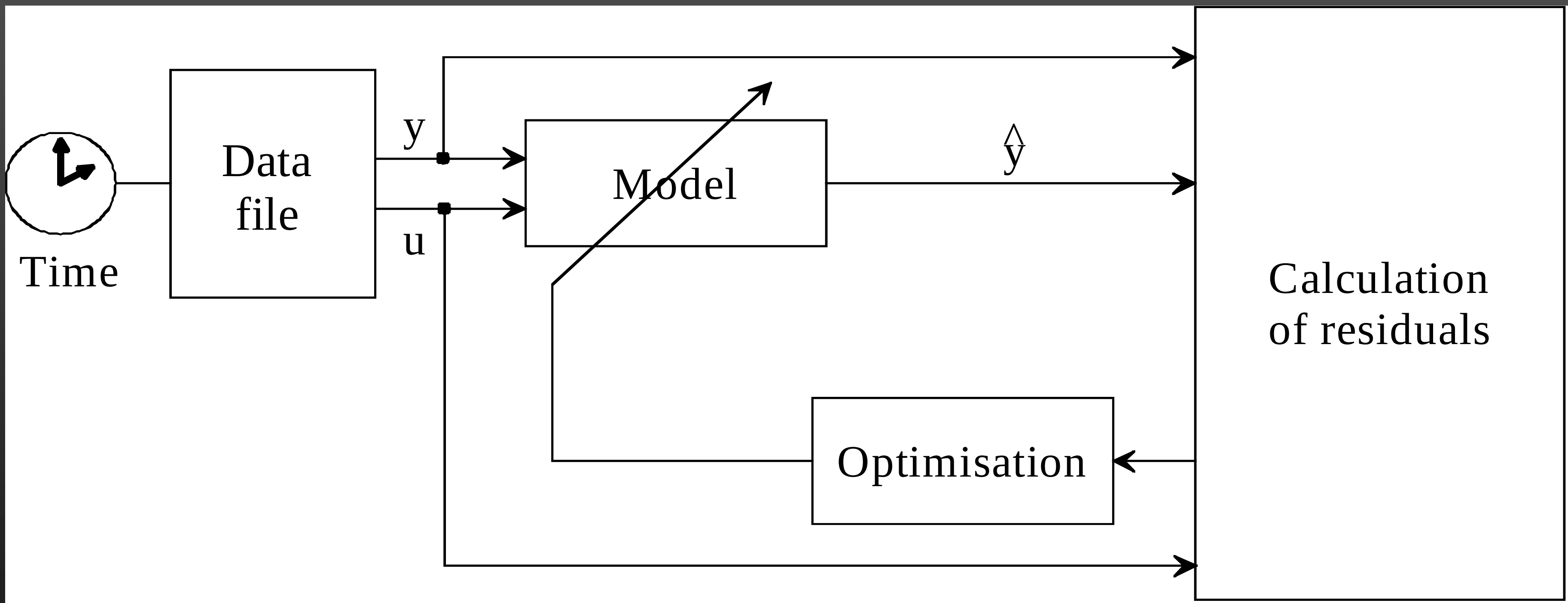
Parametric models

- *linear and nonlinear models*
- *models with explicit parameters*
  - differential equations
  - difference equations
  - transfer functions
  - state-space functions
- *model structure:*
  - system order
  - regressors

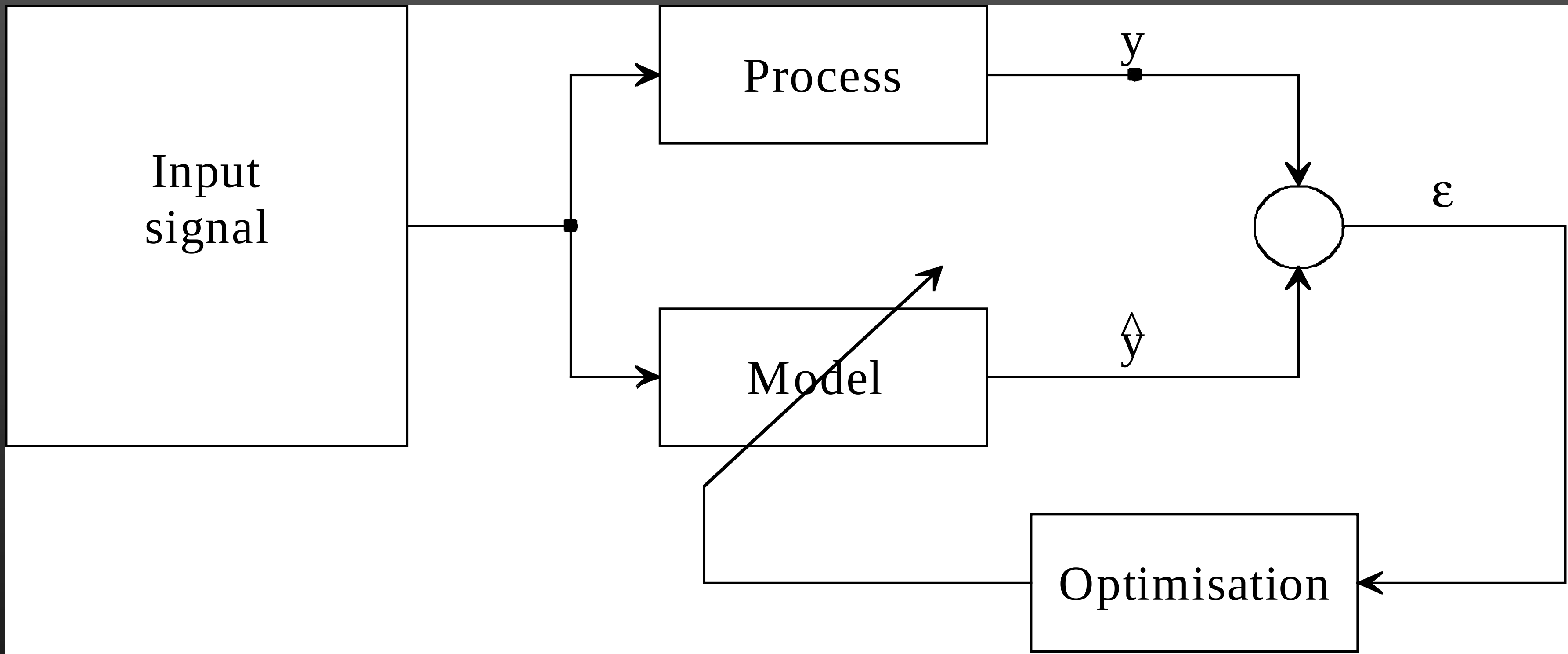
On-line model fitting



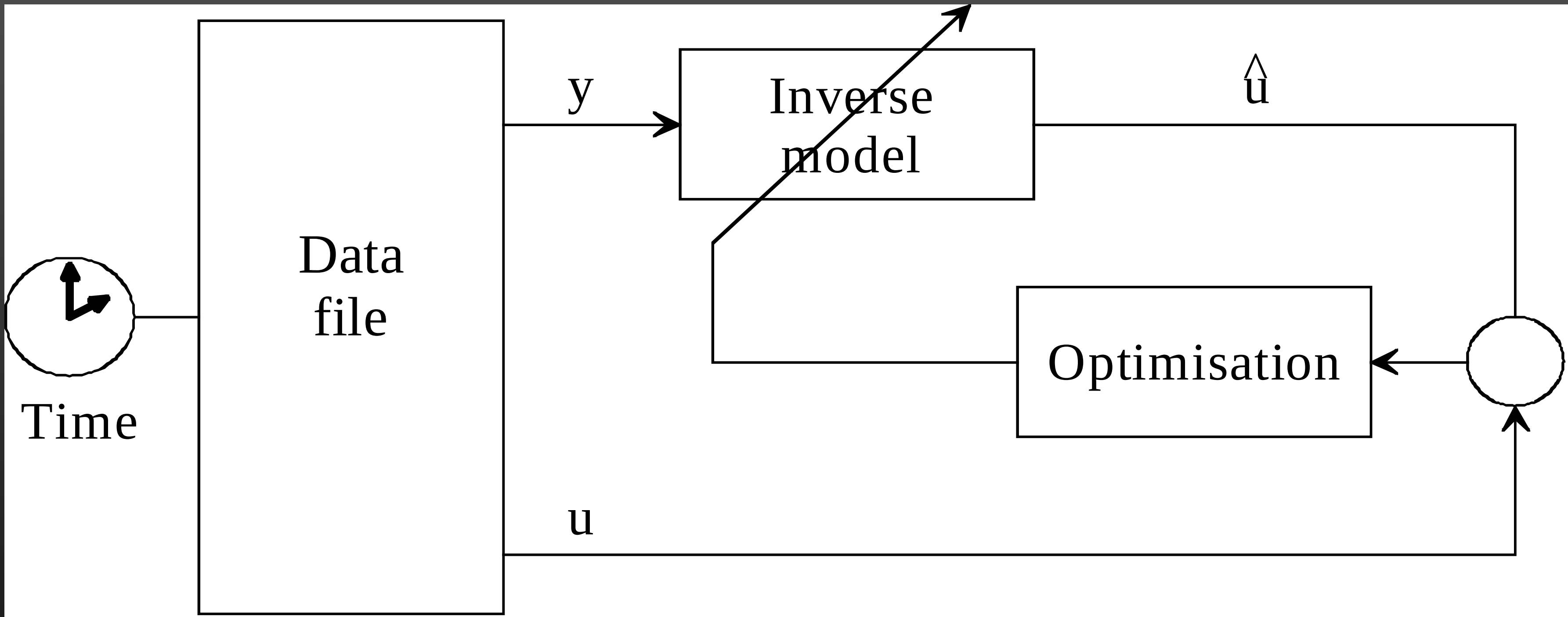
Off-line model fitting



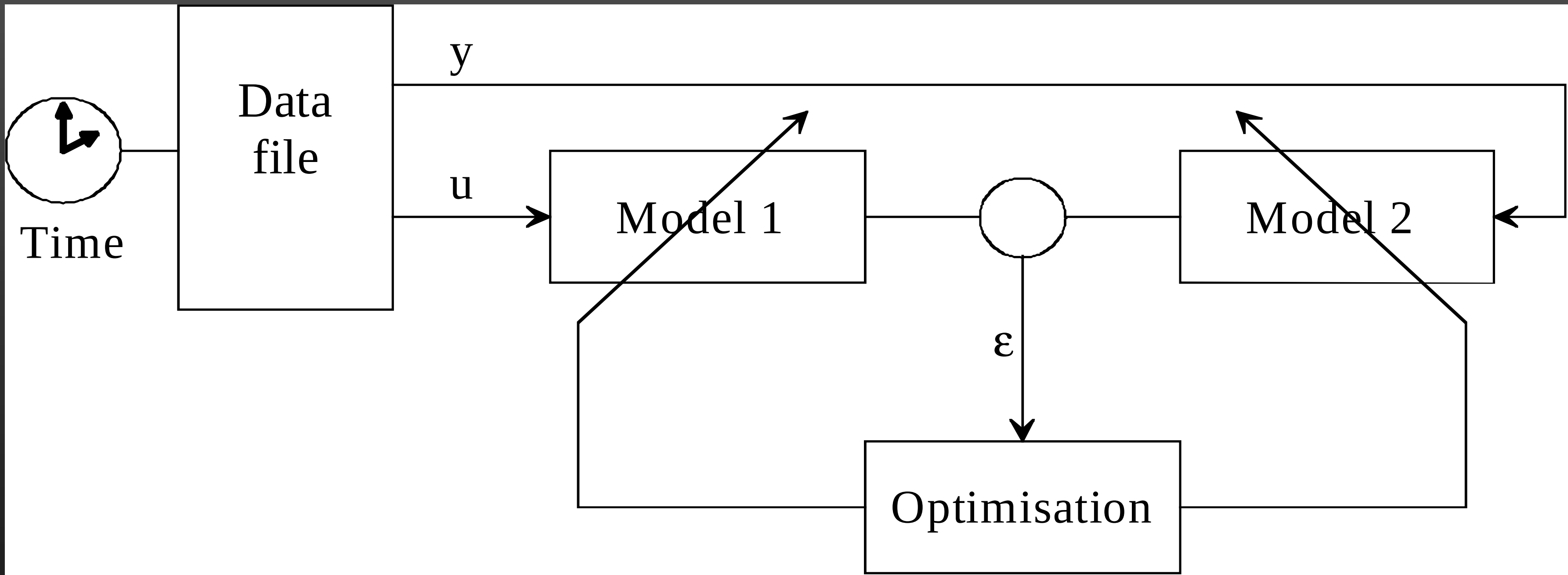
Output error model



Input error model



Equation error model



Methods for parametric models identification (System Identification Toolbox)

- linear systems

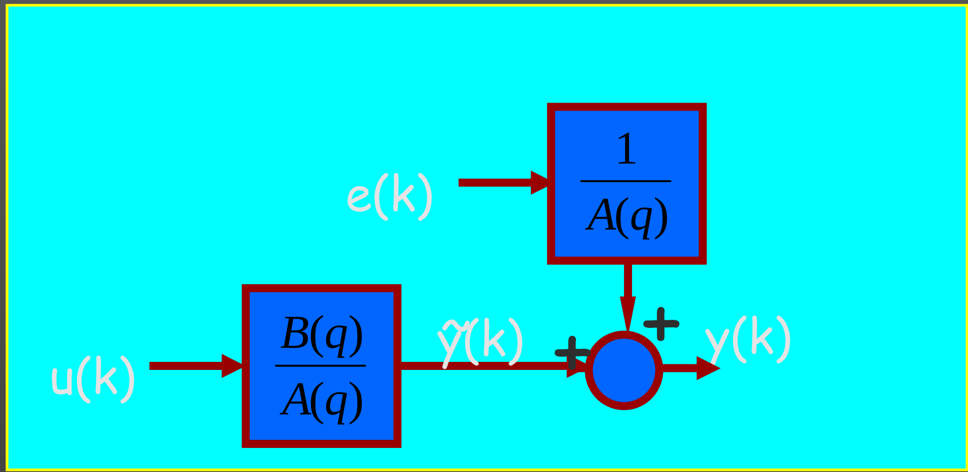
$$A(z^{-1})y(z^{-1}) = \frac{B(z^{-1})}{F(z^{-1})}u(z^{-1}) + \frac{C(z^{-1})}{D(z^{-1})}e(z^{-1})$$

  - FIR (A=F=D=1,C=0)
  - ARX (F=C=D=1)
  - OE (A=C=D=1)
  - ARMAX (F=D=1)
  - BJ (A=1)

REGRESSORS!

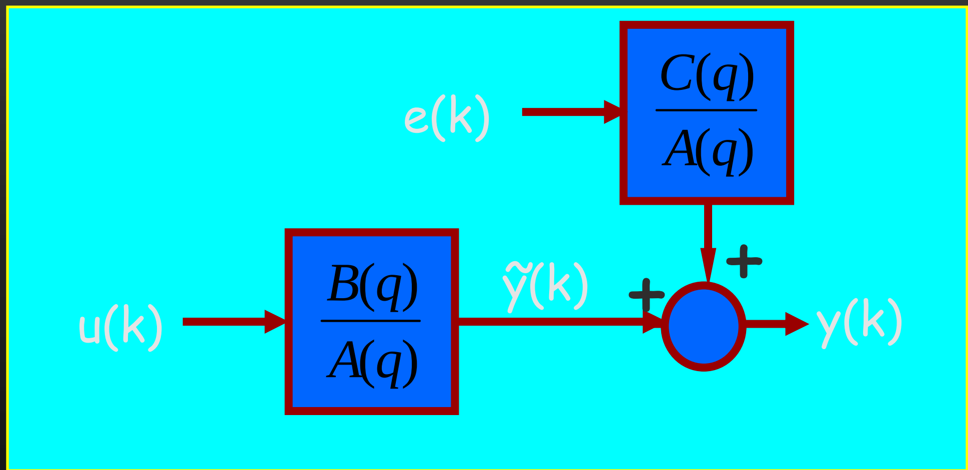
- Autoregressive model with exogenous inputs (ARX)

$y(k)=a_1y(k-1)+a_2y(k-2)+b_1u(k-1)+b_2u(k-2)+e(k)$



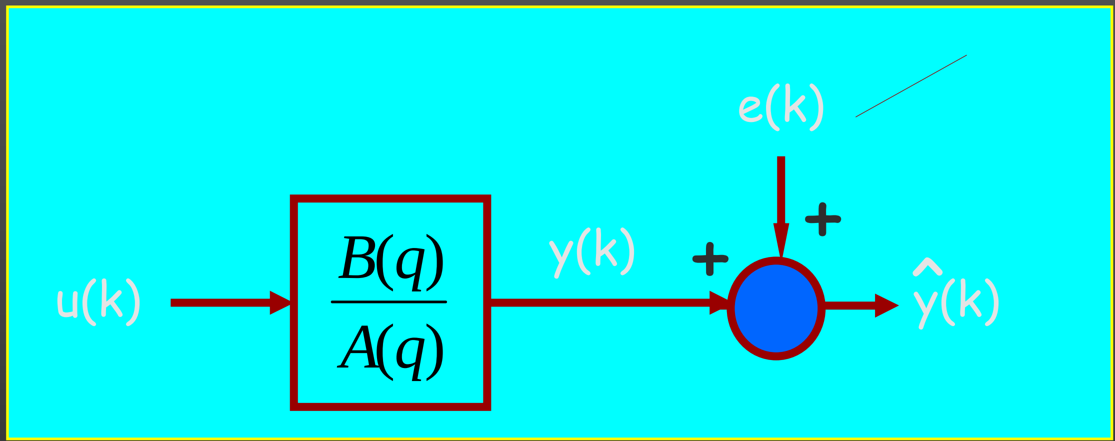
- Autoregressive moving average model with exogenous inputs model (ARMAX)

$y(k)=a_1y(k-1)+a_2y(k-2)+b_1u(k-1)+b_2u(k-2)+ e(k) +c_1e(k-1)+c_2e(k-2)$



Output error model (OE)

$$y(k)=a_1[y(k-1)-e(k-1)]+a_2[y(k-2)-e(k-2)]+b_1u(k-1)+b_2u(k-2)+e(k)$$



What are we doing in identification?

- Example: the first order dynamic system

$$y(k) = 0.9512y(k-1) + 0.09754u(k-1)$$

  - *1st order*
  - *Regressors: y(k-1),u(k-1)*
  - $y(k) = -a_1 y(k-1) + b_1 u(k-1)$

$$H(z) = \frac{0.09754z^{-1}}{1 - 0.9512z^{-1}}$$

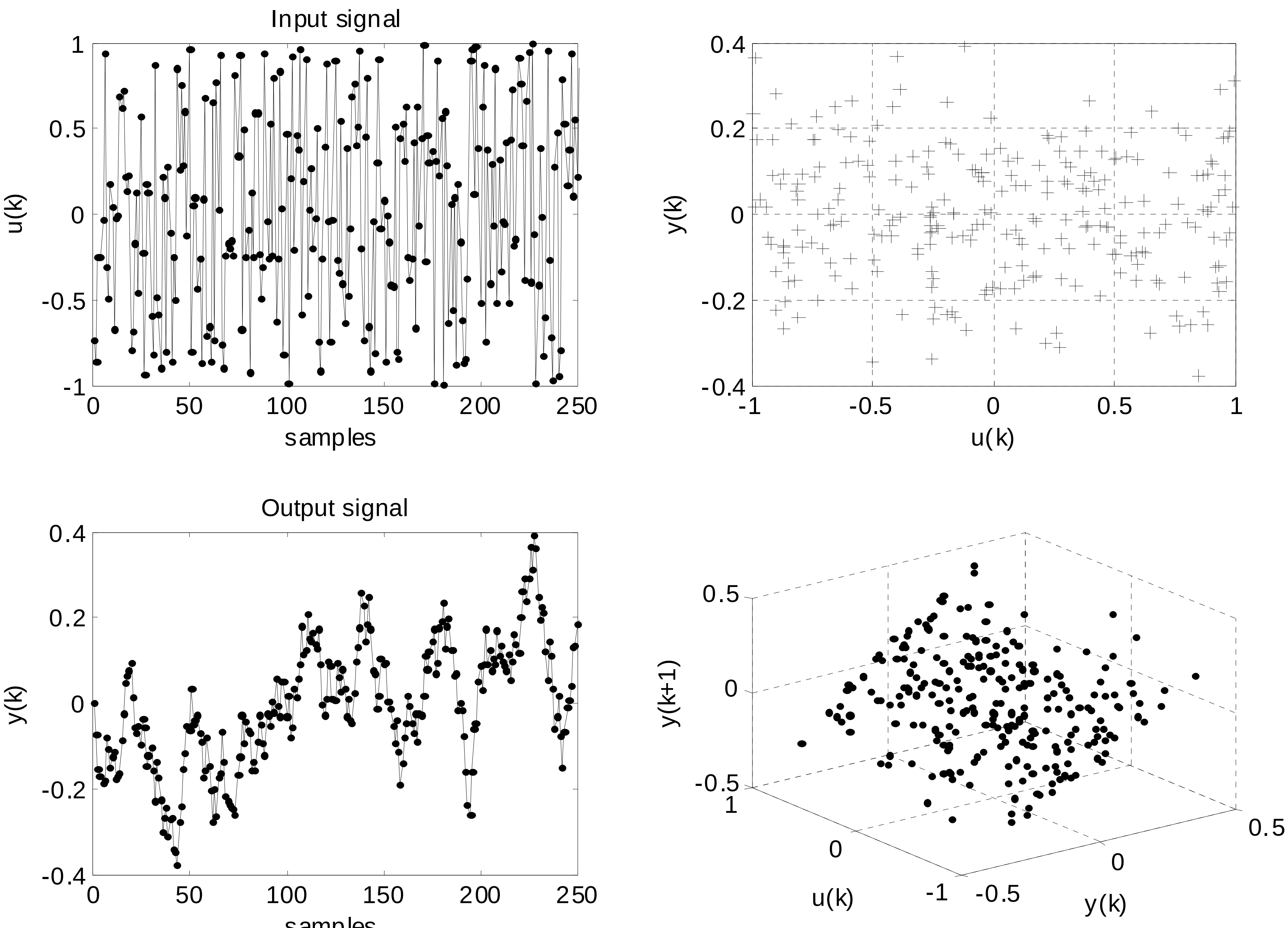
  - $$\begin{bmatrix} y(2) \\ y(3) \\ \dots \end{bmatrix} = \begin{bmatrix} -y(1) & u(1) \\ -y(2) & u(2) \\ \dots & \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}$$

$$\mathbf{y} = \Psi\theta$$

  - Order of rows and columns can be changed!!
  - Optimal solution by least squares cost function

$$\theta = (\Psi^T\Psi)^{-1}\Psi^T \mathbf{y}$$

  - Parameters are optimal for one-step-ahead prediction, validation is done with simulation (multi-step-ahead prediction).


Input signal
u(k)
samples
1
0.5
0
-0.5
-1
0
50
100
150
200
250
y(k)
u(k)
0.4
0.2
0
-0.2
-0.4
-1
-0.5
0
0.5
1
Output signal
y(k)
samples
0.4
0.2
0
-0.2
-0.4
0
50
100
150
200
250
y(k+1)
u(k)
y(k)
0.5
0
-0.5
1
0
-1
-0.5
0
0.5

Model